CORNELL
UNIVERSITY
LIBRARY

Cornell University Library
arV10368

Practical surveying;

3 1924 031 298 148
olin,anx

Cornell University Library

The original of this book is in
the Cornell University Library.

There are no known copyright restrictions in
the United States on the use of the text.

http://www.archive.org/details/cu31924031298148

PRACTICAL SURVEYING

FOR ADVANCED STUDENTS.

Demy 8vo, 532 pages, cloth, price 10*s*. 6*d*. *net.*

SURVEYING AS PRACTISED BY CIVIL ENGINEERS AND SURVEYORS.

Including the Setting-Out of Works for Construction and Surveys Abroad, with many Examples taken from Actual Practice. A Handbook for use in the Field and the Office, intended also as a Text-Book for Students. By JOHN WHITELAW, Jun., A.M.Inst.C.E. With about 260 Illustrations.

"This work is written with admirable lucidity, and will certainly be found of distinct value both to students and to those engaged in actual practice."—*The Builder.*

"Is well adapted to meet the requirements of the student of engineering and the modern surveyor."—*The Engineer.*

LONDON: CROSBY LOCKWOOD & SON.

PRACTICAL SURVEYING

A TEXT-BOOK FOR
STUDENTS PREPARING FOR EXAMINATIONS OR
FOR SURVEY WORK IN THE COLONIES

BY
GEORGE WM. USILL, ASSOC.-M.INST.C.E.

Eighth Edition, thoroughly Revised and Enlarged

BY
ALEXANDER BEAZELEY, M.INST.C.E.
AUTHOR OF
"THE RECLAMATION OF LAND FROM TIDAL WATERS," ETC. ETC.

WITH FOUR LITHOGRAPHIC PLATES
AND THREE-HUNDRED-AND-SIXTY ILLUSTRATIONS

LONDON
CROSBY LOCKWOOD AND SON
7, STATIONERS' HALL COURT, LUDGATE HILL
1904

PRINTED BY
WILLIAM CLOWES AND SONS, LIMITED
LONDON AND BECCLES.

AUTHOR'S PREFACE.

In submitting this little work to the Public, I take the opportunity of saying a few words in explanation of my object in compiling it.

My experience during several years past in delivering courses of lectures on Surveying and kindred subjects, and in preparing gentlemen for the Colonies, has shown me that, however excellent and comprehensive many existing text-books may be, they are in some points not sufficiently explicit nor in others sufficiently concise to enable the student, especially in cases of self-instruction, to grasp with readiness the subjects of which they treat. A text-book of somewhat different character seems, therefore, to be called for.

In the present work I have endeavoured to make each chapter complete in itself, and to let the chapters follow in progressive order.

I have also considered it better to explain the various instruments required in Surveying and their adjustment, before proceeding to describe their use and manipulation in the field. I follow then with a chapter devoted to a graphic treatment of Trigonometry as applied to Surveying; and the several succeeding chapters are intended to briefly explain the *modus operandi* of Theodolite Surveying, Traversing, Town Surveying, Levelling, Contouring, Setting-out Curves, Office Work, and the Computation of Land Quantities.

In preparing the matter here presented, I have not only drawn upon my own experience, but have consulted many of the chief works upon the subjects in question, and I desire to acknowledge my indebtedness to the authors of the works thus consulted. I have also to express my obligations to the Astronomer-Royal, to Messrs. Troughton and Simms, and to Mr. J. H. Steward, for valuable information and assistance; whilst to my former pupils, Messrs. H. S. Fearon and James Holden, I am indebted for the surveys of Wimbledon Park and Cardiff. I wish also to acknowledge the assistance I have received from Mr. John F. Curwen, in the revision of the mathematical portions of the work as they passed through the press.

GEO. WM. USILL.

LONDON,
October, 1888.

EDITOR'S PREFACE

TO THE EIGHTH EDITION.

THE late Mr. Usill's "Practical Surveying" has long occupied a recognized position as a text-book for students; and the exhaustion of the seventh edition appeared to the Publishers a suitable occasion for supplying some deficiencies, and for the correction of some typographical and other errors, overlooked in former editions. The work has now been revised throughout, the original text being as far as reasonably possible preserved. Some portions have been re-cast and amplified, and others entirely re-written; and it has been my aim to render the work as a whole more valuable to the student, and to maintain it not only abreast but even somewhat ahead of its already deserved reputation.

The volume having hitherto borne upon its title-page a reference to students preparing for practice in the Colonies, a chapter has been added containing the latest regulations under which licences for such practice are issued: a feature not to be found, so far as I am aware, in any text-book on Surveying, yet one of considerable importance to persons contemplating Colonial pursuit of that profession, by reason of the exclusiveness—presumably needful, but certainly stringent—of the restrictions wherewith the granting of licences is in most cases hedged about.

In Chapter IX. are inserted Rules, and in the Appendix Tables, for ascertainment of heights by the hypsometer, by courteous permission extracted from the Royal Geographical Society's " Hints to Travellers."

The Tables of Natural Sines, Tangents, and Secants, with their complements, were added by the Author as an Appendix to the Third Edition.

ALEXR. BEAZELEY.

WEYMOUTH,
May, 1904.

NOTE.—To promote the applicability of the work for use in the field, the Publishers are issuing an edition printed on thin paper and bound in limp leather covers, so as to go conveniently into the pocket.

CONTENTS.

PAGE

INDEX TO ILLUSTRATIONS . . . . . . xv—xx

CHAPTER I.

INTRODUCTION.

Subjects necessary to be known—Standards of Measure—Chains—Advantages of 100-feet Chain—Gunter's, or 66-feet Chain—Divisions of Gunter's Chain—Décamètre Chain—Arrows—Offset Staff—33-feet Tape—Poles—Ranging Rods—Bundle of Laths—Whites—Equipment of a Surveyor—Field Book . . . . 1—6

CHAPTER II.

ORDINARY SURVEYING.

Reconnoitre—Sketch Map—Stations—Main Stations—Subsidiary Stations—Testing the Chain—Test Gauge—Chain and Arrows: throwing out and doing up—Chain-men—Leader's Duties—Duty of Follower—How to use the Chain—Crossing Hedges—Hedge and Ditch—How to measure Fence—Foot-set Hedges—Offsets—Optical Square and how to use it—As to Buildings—Corners of Fields—To fix Position of an Intersection—Limit of Offsets—Pacing—Objections to Tapes for Offsets—Offset Staff—Ranging-out Lines—What is Level Ground—Observing Angle of Slope—Adjusting the Allowance for Slope—Stepping—Base-Lines—Chain-Angles—Inaccessible Distances . 7—28

CHAPTER III.

SURVEYING INSTRUMENTS.

Cross Staff—Optical Square — Line Ranger — Clinometer—Merrett's Quadrant—Clinometer Rule—Abney Level—Reflecting Clinometer

PAGE

Scale—Combined Telescope Clinometer and Prismatic Compass—Prismatic Compass—The Circumferentor—The Theodolite—Adjustment of the Theodolite—Adjusting the Vertical Axis—Centering Plates—Parallel Plates—Parallel Plate Screws—Ball-and-Socket Arrangement—The Limb or Lower Plate—The Upper or Vernier Plate—Clamps—Troughton's Clamp and Tangent Arrangement—Levelling Plates—Vertical Arc—Compass—Telescope—The Diaphragm—The Vernier—Transit Theodolite—Triangular Plate—Everest Theodolite—Box Sextant—How to use Box Sextant—Hughes's Improved Double Sextant—Plane Table—Telemeter—Telemeter for the Laying out the Base—The Use of the Telemeter as a Surveying Instrument—To measure the Width of a River—The Level—The "Y" Level—The Dumpy Level—Telescope—Object-glass—Eye-piece—The Diaphragm—The Cross-Wires—Line of Collimation—Adjustment of the Level—Adjustment for Collimation—Adjustment for Parallax—Reflecting Mirror—Levelling Staff—Aneroid Barometer—Stadiometer—Omnimeter—The Dredge-Steward Omni-telemeter—The Francis Surveying Compass and Clinometer—Stanley's Model Theodolite—Trotter's Curve-Ranger—Dalrymple-Hay's Curve-Ranger—Stanley's Ray-Shade—Johnson's Tripod-Head—Erskine's Altazimuth Theodolite—Cary's Improved Eye-piece . . . . 29—101

CHAPTER IV.

TRIGONOMETRY REQUIRED IN SURVEYING.

Plane Surface—Plane Angle—Plane Rectilineal Angle—Perpendicular—Obtuse Angle—Acute Angle—Circle—Centre of Circle—Diameter of Circle—Semi-circle—Segment of Circle—Rectilineal Figures—Trilateral Figures—Quadrilateral Figures—Multilateral Figures—Equilateral Triangle—Isosceles Triangle—Scalene Triangle—Right-angled Triangle—Obtuse-angled Triangle—Acute-angled Triangle—Theorems—Theory of Parallel Lines—Of the Circle—Trigonometrical Canon—Complement and Supplement of Angles—Trigonometrical Ratios—Sine—Tangent—Secant—Cosine—Cotangent—Cosecant—Versed Sine—Coversed Sine—Chord—Relation of Hypotenuse to the Other Sides of Right-angled Triangle—Comparison of Functions—Cotangent of Greater or Less Angles—Sin A in Terms of Cos A—Tan A in Terms of Sin A—Tan A in Terms of Cos A—Cos A in Terms of Tan A—Sin A in Terms of Tan A—Sin A in Terms of Sec A—Cos A in Terms of Cosec A—Cot A in Terms of Sec A—Complemental Angles—The Sine, &c., and its Complement—Supplemental Angles—Use of + and − Signs—Positive and Negative

PAGE

Condition of each Quadrant—Radius Unity—Basis of Formulæ—Sines, &c., for 45 Degrees—for 60 Degrees—for 30 Degrees—for 18 Degrees—for 120 Degrees—for 225 Degrees—Ratio of Radius—Solution of Right-angled Triangles—Trigonometrical Ratios of Two Angles—Sum and Difference of Sines and Cosines—Sine and Cosine of Twice an Angle in Terms of Sine and Cosine of the Angle—Sine and Cosine of Twice an Angle in Terms of Sine and Cosine of Half the Angle—Sine and Cosine of Twice an Angle in Terms of Sine and Cosine of the Sum of Three Angles—Sine and Cosine of Twice an Angle in Terms of Sine and Cosine of Three Times one Angle—Oblique-angled Triangles—Sines and Cosines in Terms of Sides—Sines and Cosines of Semi-angles—Logarithms—The Characteristic—The Mantissa—Multiplication by Logarithms—Division by Logarithms—Proportion by Logarithms—Arithmetical Complement—Involution by Logarithms—Evolution by Logarithms—Natural and Logarithmic Sines, &c.—Arithmetical Computation—Solution of Triangles by Natural Sines—Solution of Oblique Triangles by Logarithms—Heights and Distances . . 102—156

CHAPTER V.

CHAIN-SURVEYING.

Surveying with Chain only—Field-book—Ordnance Field-book—Necessity for Reconnoitre—Survey Lines to be numbered consecutively—Conventional Signs for Ditch and Hedge—for Post-and-rail Fence—for Close-paling—for Walls—for Gates—for Footpaths—for Cart-tracks or Bridle-paths—for Trees—for Orchards—for Woods—for Brushwood—for Marshy Ground—for Heath or Gorse—for Railways—for Railway Embankments—for Railway Cuttings—for Broken Ground—for Parish Boundaries—for County Boundaries—for Surveying Stations—for Direction of Line—Field-book—Best Size Field-book—Single Line preferable to Double Column—Chain Survey of Wimbledon Park—Few Lines as possible—Tape not to be used for Offsets—Instructions to Chain-men—Enter every Ten Chains in Field-book—Boning-out Lines—Best Form of Stations—How to keep Field-book—Separate Page for each Line—As to marking Intersection of Lines—Best Form of Base-lines—Foot-paths and Cart-tracks—Gates—Hedge and Ditch—Avoid as much as possible crossing Fences—Avoid cutting Fences unnecessarily—Avoid cutting Trees—Clear up the Ground after Survey—Cautions . . 157—170

CHAPTER VI.

THEODOLITE-SURVEYING.

PAGE

Check-lines obviated—Accurately mark Station—When to take Angles—The Necessary Number of Angles—Angles necessary—Surveyors' Institution Examination—First a Chain Survey—What to avoid—Surveying a River—Don't spare the Use of the Theodolite—Corroboration of Observation—Hints on the Use of the Theodolite . 171—179

CHAPTER VII.

TRAVERSING.

Traversing with Chain—Traversing by Included Angles—Guarding against Metallic Attractions whilst Traversing—Plenty of Assistance required in Traversing—Northings and Southings in Traversing—As to closing a Traverse—Necessity for Care in checking—Relative Positions of Bearings—Magnetic Variation or Declination . 180—185

CHAPTER VIII.

TOWN-SURVEYING.

As to marking Stations in Streets—Taking Angles—Objections to Lamp-posts, &c.—Taking Observations in Crooked Streets—As to the Chain—When to take Angles—Do not erase Figures in Book—Use Arrows for counting—As to Buildings—Lamp-posts, Gullies, etc.—As to Streams—As to Plotting . . . . 186—196

CHAPTER IX.

LEVELLING.

Definition—Curvature of the Earth—Allowance for the Earth's Curvature—Refraction—Necessary Adjustments—Simple Levelling—Compound Levelling—Datum—Ordnance Datum—Bench-marks—Position of Bench-marks—Different Kinds of Levelling—Level-book—

PAGE

Foot-plates—Keeping the Level-book—Making up the Level-book—Levelling-staff, how to use it—As to Distances—Measuring across Streams—Providing for Curvature, etc.—Instructions to Staff-holder—Plenty of Information—Taking Level of Water—Levelling with Theodolite—Levelling with Aneroid—Levelling with Hypsometer—Cross-sections—Ticketing . . . . 197—230

CHAPTER X.

CONTOURING.

Vertical Intervals and Horizontal Equivalents—Hypotenusal Allowance—Table of Horizontal Equivalents—Datum—Bench-marks—Keeping the Field-book . . . . 231—236

CHAPTER XI.

SETTING OUT CURVES.

General Principles—Limit of Radii—Preliminary Operations—Tangent Points—Tangential Angle—Length of Curve—Impeded Point in Curve—Apex Inaccessible—Setting out with two Theodolites—Curves of Different Radii—Curves of Contraflexure—Formula for Curves—Table of Tangential Angles—Setting out by Offsets—Curve Offset Rule—Setting out from Same Tangent—Setting out by means of Ordinates . . . . 237—252

CHAPTER XII.

OFFICE WORK.

Necessity for System—Roughly plot the Survey-lines—Let the Paper be well seasoned—Draw Scale on Paper before commencing—Boxwood Scales best—Plot Survey North and South—Keep Paper Perfectly Flat—Laying down Survey-lines on Paper—Check Measurements—Marking Stations—Straight-edge—Never plot from Pencil Lines—As to plotting from Long Lines—Plot all Survey-lines first—Plot each Day's Work as soon as possible—Equipment of Office—Drawing-tables— Scales — Pricker — Pencils — Points of Pencils—Protractors—Beam Compasses—How to use Beam Compasses—Pricker or Needle-holder—Parallel Rules—Set-squares—Offsets—

PAGE

Curves—Drawing-pens—Dividers—Spring-bows—Drawing Instruments—Proportional Compasses—India Rubber—Indian Ink—Colours—Conventional Signs and Colours—Model Plan—Inking-in—Reference—As to Position of Plan on Paper—Boundaries of Different Properties—Paint Brushes and Pencils—Precautions in Colouring—North Points—Borders—Printing and Writing on Plans—Scales—Enlarging and reducing Plans—Pantagraph—Eidograph—How to adjust the Eidograph—Enlarging and Reducing by Squares—Copying a Plan—General Hints . . . . . . 253—280

CHAPTER XIII.

LAND QUANTITIES.

Table of Superficial Measures—Averages of Fences—Computing by Triangles—To ascertain Areas upon the Ground—The Computation Scale—How to use Computation Scale—Various Kinds of Computation Scales—Areas by Different Scales to Plan—Planimeter 281—290

CHAPTER XIV.

COLONIAL LICENSING REGULATIONS 291–304

APPENDIX.

TABLES.

Hypsometer Tables 307
Introduction to Tables of Natural Sines, etc. . . . 311
Natural Sines and Cosines . . . . . . 312—326
Natural Tangents and Co-tangents . 327—339
Natural Secants and Co-secants . . . . . 340—352

INDEX . . . . 353—358

INDEX TO ILLUSTRATIONS.

Description of Illustration.	No. of Figure.	No. of Page.
A.		
Abney Level	57, 58, 59, 60	33, 34
Acute Angle	Sketch	103
Acute-angled Triangle	Sketch	104
Altazimuth Theodolite	165	99
Aneroid Barometer	141, 142	80, 81
Angle A	168, 169, 170, 171, 172, 173, 174, 176, 178, 179, 180	107, 108, 109, 111, 113, 117
Angle of Slope	20, 30, 32	20, 21
Areas of Land	355, 356, 357	283, 284, 286
Averages of Fence Lines	353, 354, 356, 357	282, 284, 286
B.		
Back and Fore Sight	248, 249, 259, 263, 273, 274	200, 201, 208, 210, 219, 220
Base-Lines	35	24
Beam Compass	316, 317	259
Bench-marks	251, 252, 253, 254, 255, 256, 257, 258	204, 205
Borders of Plans	339, 340, 341, 342, 343, 344, 345	270
Boundaries	Sketch	160
Boundaries	13, 14, 15	13
Box Sextant	108, 109, 110, 111, 112, 113	60, 61, 62
Broken Ground	Sketch	160
Brushwood	Sketch	150
Buildings, to Survey	22, 23, 26	16, 17
C.		
Caps.	84, 85	45
Centering-plates for Theodolite	75, 76, 77, 78, 79, 80, 81, 82	42, 43, 44

Description of Illustration.	No. of Figure.	No. of Page.
CHAIN AND ARROWS	10	10
CHAIN ANGLES	39	25
CHAINING THROUGH HEDGES	12	12
CHECK LINES	36, 37, 38	24
CIRCLE	Sketch	103
CIRCUMFERENTOR	72	40
CLAMPING ARRANGEMENT	91, 92	48, 49
CLINOMETER RULE	56	33
CLIPS	95	50
CLOSING A TRAVERSE	235	185
CLOSE-PALED FENCE	Sketch	158
COLLIMATION	132, 133, 248	74, 200
COMBINED CLINOMETER AND PRISMATIC COMPASS	62, 63, 64, 65, 66, 67, 68, 151 to 155	35, 36, 37, 38, 89 to 91
COMPLEMENTAL ANGLE	169, 170, 171, 180	107, 108, 117
COMPOUND LEVELLING	249	201
COMPUTING SCALE	358	288
CONTOURING	283, 284, 285, 286, 287, 288, 289, 290, 291	232, 233, 234, 235, 236
CORNERS OF FIELDS	24	17
CORROBORATION OF OBSERVATION	229	178
CROSS SECTIONS	278, 279, 280, 281	229, 230
CROSS STAFF	47, 48, 49	29, 30
CROSS WIRES	99, 130, 131	51, 73
CURVATURE	247	198
CURVE RANGER	157, 158, 159	93, 94, 95
CURVES, OFFICE	323	261
CURVES, SETTING OUT	292 to 308	237 to 251
D.		
DATUM	250	203
DATUM, ORDNANCE MARK	258	205
DIAMETER OF CIRCLE	Sketch	103
DIAPHRAGM	98, 99, 130	51, 73
DIRECTION OF LINE	Sketch	160
DITCH	Sketch	158
DIVIDERS	326, 327	263, 264
DIVISIONS OF CHAIN	Sketch	3
DOUBLE SEXTANT	113a	63
DRAWING INSTRUMENTS	328	264
DRAWING-PENS	324, 325	261, 262
DRAWING-TABLE	309, 310	255
E.		
EIDOGRAPH	348, 349, 350	274, 277
ENLARGING OR REDUCING PLANS	351, 352	278
EQUILATERAL TRIANGLE	Sketch	104
EYE-PIECE	166	100

Description of Illustration.	No. of Figure.	No. of Page.
F.		
Field Work	213, 214, 215, 216, 217, 218	161, 162
Fixed Station	5	8
Footpaths	Sketch	158
Footplates	261, 262	209
G.		
Gates	Sketch	158
H.		
Heath or Gorse	Sketch	159
Hedge, Boundary of	Sketch	158
How to hold the Chain	11	11
How to hold the Drawing-pen	324	261
Hypsometer	277	227
I.		
Inaccessible Points	41, 42, 43, 44, 45, 46, 123, 211	26, 27, 28, 69, 153
Intermediate Sights	263, 268, 269, 270, 271, 273, 274	210, 217, 219, 220
Isosceles Triangle	Sketch	104
L.		
Lettering Template	346	271
Level	96	51
Level, "Dumpy"	125, 127, 128, 129	70, 71, 72
Level, "Y"	124, 126	69, 71
Levelling	246, 248, 249, 259, 263	197, 200, 201, 208, 210
Levelling the Bubble	133	74
Levelling the Plates	93	49
Levelling with Theodolite	276	223
Level Staff	136, 137, 138, 139, 140, 264, 265, 266, 267	78, 79, 216
Line Ranger	55	32
M.		
Marshy Ground	Sketch	159
Measuring across Streams	271, 272	217, 218
Model Plan	334	267

Description of Illustration.	No. of Figure.	No. of Page.
N.		
North Points	336, 337, 338	269, 270
Number of Angles	219, 220, 221, 222, 223, 224	172
O.		
Oblique-angled Triangles	197, 198, 199, 200	145, 146, 147
Obtuse Angle	Sketch	103
Obtuse-angled Triangle	Sketch	104
Offsets	17, 18, 21, 243	14, 15, 16, 192
Offsets with Optical Square	19, 20	15, 16
Offset Scale	322	261
Offset Staff	1, 2	4
Omnimeter	144	83
Omni-telemeter	145 to 150	84 to 88
Optical Square	50, 51, 52, 53, 54	30, 31
Orchards	Sketch	159
Ordnance Bench-mark	258	205
P.		
Pantagraph	347	272
Parallel Lines	Sketch	106
Parallel Plates	86, 87	45
Parallel Plate Screws	88	46
Parallel Ruler	319	260
Perpendicular	Sketch	103
Plane Table	114, 115, 116, 161, 162	63, 64, 97
Plus and Minus Signs	181, 182, 183, 184, 185, 186, 187	118, 119, 120
Post-and-rail Fence	Sketch	158
Pricker	318	260
Prismatic Compass	69, 70, 71	38, 39
Proportional Compass	329, 330	265
Protractors	311 to 314	256 to 258
Q.		
Quadrants of Circle	167, 168	107
R.		
Radius of Circle	Sketch	103
Railways	Sketch	160
Ranging-out Lines	28	19
Ratios of Two Angles	192, 193	129

Description of Illustration.	No. of Figure.	No. of Page.
Ratio of Radius	191	127
Ray-shade	160	96
Reference	335	268
Reflecting Clinometer	61	35
Relations of Lines to Functions of a Right-angled Triangle	188	122
Relation of Hypotenuse of Right-angled Triangle	175, 176	111
Right-angles with Chain	40	26
Right-angled Triangle	Sketch	104
Road	Sketch	159

S.

Saucers	331, 332, 333	266
Scalene Triangle	Sketch	104
Set-square	320, 321	260
Setting out Curves with Theodolite	292 to 303	237 to 248
Setting out Curves by Offsets and Ordinates	304 to 308	248 to 251
Sextant, Double	113A	63
Sines, Cosines, etc.	188, 189, 190	122, 123, 125
Skew-chaining	16	14
Slope Staff	31	21
Solution of Oblique Triangle	197 to 200	145 to 147
Solution of Right-angled Triangle	196	142
Stadiometer	143	82
Standards of Theodolite	94	50
Station Marks	6, 8	8
Station Peg	7	8
Station Poles	3	4
Stations	4, Sketch	7, 160
Stepping Slopes	33, 34	23
Stock-head	83	45
Subsidiary Stations	9	9
Supplemental Angles	177, 181, 182, 183	111, 118, 119
Surveying a River	228	177

T.

Taking an Angle	230	179
Tangent Screws	91, 92	48, 49
Telemeter	117, 118, 119, 120, 121, 122, 123	65, 66, 68, 69
Telescope	96, 97	51

Description of Illustration.	No. of Figure.	No. of Page.
TESTING A CHAIN	Sketch	9
THEODOLITE, "EVEREST"	106, 107	58, 59
THEODOLITE, "IMPROVED"	156	92
THEODOLITE, "TRANSIT"	103, 104, 105	55, 56, 57
THEODOLITE, "Y"	89, 90	47, 48
TICKETING	280, 281	230
TOWN-SURVEYING	236 to 245	186 to 194
TRAVERSING WITH CHAIN	231	180
TRAVERSING WITH THEODOLITE	232, 233, 234	181, 182
TREES	Sketch	159
TRIANGLE	176, 178, 179	111, 113
TRIANGLE FOR FENCES	25, 27	17, 19
TRIGONOMETRICAL CANON	172, 173, 174	108, 109
TRIPOD HEAD	163, 164	98
TRIPOD LEGS	73, 74	41, 42
V.		
VERNIER	100, 101, 102	53
VERTICAL INTERVALS AND HORIZONTAL EQUIVALENTS	282 to 289	231 to 234
W.		
WALLS	Sketch	158
WHAT TO AVOID	225, 226, 227	174, 175
WOODS	Sketch	159

PLATE No. 1. SURVEY OF PART OF WIMBLEDON PARK . *facing* p. 163
,, No. 2. FIELD-BOOK OF WIMBLEDON PARK . . ,, 165
,, No. 3. SURVEY AT CARDIFF . . . . . ,, 167
,, No. 4. COMPUTATION OF LAND QUANTITIES . . ,, 289

PRACTICAL SURVEYING.

CHAPTER I.

INTRODUCTION.

"SURVEYING is the art of ascertaining, by measurement, the shape and size of any portion of the earth's surface, and representing the same, on a reduced scale, in a conventional manner, so as to bring the whole under the eye at once."

Subjects necessary to be known.—Such being the concise description of the science of surveying by an ancient writer, I am induced to inaugurate these pages with it. A more recent treatise on the subject says, "Considered as a branch of practical Mathematics, Surveying depends for its principle on Geometry and Trigonometry;" and further, "It may be proper to mention the previous knowledge which a surveyor ought to possess, and to notice the instruments which he is to employ in his operations. As a surveyor has perpetual occasion for calculation, it is necessary that he be familiar with the first four rules of Arithmetic, and the rule of Proportion, both in Whole Numbers and in Fractions, especially Decimals, with the nature of Logarithms and the use of Logarithmic Tables, and with at least Algebraic Notation. As it is his business to investigate and measure lines and angles, and to describe them on paper, he should be well acquainted with the elements of Geometry and Trigonometry, and with the application of these principles to the mensuration of Heights, Distances, and Surfaces. In particular, he should be familiar with the best practical methods of solving the ordinary geometric problems, and should be expert in drawing lines and describing figures. He should be acquainted with the principles and practice of Levelling; he should know something of the principles of Optics and Magnetism, and should possess at least a smattering of the arts of Drawing and Painting."

The foregoing list of requisite acquirements represents more forcibly than any words of mine could the range of subjects which demands the attention of the student, and it will be my endeavour in the following pages to give them practical effect.

It is necessary, however, that I should traverse to some extent familiar ground, which I shall avoid where practicable; but I wish to make this work as complete as possible, and would therefore claim the indulgence of the reader if I seem inclined to be too elementary.

Standards of Measure.—In this country we are accustomed to what is known as the duodecimal system of measuring, whereof the foot of twelve inches is the basis. I do not propose to question the wisdom of continuing this standard in the face of the almost universal adoption of the metric system upon the Continent, and indeed nearly all over the globe; but I am bound to confess that the latter method, apart from its universality, offers greater facilities both in practical and theoretical application to surveying.

Chains.—For surveying purposes in England we have two kinds of chains, viz. the 100-feet and Gunter's. These chains, made of stout iron or steel wire, are composed each of 100 links; in the former case each link being equal to one foot in length, and in the latter 7·92 in., or 1-100th part of 66 feet, being the length of the link.

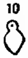
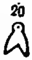
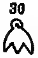
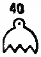
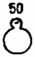
It will be manifest that the 100-feet chain has many great advantages, the chief being that it is so easily understood; and it is further argued that its increased length over Gunter is more conducive to accuracy in its use in the field.

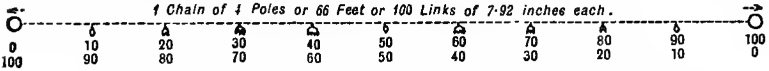
Advantages of 100-feet Chains.—For large plans of estates, especially those destined for building operations, where every inch is of consequence, or for works of construction, the 100-feet chain will prove to be invaluable. But in the operations of surveying proper, for many potent reasons, pending the complete revolution in our system of mensuration, I must admit my preference for Gunter's chain.

Gunter's, or 66-feet Chain.—This instrument, if I may so call it, was invented in the early part of the seventeenth century by the Rev. Edmund Gunter, an eminent professor of astronomy at Gresham College. It is also called a four-pole chain. It is 66 ft. long (or four poles of 16½ ft.*), composed of 100 links of strong iron or steel wire, each link being 7·92 in. or 1-100th part of 66 ft. At every 10 links is fastened a brass tablet of different shapes to denote its value in tens, whilst at each end is a conveniently constructed brass handle.

---

* Poles, sometimes called perches or rods, in different parts of the kingdom, were formerly (by custom) of various lengths; as, of 15 ft. or 5 yds., 7 yds., 8 yds., &c. All these are now obsolete, and the statute acre (35th year of the reign of Edward I.), consisting of 160 square perches (of 272¼ square feet each), is general throughout England.

Divisions of Gunter's Chain.—The first 10 links is distinguished by a tablet like this ; the 20 thus, ; the 30 thus, ; the 40 thus, ; and 50 links or the centre of the chain (33 ft.) by a circular tablet thus, ; so that from each end of the chain are tablets of similar shape and position, and the number of links is counted therefrom. But it is necessary to explain that, having reached the centre of the chain, or 50 links from one end, in proceeding to the other extremity, what represents 40 links from that end is really 60 from the commencement, and similarly 30 is 70, 20 is 80, and 10 is 90, whilst the handle represents 100 links. The following sketch may serve to illustrate this.


1 Chain of 4 Poles or 66 Feet or 100 Links of 7·92 inches each.
0 100
10 90
20 80
30 70
40 60
50 50
60 40
70 30
80 20
90 10
100 0

So that the 1st, 2nd, 3rd, 4th, and 5th labels represent 10, 20, 30, 40, and 50 links respectively from either end. A very little practice enables one to acquire a perfect facility in reading the chain.

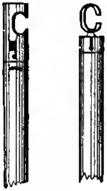
Décamètre Chain.—The décamètre chain is similar in construction to the Gunter, being divided into 100 links. Each 10 links equal a metre, or 3·2809 ft., so that a décamètre chain is 32·809 ft., or nearly the length of half of our Gunter.

Arrows.—Accompanying each chain are 10 arrows, or skewers, about 9 in. long, pointed at one end and having a ring * at the other for greater facility in carrying. These arrows are made of stout wire, and are used to mark upon the ground the end of each chain. The reason why ten is the number adopted is that ten chains (66 ft.) equal one furlong, and eight furlongs or eighty chains equal one mile. Again, an acre of land is ten square chains.

Offset Staff.—Besides the chain, the surveyor should be provided with a small staff or rod (called an offset staff), 6 ft. 7·20 in. long, divided into 10 parts or links. This staff should be made of well-seasoned wood, painted in link lengths black and white alternately; it should have an iron spike at one end and at the other a stout open ring (as sketch, Figs. 1 and 2) for forcing or drawing the chain through a hedge.

---

* It is usual to tie a piece of red cloth or tape round the handle of the arrows, so that they may be the more easily distinguishable when stuck in the midst of grass or plants, &c.



Figs. 1 and 2.—Offset Staff.

33-feet Tape.—It is also advisable that the surveyor should carry in his pocket a small tape, say 33 ft. long, to be used *only* under circumstances when absolutely necessary. These tapes are divided into 50 links, similar to the chain.

Poles.—In order to mark out upon the ground any lines necessary for surveying purposes, poles from 10 to 20 ft. long, according to circumstances, must be provided. They should be 2½ or 3 in. thick at the bottom, and taper to about 1 in. at the top. They should be shod with an iron shoe, pointed so as to easily penetrate the ground.

These poles should be made of well-seasoned deal, free from knots, and perfectly straight. Although it is an unquestionable advantage to have them painted (white, or alternate white and red, or black and white, according to fancy), yet it is not a matter of very much consequence, unless they are intended to be used again upon another survey, in which case the paint is a protection.

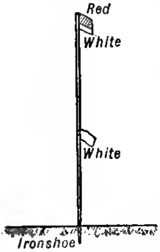


Fig. 3.—Station Pole.

I prefer to surmount these poles with a flag about 18 in. by 14 in. square, of red and white bunting, and it will be found extremely useful, especially for long distances, if a piece of white canvas is fastened by tapes half-way up the rod (see Fig. 3). These poles are chiefly used for stations at the end of long lines. In some cases even these will not be long enough, when of course arrangements must be made according to circumstances, as will be hereafter explained.

Ranging Rods.—No surveyor should be provided with less than about a dozen (or more if necessary) ranging rods, equally very straight and well seasoned to ensure against warping. They should be 6 ft. 7·20 in. long, with iron shoes at the bottom, and tapering from 1½ in. to ⅝ths of an in. in diameter,* and should be divided into ten equal parts (one link each), and painted alternately black and white, or black, white, and red, or red and white, and I have known them to be painted blue and white (this of course is entirely a matter of fancy). Red and white flags should be fastened at the top and white flags tied half-way down.

---

* I have a strong preference for my rods to be octagonal in section in preference to circular, as I think the arris of the former is of great assistance in ranging out lines.

The reason why I recommend them to be 6 ft. 7·20 in. long is that they are none the worse for being a little longer (some surveyors have their rods only 5 ft. long), and in the absence of an offset staff they may be used for all such purposes.

Bundle of Laths.—I always instruct my men to provide a bundle of laths, as not only are they light in bulk, but are "cheap and plenty," and have the advantage (if judiciously selected) of being fairly straight, easily sharpened to a point, and your chainman will not object to carry a dozen or so about with him. For ranging out a long base or other line, especially over very uneven ground, they are simply invaluable. Being white, they can be seen at a great distance, and when done with, if left on the ground, it is not a very serious loss.

Whites.—These are very necessary adjuncts to a survey. Varying from 15 in. to 3 ft. in length, they are simply thin sticks cut from a wood or hedge, as straight as possible, pointed at one end and having a cleft cut in the other for the purpose of inserting pieces of white paper. These are very useful in ranging out lines or for establishing stations.

Equipment of a Surveyor.—I do not know whether it is at all necessary that I should offer any suggestions as to the personal *habillement* of a surveyor, for my own experience has been that I have found the oldest clothes (sound, of course) the most suitable, as it leaves one enjoyably indifferent to the accidents that frequently happen to one's garments. Climbing over fences and walls, crawling through hedges, ascending trees, or fording a stream, are not calculated to improve one's clothes, should they be required for further use.

I may say, however, that, presuming it to be absolutely necessary to provide an outfit for surveying, the following may be a useful guide. First and foremost, good, strong, and unquestionably watertight boots, with plenty of hob-nails, are imperative, for, apart from risk to health, wet feet for a greater part of the day, especially in winter, do not conduce to comfort or improve the temper. High-top boots are a great mistake, for not only do they impede the free action of the feet and are clumsy, especially in heavy land, but if one has to work in water, the act of stooping to obtain a better sight of an object may defeat the original intention of keeping the legs and feet protected from wet. Leggings are also a mistake, as they may keep the external wet out, but they also keep the perspiration in. Woollen cord trousers with leather gaiters are the most suitable for any field work. A jacket of good pilot cloth with plenty of pockets is better than anything, for it will stand many hours' exposure to wet. On no account do I recommend the use of a mackintosh, as it is always in the way if not wanted for wear, and is constantly being torn; and I maintain that, except for driving, the mackintosh is anything but desirable. I will conclude my personal remarks by advising the use of a soft felt or cloth cap with as little brim as possible, as hat-brims are found, especially in instrumental observations, to be not only a nuisance, but often dangerous.

Field Book.—The surveyor should be provided with a good field-book, for which stout blue paper is generally the best. Some surveyors prefer an oblong book about 8 in. by 4½ in., ruled with two lines down the middle forming a central column, on either side of which may be made sketches of fences, buildings, or other objects right and left of the chain-line; but I prefer a quarto book about 7½ in. by 6 in., of which I shall have more to say presently. It is advisable to carry several spare pencils (F) in the pocket in case the one being used is lost; but to guard against such a contingency it is useful to tie a piece of string at the end (taking care to cut a notch round the head), and fasten it to the button-hole of your coat, with a sufficient length of string to enable you to manipulate the pencil. The same advice applies to a small piece of india-rubber, which it is always necessary to have. A good clasp pocket-knife is indispensable, not only for sharpening the pencil, but is very useful for cutting sticks, &c. I strongly recommend the young surveyor to carry a pair of good field-glasses, slung by a strap over his shoulder, as he will find them exceedingly useful, indeed on a large survey absolutely necessary. In the absence of a prismatic compass (to which, of course, preference is given) it is desirable to have a pocket-compass, to determine the bearings of points of the survey. It is quite as well to carry a scale, 6 in. long, of say two chains to an inch; a pair of pocket-compasses, a plentiful supply of white paper, string, a few nails, a lump of chalk, and last, but not least, a fairly-sized plumb-bob. I may also say that I have found a pocket-whistle exceedingly useful to attract the attention of my men when beyond the range of one's voice.

It is sometimes difficult to communicate vocally with one's men to windward, even if within ordinary earshot. By directing one's voice towards the ground about half-way to them (especially if a hand slightly concaved be placed at each side of the mouth), a verbal message may be sent against a fresh wind, to a distance at which point-blank shouting would fail to reach them.

CHAPTER II.

ORDINARY SURVEYING.

BEFORE proceeding to describe the *modus operandi* of surveying in the field, I wish to offer a few remarks upon the important question of *reconnoitre.*

Reconnoitre.—It is absolutely essential that the surveyor should, as a first step, make himself thoroughly conversant with the surroundings of the ground he has to survey, by walking all over the estate, whereby he not only gains an intimate knowledge of the various boundaries, the position of buildings, streams, &c., but is enabled to form an accurate idea of the best routes for his principal lines. It has, indeed, been argued that such a proceeding is unnecessary, occupying as it does valuable time; but the question is whether it is not an absolute saving of time to lay out the work so systematically, that, when chaining operations commence, there is likely to be no hitch or delay, by reason of encountering obstacles not previously observed which involve extra work or, possibly, the abandonment of an important line in consequence. One thing is surely important, and that is, to establish the principal stations, which can only be done after a careful examination of the ground.

Sketch Map.—In making a reconnaissance of a proposed survey, it is desirable to make a neat sketch of all the chief features, so that, having determined the routes of your base and other lines, you may delineate them upon this sketch and number them consecutively, which will be found to be of the greatest assistance, not only in subsequent field operations, but in plotting the survey.

Stations.—To make a survey of even a simple field, equally with an extensive estate, it is necessary to establish stations at those points to which it may be desirable to run lines. Thus A B C and D (Fig. 4) represent stations which comprehend a complete investiture of this figure, whereby lines from A to B, B to C, C to D, and D to A will be necessary to enable the boundaries of the field to be taken.

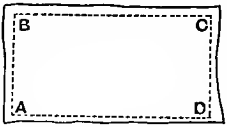


Fig. 4.—Stations.

Main Stations.—Stations are of a twofold character, main and subsidiary. Main stations represent those chief points which, whether the figure to be surveyed be regular or irregular, embrace such lines as will command the boundaries of the survey. These stations are shown in various ways, according to circumstances. If the survey is of only a temporary character (such as can be executed in a single day) then poles or ranging-rods may be fixed for the purpose, but if required for an extensive survey, then stout pegs should be driven into the ground, whilst in some cases special posts, built up and well strutted into the ground (see sketch, Fig. 5), may be necessary. If pegs are used they should be 5 in. to 8 in. long and $1\frac{1}{4}$ in. square, driven with about $1\frac{1}{2}$ in. standing out of the ground, and in pasture land the turf should be cut round them in the form of a triangle (see sketch, Fig. 6). In order to easily identify these pegs I usually cut off a corner of the top (see Fig. 7) and mark the top with a letter corresponding with the sketch plan. Upon an extensive survey a large quantity of pegs will be found necessary—any local carpenter will gladly make them for a shilling or fifteenpence per dozen—and their value is incalculable. Temporary stations (required the same day) may be established by whites or marks on the ground. In pasture land, it is customary to cut the turf in some conventional form (such as shown in sketch in Fig. 8); but under all circumstances I confess to a predilection for pegs. If pegs are placed in the ground to denote stations where a line is to be run thereto, the peg should in due course be drawn and a ranging-rod or pole put in its place.

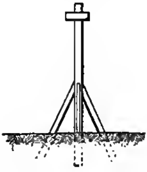


Fig. 5.—Fixed Station.

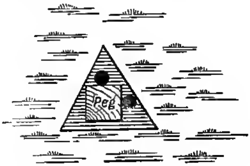


Fig. 6.—Station Mark.

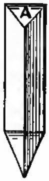


Fig. 7.—Station Peg.

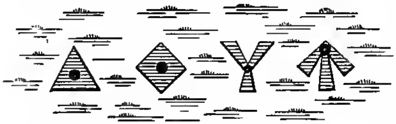


Fig. 8.—Station Marks.

Subsidiary Stations.—Subsidiary stations have reference to those points upon the base or other main survey-lines, where it is necessary to run auxiliary lines, to pick up the boundaries of internal fences, &c., and are determined according to circumstances, as the process of chaining the main lines is carried on. If in the case of an ordinary field (Fig. 9), when after chaining A B and B C, we proceed to take up C D, it will be necessary at *e* to have a station, and similarly on line D E to do the same at *d*, for the purpose of measuring the "tie" or "check" line *d e*. Anticipating my remarks upon the field-book, each station should be marked round with a circle or oval.

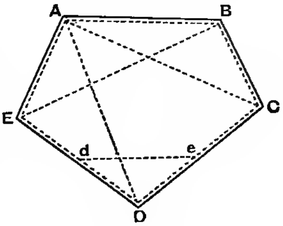
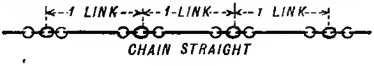
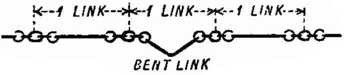


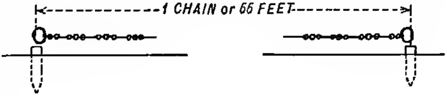
Fig. 9.

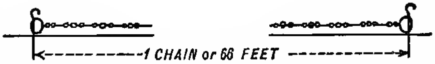
Testing the Chain.—Before commencing chaining, the surveyor should satisfy himself as to the accuracy of his chain, as, if it has been used before, either from constant pulling through fences, or other causes, it may become elongated, or, in going over rough ground, by treading upon some of the links they may become bent, and consequently shortened, as in the accompanying sketch.





Test Gauge.—To form a test gauge upon an even surface, preferably a pavement, it is desirable to measure accurately with a rod (the longer the rod the better) 33 ft. and 66 ft. in the same line. These lengths should be tested by measurement from the other end, and having been determined, marks should be cut in the pavement with a hammer and chisel, at each end and in the centre. In the absence of pavement, upon level ground drive in stout pegs, 66 ft. and 33 ft. apart, and having accurately gauged the two lengths, drive nails into the pegs to mark the exact points. A test gauge should be established in close proximity to every surveyor's office for constant comparison; but in a large survey it is desirable to make one close to the scene of operations, so that each day before commencing work the chain may be applied, and if longer may be adjusted by removing one or more of





the connecting links, or, if short, by straightening the wire links.

It may be stated that a Government standard of all kinds of English measures has been established in Trafalgar Square, by means of permanent bronze marks, let into the granite plinth of the terrace wall in front of the National Gallery. There is also a standard in the Guildhall, belonging to the Corporation of London; and in nearly every city and town in the kingdom, the Borough Surveyor has arranged certain marks wherewith to test his chains, and these, on a courteous request, will doubtless be put at the service of any surveyor whose avocations may call him into the neighbourhood.

There is an art in doing up and throwing out the chain. In the former case, the chain should be taken at its centre (with the circular tablet) and gradually each pair of links towards the end should be cylindrically folded diagonally over the last until the handles are reached, so that when tied up, the chain represents almost a wheatsheaf. The accompanying sketch (Fig. 10) shows the chain folded up and the arrows.

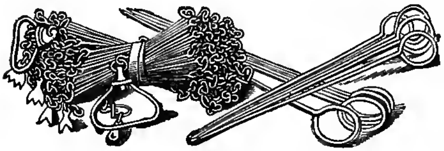


Fig. 10.—Chain and Arrows.

In throwing out, the handles should be held in the left hand with a few links loose, whilst the rest of the folded chain is held with the right, and by this means thrown smartly away, retaining hold of the two handles,

Chain-men.—Now, in all chaining operations, there is one person to drag the chain, called the leader, and another to follow, called the follower. Of these two (supposing two men are employed to assist the surveyor) the follower should be the more intelligent and trustworthy.

I would here say, that in all organised surveys there should always be ample assistance. I mean, that two men at least are requisite, so that the surveyor may be free to make observations, sketch, and enter measurements in his field-book, and generally superintend operations. Indeed I go further, and express a firm conviction, that it is real economy to have a third man, or an intelligent lad, to fetch and carry rods, to take charge of plans, books, &c., and generally to act as aide-de-camp.

Leader's Duties.—Reverting to the leader and follower, it is necessary to instruct each in their respective duties. To the leader should be explained, that, at the commencement of work he is to receive (and count for his own satisfaction) the arrows, for which he will be held responsible. His duty is to precede the follower in a direction indicated and to draw the chain gently after, and upon reaching the limit of its length he is to turn half round to face the follower, holding the handle of the chain in his hand with one of the arrows between the inside of the handle and the inside of his fingers thus (Fig. 11), and to watch for a signal from the follower as to how he should move laterally right or left, taking care (on his part) to keep the chain straight, by gentle shaking up.

Some surveyors hold that the leader should completely face the follower at the end of each chain, but my experience has been that, by so doing, his body often obscures a forward point, and by very little practice, he can be made to do the work as well sideways. It is necessary that he should hold the arrow perfectly upright, and only move it gradually right or left, so as to mark the exact spot indicated by the follower.

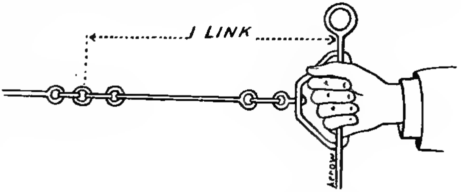


Fig. 11.—How to hold Chain.

Here I may say, that it is useful to range out several points in a line by means of laths or whites, which will be useful in guiding the leader to keep in the direction it is necessary to go. The surveyor must impress this upon him, as I have sometimes found that the leader will elect to walk in a certain direction, apparently to his own satisfaction, which has the disadvantage of being considerably out of the line.

Duty of Follower.—The duty of the follower, having previously had the destination of the line explained, is to retain the other end of the chain in hand, and to direct the leader as to the direction he should take; to call out when the chain is at its full length; to hold the extremity of the handle against the centre of the station whence the line starts, or against the arrow which had been previously placed in the ground (taking care to hold the outside of the handle against the point); to see that the chain is stretched perfectly straight and lies evenly in a true line with the forward station; to direct the leader to move his body altogether right or left, and when approximately in line, to instruct him by slight lateral movement of his hand, right or left, until the exact point is obtained. If within hearing range he should call "To you" or "From you," or if beyond earshot, by moving the head right or left; and to convey to the leader that he is right, and it is necessary to fix an arrow in the ground to mark the spot, either call out "Mark," or convey that meaning by a nod.

In the event of its being found impossible to make the leader hear your directions or those of the follower, if you want him to move to the right, wave your right arm backwards and forwards, and if to the left, similarly with your left arm; and to indicate that he is in a right position, bring both arms smartly to attention.

How to use the Chain.—It should here be explained that as the chain measures 66 ft., or 100 links, between the ends of the handles, it would not be right to hold one extremity against the arrows or pegs at each end, for by so doing, the length of the line is diminished by the number of half-thicknesses of the arrows or pegs, corresponding with as many chain-lengths as have been measured. But when pegs are used, if the end of each handle is held in the centre—or with arrows, if the leader holds the inside of his handle against the arrow, whilst the follower holds his handle (outside) against the arrow at his end—by these means the proper length may be adjusted.

After placing an arrow in the ground at the end of the first chain, the leader proceeds in direction of the goal, until he has reached the limit of the chain. The follower, having walked to the first arrow, and held his end of the chain thereto, now directs the leader so as to mark the second chain, which having been duly accomplished, the men go forward (the follower having previously picked up the first arrow), and so they continue, until the leader has expended all his arrows, when, having placed his last in the ground, he calls out "Ten," which should be acknowledged by the surveyor and booked accordingly. The surveyor now proceeds to the tenth arrow, and putting his offset staff in the place of the tenth arrow, the follower, having reached this point, picks up the tenth arrow, and counts the ten arrows before handing them over to the leader, who on his part again counts them to see that he receives the right number.

The foregoing is a description of the method of chaining a simple line between the points, supposing it to be necessary only to ascertain the length of a line, but it seldom happens even in a check-line that such an operation can be performed without crossing through hedges or fences of some description.

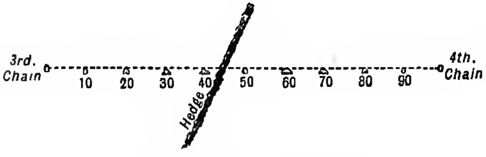


Fig. 12.—Chaining through Hedges.

Crossing Hedges, &c. (Fig. 12).—In these cases the leader and follower must wait before moving forward, to allow the surveyor to note the chainage of each intersection. For instance, if after three chains have been measured a hedge intervene between the third and fourth chain, then the follower, noting at what point the leader's end of the chain should pass through the hedge, gives the necessary directions, which having been done, the chain is now pulled tight, and a fourth arrow having been adjusted in place, the chain is allowed to rest until the number of links is ascertained where the fence crosses the chain. In the case supposed (Fig. 12) the number of links is 47, so that the crossing of the hedge on our chain-line should be booked 3·47—that is, 3 chains 47 links.

Hedge and Ditch.—Here it may be well that I should speak of hedge and ditch, which appears to be a question somewhat enveloped in mystery. If I stand in a field with the ditch on my side of the hedge, then I know that the field in which I am standing reaches only up to the edge of that ditch, and that both the ditch and hedge belong to the field on the other side, as per sketch (Fig. 13). Thus the boundary of A is the edge of the ditch on the left, whilst the ditch and hedge belong to B. In illustration of this, when a railway is staked out through a district, it is usual for the contractor to fence-in the land required for the works by means of what is called a "post-and-rail fence" (see Figs. 14 and 15), which represents the extremity (on either side) of the property acquired by the company; and one of the last things done before the completion of the railway, is for the contractor to cut a grip or ditch, on the inside of the fencing, and with the excavated soil to form a mound in which "quicks" are set. The life of the larch post-and-rail fence is supposed to be long enough to enable the quick to develop into a hedge. And in future years, when decay shall have removed the wooden fence, a surveyor will make the necessary allowance outside the hedge for the real boundary of the railway.

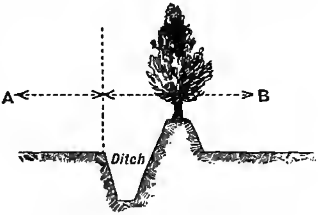


Fig. 13.—Boundary of Field.

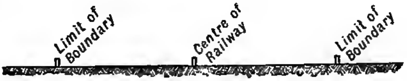


Fig. 14.—Boundaries.

Fig. 15.—Boundaries.

How to measure Fence.—Here I would say, that it has been found to be more convenient to take all measurements to the centre, or root, of the hedge, and make the necessary allowance for the edge of the ditch therefrom. The usual allowance is six links, but in different counties in England this length varies according to custom, and it will be prudent of the surveyor to make inquiries in the locality as to that custom. This allowance of six links is, of course, upon the square—as A B (Fig. 16)—for, if the chain crosses in an oblique direction (as C D), then the distance will be greater. For instance, suppose the edge of the ditch on the square is six links, as A B, but the chain crosses instead at an angle of 60°, then the length from the hedge to the edge of the ditch will be nearly seven links instead of six.

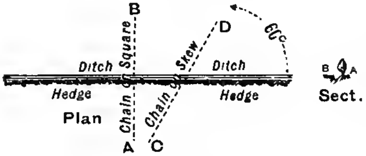


Fig. 16.—Skew Chaining.

Foot-set Hedges.—It may happen that a hedge has a ditch on either side, or none at all, and yet divides two properties, and in such a case the centre or root of the hedge should be taken.

Offsets.—The process of surveying, after the necessary lines have been laid out, consists of determining the various boundaries, buildings, &c., by means of lateral measurements, to such points right or left of the chain-line, as may distinguish any alteration in shape of the fence, or the angles of the buildings.

These lateral measurements are called offsets, and strictly speaking are always taken at right angles to the chain-line. As it is possible upon the ground, no matter how uneven, to lay out a straight line, which on paper is drawn with a pencil and straightedge, so it is possible also upon the ground to set out a right angle. Under the head of "Instruments" (Chap. III.) I have described the cross-staff (p. 29) and optical square. I have described these appliances for setting out a right angle; and for taking offsets the latter will be found to be the most useful and accurate. But for general work, the surveyor soon gets accustomed, with the eye alone, to find the exact position on the chain at right angles to any clearly defined point. A greater help is to lay down the offset staff as nearly square with the chain as it is possible to judge, and then, looking along the rod, to mark with the eye any point in line therewith in the fence, as shown by the dotted line D E (Fig. 17)

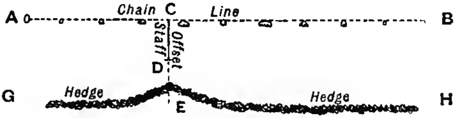


Fig. 17.—Offset.

when A B is the chain, G H the fence, and E a point to which it is necessary to take an offset; C D is the staff, and C E the offset. In using a cross-staff, great care has to be observed that the rod on which it is fixed is in a vertical position,* and exactly upon the chain-line. The box is directed so that two of the slots are in line with the chain-line, as *a b* (Fig. 18), when, by looking through *c d* in direction C D, we have a right angle with A B.

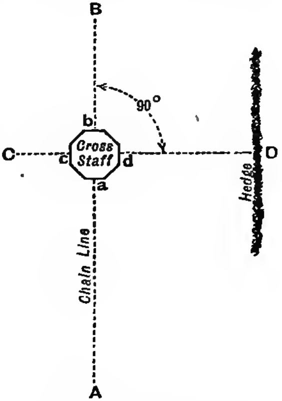


Fig. 18.—Offset with Staff.

Using the Optical Square.—In the case of the optical square, the operator holds the instrument in his left hand, and having placed a flag at D, or a piece of paper in the hedge, walks along the chain-line keeping his eye upon the advanced flag B until the flag or mark at D becomes coincident with the flag B (as in Fig. 20), when C D is at right angles with A B. Fig. 19 illustrates the *modus operandi* of taking an offset at the intersection of two hedges D, with an optical square, where A and B are the flags on the chain-line and C the point of observation.

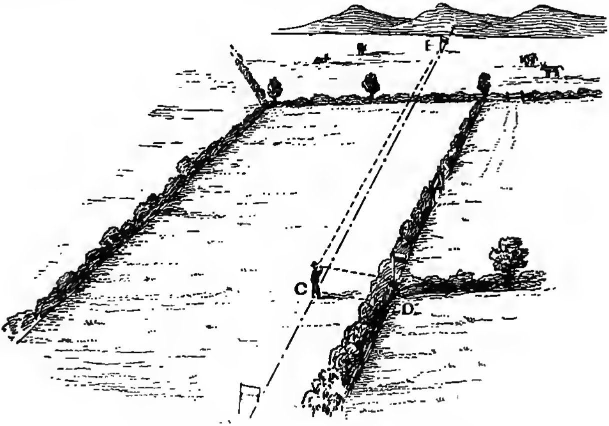


Fig. 19.—Offset with Optical Square.

---

* Equally with the necessity to fix the cross-staff perfectly vertical, so should all ranging rods be made perpendicular, which is best effected by using a plumb-bob.

Offsets should be taken at all points of divergence in the line of fences, or at angles formed by two fences. It is not necessary to take offsets at every chain if the hedge is fairly straight, but may be done every second or third, but when there is any appreciable bend or kink, as in Fig. 21, it will be desirable to take offsets at 1*a*, 2*b*, 3*c*, 4*d*, and 9*f* on the right-hand side of the chain, and 6*e* on the left. It will be seen, that the fence from *d* to *e* crosses the chain diagonally, as does that from *e* to *f*, and in addition to the offsets 4 *d*, 6 *e*, and 9 *f*, the distances along the fence 5 *d*, *e* 5, *e* 7, should also be measured, and to fix the corner *f* a temporary station in the chain-line, as at 8, should be noted, and the distance 8 *f* measured as a check. If the ditch is on the other side of the hedge to the chain-line, then it is customary to take the offset to the centre, or root, of the hedge and add six links for the edge of the ditch, and if the ditch is on the same side, either to take the offset to the edge, or to measure to the root of hedge, and deduct six links. I may here say that unless the ditch be very wide, or the hedge inaccessible, I always prefer to measure to the hedge and deduct for the ditch, as denudation, or other cause, renders the edge of the ditch of a very undefined character, and if strictly taken in offsetting would not fairly represent the true boundary.

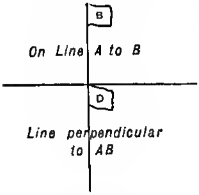


Fig. 20.

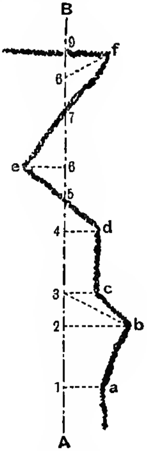


Fig. 21.

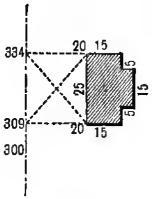


Fig. 22.

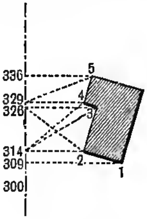


Fig. 23.

As to Buildings.—Buildings require to be very carefully taken at each angle, and the right angle must be very accurately set out; in the case of Fig. 22, when a building is square with the chain-line, it is only necessary to take offsets to the face of the building. It will be seen that after the third chain, at 309 and

334 are points whence the two corners of the building run, and the difference between 309 and 334 should be the same as measuring along the face of the building, viz. 25 links. All that is necessary is to measure the depth of the building together with any projections that may occur, as in Fig. 22. In the example shown by Fig. 23, keeping the same points on our chain-line, it will be seen that the first offset at 309 is to 1, which is the angle of the back of the building, whilst 314 to 2 is the front corner, 326 is the termination of that same plane, 329 the angle formed by the projection at 4, and 336 the other angle of the same plane. The lengths of the frontage, sides, back, and projection, should be measured carefully, and the various angles of the buildings should be fixed by diagonal tie-lines, as shown in Figs. 22 and 23.

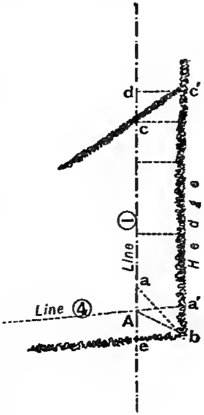


Fig. 24.

As to taking Corners of Fields.—In the case of commencing a chain-line in the corner of a field, as in Fig. 24, it is not sufficient to take one offset from A to *a′* on line 1, and one from A to *e* on line 4, to obtain the angle *b* formed by the two fences, but the diagonals A *b* and *a b* are necessary to accurately fix the point of intersection. Equally, when the chain-line crosses the fence at *c* it is not only necessary to take one offset at *d* to *c′*, but the length *c c′* along the hedge should be measured, so that with the length *c d* on the chain we have a triangle to fix the exact position of *c′*.

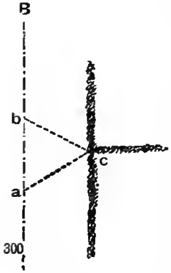


Fig. 25.

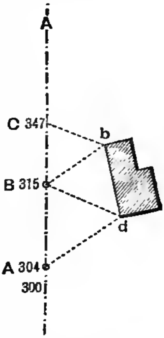


Fig. 26.

To fix Position of an Intersection.—It may happen that the intersection of a fence on the other side of the hedge requires to be accurately determined, for which purpose a simple offset would hardly be sufficient. Set out a triangle, with one side on the chain, as in Fig. 25, as *a* at 320 and *b* at 337, and then measure the length *a c* and *b c*. And again, to fix the angles of a building when a right angle is deemed insufficient, as in Fig. 26, leave stations A B and C at 304, 315, 347, from which measure the lengths A *d*, B *d*, B *b*, and C *b* to the corners of the building.

Limit of Offsets.—Now as to the limit of offsets, I may say that I do not agree with many writers who recommend offsets of 100 or 200 links, or even more, nor do I approve of the use of a tape for such a purpose, except under exceptional circumstances. In a well-known work on surveying I was surprised to read that "Offsets may be measured by pacing, with a tape, or with one offset staff. We prefer the last, although for preliminary or parliamentary work we generally measure by pacing, and the student will find that after a little practice he can measure his offset *by pacing quite as near as he can plot the work.* Of course it is understood that we have to get ourselves into the habit of pacing a yard at every step." Now, I need hardly say, that I most emphatically condemn every word of the foregoing advice, as being entirely contrary to what is required of the surveyor of to-day.

Pacing.—It is true that military surveyors are in the habit of pacing and sketching to a very great extent, and even for "cadastral" purposes have been known so to train their horses that a cavalry man can form a very fair approximation of distance by counting the number of paces the animal makes. I elect to quote from an eminent military surveyor* upon the subject of pacing, who says: "In such surveying as an officer is generally called upon to perform, sketches of small positions, reconnaissances, &c., he will of course be unprovided with a chain, and must determine the length of the base by pacing or counting the paces of his horse." But even this recommendation is qualified by the remark that approximation is sufficient. I certainly have yet to learn with what degree of satisfaction, not to say accuracy, the offsets for a survey of any importance can be done by pacing, even upon perfectly level ground. I recommend the student in surveying of to-day to keep forcibly in mind the maxim that "a thing that is worth doing at all is worth doing well," and any trouble involved in taking his offsets in the proper way, will be amply repaid by the accuracy with which his work is accomplished.

Objections to Tapes for Offsets.—My objection to a tape is threefold: 1st, it is conducive to laziness and long offsets; 2nd, after much use or wet it either elongates, or in windy weather it is shortened by sagging; and 3rd, it is an intolerable nuisance either to keep winding up, or to have to gather it in folds in your hand, added to which, the filthy state in which it makes your hands and book. Further, after continual usage, either by dirt or wear, the figures get indistinct, and this often leads to errors.

I have said that I do not approve of long offsets, and I think 50 links should be the maximum, unless under very exceptional circumstances. Long offsets are generally the result of laziness; for rather than set out a small triangle from a chain-line, when a considerable bend in a fence occurs, and from the sides C E and E D of this triangle take short offsets, as shown in Fig. 27, many surveyors who advocate long offsets would take to the bend direct from the chain-line A B. And here let me say that a triangle such as C D E cannot be considered correct unless a tie-line such as C′ E is measured.

---

* Major W. H. Richards.

Offset Staff.—I need hardly say that I recommend the use of an offset staff for taking offsets, feeling persuaded it is the most accurate and convenient method. And the staff is useful for determining a right angle, as well as to pull the chain through hedges, &c. To use the offset staff, lay it with one end against the chain, and looking along it, note any point in the fence where a line produced would cut, and then turn it over carefully, so that it does not slip back, to prevent which, place your toe against the end, and so on until you have reached the point. A little practice will soon render the task simple.

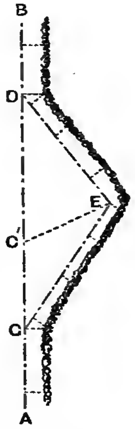


Fig. 27.

Ranging-out Lines.—Having determined the position of the main and chief subsidiary stations, it is now requisite to range-out such lines as may be necessary to proceed with the survey. Poles or rods having been placed at the extremities of lines, the lines themselves should be "boned-out," which is accomplished by sending a man forward with laths or whites, and the surveyor, placing himself at some little distance behind a rod, at the commencement of the line, is enabled to range as many intermediate points as he may deem necessary. I strongly recommend standing, say two or three yards away from the rod, as a much better sight is obtained than by being so close to the rod. It is advisable to range out a number of intermediate points, especially in undulating ground, as, not only may it not be possible to command the forward station if in a valley, but they are extremely useful in guiding both the leader and follower in the chaining operations. This is illustrated in Fig. 28. If A and B represent the stations of a line which has to pass across a valley, it is manifest that unless

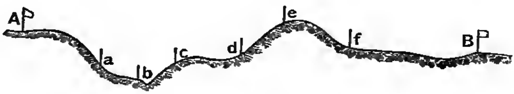


Fig. 28.

such points as *a b c d e* and *f* have been previously established, it would be impossible to chain the line A B. It sometimes happens, owing to inequality of the ground, that, in running a line, the forward mark is lost to sight while at the same time enough cannot be seen of the poles already planted to allow of the line being continued by setting-out by ranging from them. In such a case, one person (A) carrying a rod advances as well as he can judge along the intended line until he sees the forward mark; and another (B) goes further ahead until, looking back, he can just see the last rod of the line already set out, and he ranges (A) in line between himself and it. (A) then ranges (B) in line between himself and the forward mark. (B) then ranges (A), and (A) ranges (B), until no further correction is needed: the forward mark, the rods planted by (A) and (B), and the last rod of the line already set out, being then in true continuation of that line if the work has been properly performed.

What is Level Ground.—Any ground of a fairly level character may be treated as being quite level—that is, any ground whose slope does not form a greater angle with the horizon than five degrees. But beyond this, it is necessary to adopt some means of regulating our measurements. If we take a pair of compasses, as in Fig. 29, and with A as a centre and B (the foot of the slope) as a radius, and strike the arc B C until it cuts the horizontal line A C, it will be seen that the line A C is greater than A *b*, *b* being a point whence a perpendicular is let drop to cut the foot of the slope. Now, it is well understood that in surveying operations all measurements upon the ground are reduced to the horizontal, as, "when plotted the survey represents a perfect plane, and in chaining the lines they should be so conducted, that every length should be a point exactly equal to the base of a right-angled triangle."

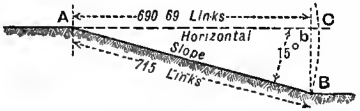


Fig. 29.

In the case of Fig. 29, if we plotted the line A B exactly as measured along the slope, which in this case is 715 links, we should make our line 24·31 links longer than it should appear, and consequently our plan would be inaccurate. I make no apology for reproducing the following well-worn simile to illustrate my meaning. If we take a staircase composed of 30 steps, each tread being 12 inches wide, and each rise 6 inches, strictly speaking we could only show them as the plan of a house by a length of 30 feet, whereas if we measure the string of the staircase it will prove to be 33·54 feet long. Thus I do not think any more need be said to emphasise the necessity of reducing *all* measurements to the horizontal.

Now there are several ways of doing this; chiefly by reducing the hypotenusal measure by calculation, having obtained the angle of slopes, and by "stepping." Of the former, much may be argued for and against, and I propose to say a few words on both sides. Of such a method there can be no doubt that for expedition a great deal may be said in its favour. With an Abney level or clinometer it is very simple indeed to observe the angle of slope and to make the necessary reduction in the chainage as the work proceeds. But the very greatest care and discrimination is requisite in determining these angles. It very seldom happens that the slope of a hill-side is regular; on the contrary, it is often made up of constantly varying inclinations, some flat, some steep; and to accurately determine the hypotenusal correction separate angles will have to be observed at each point of variation. Fig. 30 will better illustrate my meaning, for between A and B it would not be sufficient to observe the angle formed with the horizon by A B, because, to be correct, the hypotenuse should be measured along the line A B, whereas (that being impossible) we follow the undulations of the ground between these points, such as A *b c d e* B, and use the length so measured as the multiple of the angle of slope. Thus, whereas the line A *b c d e* B measured along the surface of the ground is 720, the angle A B C (the angle of slope with the horizon) being 25°, the hypotenusal deduction would be 72·38 links, whereas strictly speaking it should only be 70·50 links, by reason of having taken the angle from A to B. So that, to be accurate, it is necessary to observe the angles of slope at A *b c a* and *e*, and for each separate angle to take the length along the slope between the points.

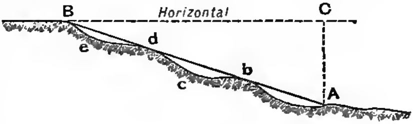


Fig. 30.

Observing Angle of Slope.—It has been suggested that to obtain the angle of slope it is sufficiently near to send a chain-man

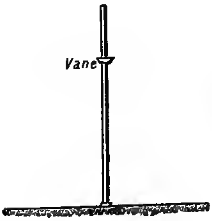


Fig. 31.—Slope Staff.

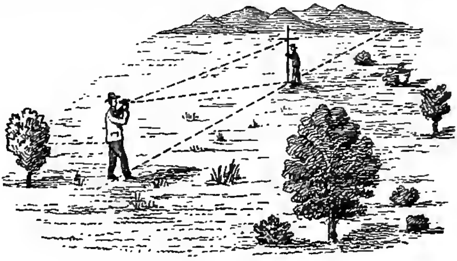


Fig. 32.—Taking Angle of Slope.

to the point at which it is desired to take the angle, and to observe when the clinometer cuts his face; but if the surveyor happens to be a short man and the chain-man tall, it is difficult to see how he is to obtain accurate results. I recommend the use of a sliding vane similar to Fig. 31, which should be adjusted to the height of the eyes of the surveyor, and this, being held perpendicular at any point, will give a true line parallel with the slope of the ground. Fig. 32 represents my meaning.

The following is a table of allowance to be made for the difference between hypotenusal and horizontal measurement:—

Degrees.	Links.	Degrees.	Links.	Degrees.	Links.	Degrees.	Links.
5	00·4	14	03·0	23	07·9	32	15·2
6	00·6	15	03·4	24	08·6	33	16·1
7	00·7	16	03·9	25	09·4	34	17·1
8	01·0	17	04·4	26	10·1	35	18·1
9	01·2	18	04·9	27	10·9	36	19·1
10	01·5	19	05·4	28	11·7	37	20·1
11	01·8	20	06·0	29	12·5	38	21·2
12	02·2	21	06·6	30	13·4	39	22·3
13	02·6	22	07·3	31	14·3	40	23·4

Adjusting the Allowance for Slope.—It should be here explained that many surveyors, having calculated or obtained the necessary allowance, either move the arrows in accordance with the reduction from the length of slope, or make the alteration in the field-book; the former method, however, is best, as any offsets that may be required will be more favourably effected than by the latter. To use the clinometer a very steady hand is required, and possibly the best instrument for the purpose is the Abney level (described in Chap. III. p. 33); but a primitive and very useful little clinometer may be made by cutting a stout piece of cardboard into the shape of a semicircle and dividing it right and left of the centre into 40 degrees, each of which may be marked with the figures given in the table. It is held in one hand and held up to the eye, and looking along the diameter of the card you note when this line cuts the vane of the staff, when a small plummet hanging from the centre marks the angle, which should be read by one of your men.

Stepping.—I venture to think, however, that if necessary care is observed, chaining up and down slopes may be accomplished with sufficient accuracy for all practical purposes by what is known as stepping, which consists of short lengths of the chain being held in a horizontal position, and the extremity transmitted to the ground by means of a plumb-bob, as shown in Figs. 33 and 34. The greater the angle of slope so much less will be the horizontal distance, and *vice versa*, and great care should be observed, not only in taking short lengths of the chain, but in accurately marking the exact point above the plumb-bob, indicating the end of the length, after it has been brought to rest.

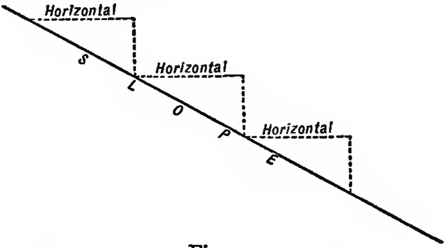


Fig. 33.

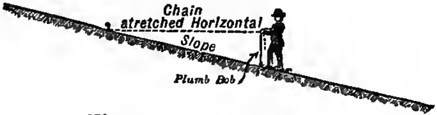


Fig. 34.—Stepping Slopes.

I am of opinion that chaining may be accomplished along sloping ground both accurately and expeditiously by this means if the necessary care is observed, and it has the advantage of indicating absolutely the true position whence an offset is taken, rather than by calculation. I have had very extensive experience in measuring along the sides of hills, and have always found this system satisfactory. But it must not be supposed for a moment that I am an advocate for substituting for a plumb-bob staves or arrows dropped from the end of the chain, which is a very frequent custom.

Base-lines.—In all surveys, large or small, there should be base-lines intersecting the figure to be surveyed. The letter X is the best form for the base-lines to take, care being observed that their direction is upon as level ground as possible, for upon the correctness of the length of these lines the accuracy of the whole of the details depends. I have said tolerably level, that is to say, with no greater undulation than say 4 deg. to 5 deg., for gentle slopes have comparatively slight effect upon linear measurements, and if the ends of the base-lines are otherwise well situated, so as to command an uninterrupted view of surrounding country, the existence of such undulations in the intervening ground need not be considered a drawback. Base-lines should be as near the centre of the survey as possible, since the liability to inaccuracy in the triangulation increases with the distance from the original base. The base-lines (and there may be more than two, and only one under certain circumstances) should form the basis of a system of triangulation which comprehends the various boundaries of the estate. The equilateral is the best form of triangle, and it should be sought to lay out this figure as much as possible, but of course this is not always practicable. The sides of the triangles formed upon these base-lines are called chain- or survey-lines, which are so arranged as to take the boundaries of the property, and from these again are subsidiary chain-lines, to pick up any of the fences or other objects that intersect the estate.

A very simple illustration of the base- and survey-lines will be seen in Fig. 35, in which A B is the main base-line and C D the other; the survey-lines are A C, C B, B D, D A. Now three sides of a triangle, however carefully measured, are no guarantee of its accuracy; there must be a proof or tie line. It has been recommended to test the accuracy of a triangle by letting drop a perpendicular from the apex to the base; this is all very well on paper, but upon the ground it is not always either practicable or expedient. In Fig. 35, quite by accident the line C D crosses the line A B from the apex of each triangle A C B and A D B at as near 90 deg. as possible, consequently the length C *b* will test the triangle A B C, and *b* D will prove A B D.

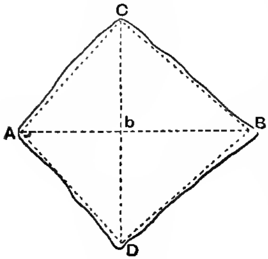


Fig. 35.

I have borrowed an excellent example (Fig. 36) from a well-known work (on surveying) which illustrates my argument exactly, where it will be seen that the property consists of two fields adjoining a road, which are together in the form of an irregular triangle. The three sides A B, B C, and C A embrace the exterior boundaries, whilst the direction of the internal fence is of a character that a line E D may serve the double purpose of taking up this hedge and acting as a check to the triangle. For if the lengths A E on line A B and A D on the line A C be carefully measured, then the length E D will be proved to fall exactly within these points after the triangle has been plotted.

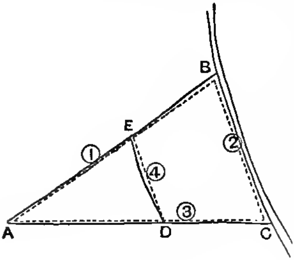


Fig. 36.

Fig. 37 shows how the irregular figure A B C D E F may be divided

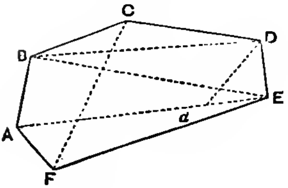


Fig. 37.

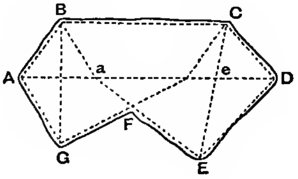


Fig. 38.

into triangles, and by C F all four triangles may be tied, although I should recommend a further check, such as D *d*.

It does not always follow that a survey must consist only of triangles, although it is always advisable to adopt this figure when possible, for, as in the case of Fig. 38, A B C D is in the form of a trapesium, and so long as the line B C is checked by such ties as B *a* C *e* the work will be all right. The line C *e* produced to E, checks the triangle *a* D E, as does a part of B G the figure A *a* F G.

Chain-angles.—We have dealt so far with simple figures, whose outlines can be ascertained by running lines in various directions to take up the boundaries and intersecting fences, which lines are checked by such means as I have briefly described; but there are cases, such as woods or ponds, which it is impossible to get through or across, where it is necessary to chain round, taking the exterior boundary, and fix the relative directions of the lines circumscribing the figure by means of what are called chain-angles.

I have already explained that three sides of a triangle measured is no proof of accuracy, to ensure which a fourth or tie-line is required. This is all the more necessary in the case under consideration, where we have, as in Fig. 39, to run our lines all round outside, and have to prove our work. Here we have to tie our lines in such a manner as to comprehend the outline of the wood, through which it is quite impossible to survey. Briefly, to prolong lines 1 and 4 and to tie their extremities by the line A *a* would not be sufficient to ensure the angle, therefore a second tie *a* a' is necessary, and, similarly, lines 3 and 4 by means of the ties D d' and D *d*. The acute angle formed by lines 1 and 2, although tied by *b* b' (which serves the double purpose of a survey-line), could hardly be trusted unless checked at the other extremity of 2 by the ties *c* c^1, *c* C, C c^2.

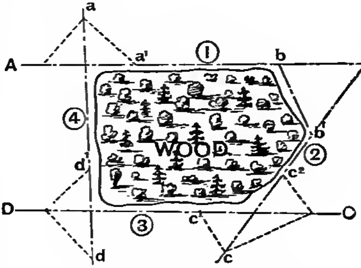


Fig. 39.—Chain-angles.

I might give numbers of instances of how such figures may be circumscribed by means of lines and chain-angles, but in these days, when instrumental observations have superseded such methods, I deem it to be unnecessary to dwell upon the subject.

Inaccessible Distances.—It rarely happens that a survey of any extent can be carried out without some difficulties being encountered, such as base- or important chain-lines being interrupted by obstacles, in the form of rivers, arms of lakes, ponds, buildings, &c., when it is necessary to resort to some means of working round in the one case, or by geometric construction or angular observation to ascertain the intervening distance. This strengthens my argument in favour of reconnoitre previous to commencing a survey, as in undulating ground a building or other obstacle which had been unobserved might come directly in the line, which by careful arrangement beforehand might have been avoided. In the absence of any instrument, such as a box sextant or optical square, a right angle may be approximately set out on level ground by the following simple method. Measure forty links on the chain-line, and put arrows, as at A and B (Fig 40), then with the end of the chain held carefully at A take eighty links and instruct another chain-man to hold the eightieth link at B; take the fiftieth link in your hand and pull from A and B until they are fairly tight, when an arrow at C will be perpendicular with the line A B, in other words A B will equal 40, B C 30, and C A 50 links.

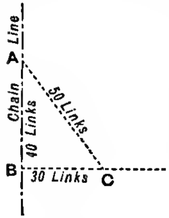


Fig. 40.

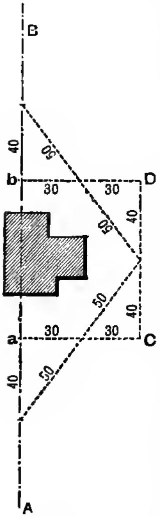


Fig. 41.

I have said this may be done approximately on level ground, but I do not recommend any reliance being placed upon a right angle set out in the manner above described if intended to overcome a difficulty such as is represented in Fig. 41, where the line A B is interrupted by a house. In this case it is assumed that if at *a*, on the line A B, a right angle be set out (as explained) and a sufficient distance *a* C, say 60 links, measured, and C D (made perpendicular to *a* C) 80 links, and D *b* at right angles to C D measuring also 60 links, and *b* B made perpendicular to *b* D, then *a b* will be within the points A and B, in other words in the same line, supposing the building did not obstruct. Thus four right angles have to be set out and measured to carry the line A B past the building. I recommend the student to practise this problem on perfectly level ground, and I venture to think he will agree with me that, unless the line *b* B has been ranged from A upon sufficiently high ground to see over the building, very little reliance must be placed in the prolongation of the line A *a* by such means as I have described, and yet there are numbers of works on surveying which give it as a practical example. I can only say that I should observe the greatest care in checking with a theodolite such work before I should trust to such a prolongation.

I have selected one or two such examples of measuring over inaccessible distances, across rivers or ponds, by the chain only, as appear to me to be capable of satisfactory results, if great care and accuracy be observed, for, unlike the case of the building, you can command all points. Suppose, as in Fig. 42, the line A B is intercepted by a river, the width of which is too great to ascertain by measurement across. We must therefore proceed to set out such a figure on one side of the stream as will enable us to range across it a line which shall so intersect the line A B, that this point of intersection shall be equidistant from a given point to another point, to which we are able to measure on the ground.

First, range the line A B across the stream, sending a man with rods to establish on the other side where directed in the first instance at B. From any convenient point *b* measure towards A such a distance as judgment tells to be greater than that across the river, say 400 links, at A the extremity of 400, and *b*, set out right angles, and from *b* measure 300 links to *b′*, and from A 600 links to *a′*. Place rods at *a′* and *b′* (having previously checked the lines A *b′* and *a′ b′*, which should respectively be 500 links); now range through *a′* and *b′* the point *c* on the line A B, then *c b′* will equal *a′ b′*, viz. 500 links, and *b c* will equal A *b*, viz. 400 links. Measure from each edge of the stream to *b* and *c*, the sum of which deduct from 400, and you have the width of the river.

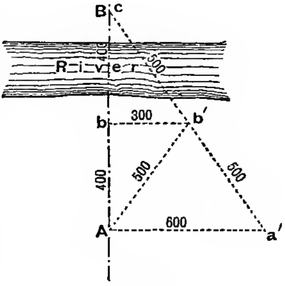


Fig. 42.

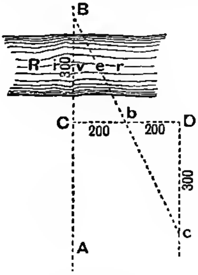


Fig. 43.

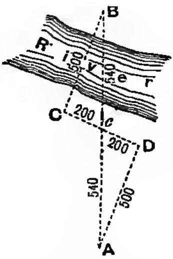


Fig. 44.

Again, in Fig. 43 at C on the line A B set out the perpendicular C D, and make it some equal number of links, say 400; bisect C D in *b*, and at D set out the right angle C D *c*, make D *c* = 300 links, place rods at *c* and *b* and range the line through until it intersects A B in B, then C B will equal D *c* = 300 links. Similarly, if the line passes obliquely (Fig. 44), set out any line parallel (approximately) with the bank of the river, as C D, measure 200 links either way, at each end set off the perpendiculars D A, C B, then will *c* B = *c* A = 540 links. Again, as in Fig. 45, measure off the perpendiculars B C, D E, ranging the point C in line with A E; then

$$\text{AB} = \frac{\text{BC} \times \text{BD}}{\text{DE} - \text{BC}}$$

All the foregoing are fairly good methods of determining inaccessible distances, in the absence of instruments for taking angles, but I need hardly say that the right angles should be set out with an optical square or other reliable appliance, and even then the very greatest care must be observed.

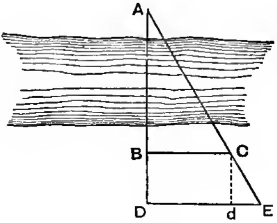


Fig. 45.

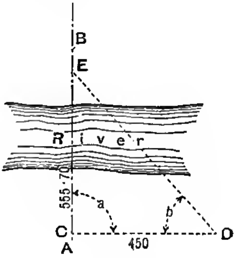


Fig. 46.

The simplest, quickest, and most reliable method of determining an inaccessible distance is as follows: (Fig. 46) at C, with a box sextant or theodolite set out the line C D at right angles with A B, measure any distance C D, and at D observe the angle E D C. Then

$$\text{CE} = \text{nat. tan. EDC} \times \text{CD}.$$

For example, the angle E D C is 51°, and the length C D = 450 links. Now nat. tan. of 51° = 1·2349.

∴ 1·2349 × 450 = 555·7050 links, which is the length C E. Should there be any doubt as to the accuracy of the observation or calculation, place the instrument at E and observe the angle C E D, which should equal 90° − 51° = 39.

I now leave this branch of my subject, as in subsequent chapters I propose to treat the whole question of field work in greater detail.

CHAPTER III.

SURVEYING INSTRUMENTS.

In the early days of surveying only very primitive instruments were available, but nowadays the science of surveying has attained such a state of perfection that we have instruments of all kinds for facilitating geodetical operations in the field.

Cross Staff.—In the foregoing chapters I have referred to the process of taking offsets with an offset staff, which for short lengths may generally be relied upon. Although I am bound in this division to refer to the cross-staff, I have no hesitation in condemning its use upon nearly every ground. I look upon such appliances as only an excuse for long offsets, against which I am very strongly opposed, and with such feelings I naturally discourage their use. Indeed, apart from this prejudice, I cannot see any feature of recommendation in the cross-staff except for approximation.

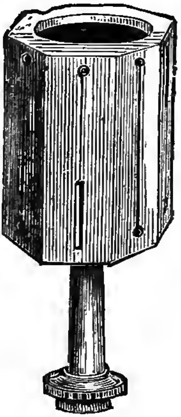


Fig. 48.

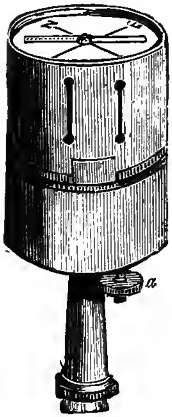


Fig. 47.

The cross-staff is made either cylindrical or octagonal in shape, about three inches in diameter (see Figs. 47 and 48) and five inches deep. It has slots placed at right angles to each other, in which are contained fine wires strained very true and vertical. In the octagonal staff there are also slots on the other four faces, which may be used for approximating an angle of 45 deg. The staff is fixed upon a rod (spiked at the end), and being placed perfectly perpendicular at a point on the line A B (Fig. 49), at which it is desired to set out a right angle, the slots *a* and *b* are adjusted so that, looking from *a* to B and back from *b* to A, the wires are coincident with the points B and A. Many cross-staves have a compass fixed at the top, as in Fig. 47, which—provided the staff is accurately adjusted in a truly vertical position on the line—may serve to take the bearing of the line with magnetic north. There is a form of cross-staff, as in Fig. 47, which is so constructed that the upper part of the cylinder may be moved round upon the lower portion with a rack and pinion movement actuated by the screw *a*. A ring on the lower member is divided into degrees and subdivisions, and, with a vernier attached to the upper cylinder, it is possible—with the greatest care—to obtain the angle of one or more points, but this can only be regarded as approximate.

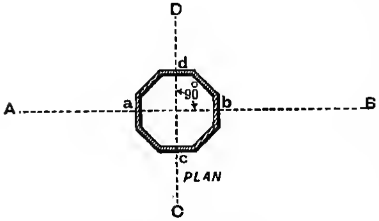


Fig. 49.—Cross-staff.

Some writers affirm that this instrument is valuable "for setting out base-lines, . . . more particularly for crossing any high ground with a sharp ridge." I can only say that I should place very little reliance on the accuracy of any survey which depended upon a cross-staff, either for the prolongation of lines across ridges or even for setting out right angles.

Optical Square.—This is at once a most accurate and useful little instrument for its purpose, but it also must be used with great caution. All appliances of this character are liable to be used to save trouble—I mean they facilitate long offsets. The optical square (Fig. 50) consists of a metal box of from 1½ to 2½ in. diameter, formed by an outer and inner tube working one within the other, so that by a slight movement right or left the slots upon the outer tube are made identical with similar slots on the inner case, but which latter are so placed in fixing the two together that although capable of a slight movement they are held in position by a screw. This enables the instrument to be protected from dust or dirt when not

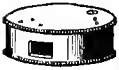


Fig. 50.

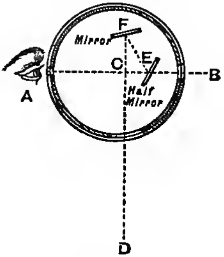


Fig. 51.

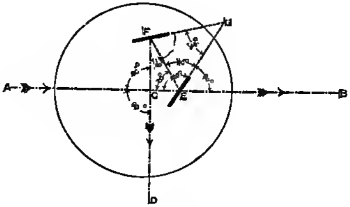


Fig. 52.

in use. Within this circular box are contained two mirrors (one of which is only half silvered, the lower portion being plain) placed at an angle of 45 deg. with each other. Referring to Figs. 51 and 52 it will be seen that the glass E is placed at an angle of 120 deg. with the line of sight or diameter of the box, and the mirror F is at 45 deg. with this. Now, by a well-known law, a ray of light in direction of A B falling on E will be reflected on to F at an angle of 60 deg. (F E C), which will be again reflected in the line F C, whereby F C is 90 deg. with A B. Thus, a person wishing to establish a point on his chain-line A B at right angles with some particular point, right or left, has simply to walk along the line in direction of B until the object at D becomes coincident with the forward station B. Thus, supposing a white flag is placed at B, Fig. 53, and another flag at some distance further ahead, say B′ (for this is most important, as will be explained hereafter), and at the point D a red flag is fixed; then, provided the observer is in absolute line with B and B′ when D appears coincident on the upper half of the mirror E, the red and white flags will be as on Fig. 54. Again, if at any point on the chain-line, as C, Fig. 53, it be necessary to establish a point at right angles, as D, instruct an assistant to move backwards and forwards until his flag is coincident with the points B and B′.

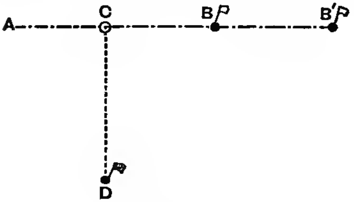


Fig. 53.

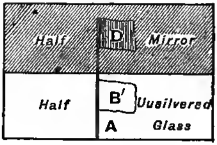


Fig. 54.

The Line Ranger.—This is a very useful little appliance for obtaining an intermediate station upon a line. It consists of two reflecting glass prisms placed one over the other, having two sides in the same plane so that the hypotenuse of the one is at right angles to that of the other. The observer holding the ranger in his hand and looking into the prisms in direction of G H (Fig. 55), if he is in a true line will see the image of a pole at B on his right hand reflected in the prism E, whilst a pole at A on his left will be reflected in the prism F, "so that when these images are in the same straight line the instrument is also exactly in the same straight line with the objects A and B."

Clinometer.—The clinometer in its primitive state was simply an appliance for ascertaining the angle of a slope with the horizon.

the most simple being a card in the form of a semicircle, divided right and left of the line of quadrant into degrees and subdivisions

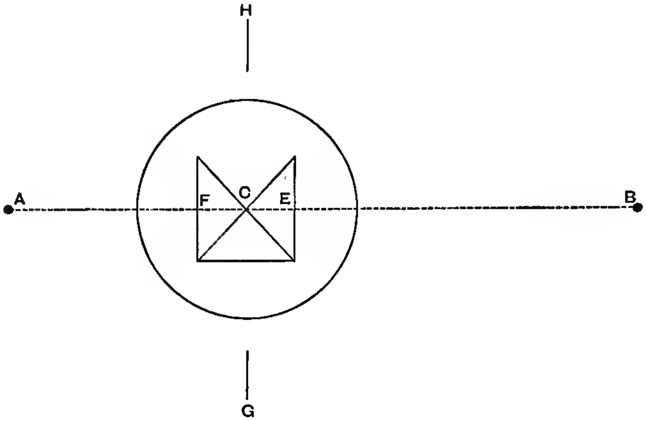


Fig. 55.

of a degree. With a small plumb-bob attached to the centre it is possible, by elevating or depressing the line of diameter, to read off the number of degrees, &c.

Merrett's Quadrant.—A modification of the quadrant was invented by the late Mr. H. S. Merrett. It is made of boxwood, having two arcs (right and left) of 90 deg., divided into degrees and half-degrees, being sufficiently clear for the purposes for which it would be required. It has two tables engraved on it, one to ascertain the height of any object, the other for shortening the hypotenusal line to the horizontal line, when required in surveying hilly districts, the same as a theodolite.

There are also angles of slopes usually adopted in railway cuttings and embankments.

It is governed in its operations by a spirit-level on the top, having two sights. The arc is divided both ways so as to enable the observer to read either for angles of acclivity or declivity. It is fixed upon a small tripod, whereby greater accuracy is obtained.

I have thought it necessary to explain briefly this instrument, but neither has it come into very general use, nor can it in fact compare with the many excellent instruments of modern days.

Clinometer Rule.—This is possibly one of the most simple and compact of the absolute instrument types. It consists (Fig. 56) of a boxwood rule in two parts with a hinged joint (like a sector), about 6 in. long when closed. In the upper arm a small spirit-level C is inserted, whilst at either end are fixed sights A and B with cross wires. Attached to the hinge is a brass quadrant F, which is divided into 90 deg. and minutes. A spirit-level D is also inserted in the upper portion of the lower arm G, whilst upon one of the sides is engraved a scale and table of hypotenusal allowances, and the more complete clinometers are accompanied by a small compass F. In using this instrument for acclivity you sight from A to B, and for declivity from B to A.

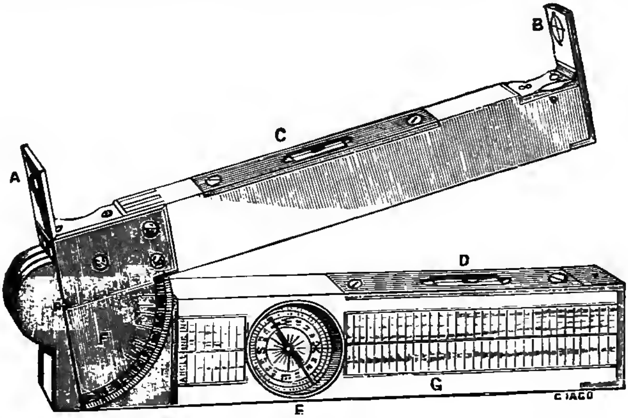


Fig. 56.

The Abney Level.—This is a reflecting level and clinometer combined, and is deservedly popular amongst engineers and

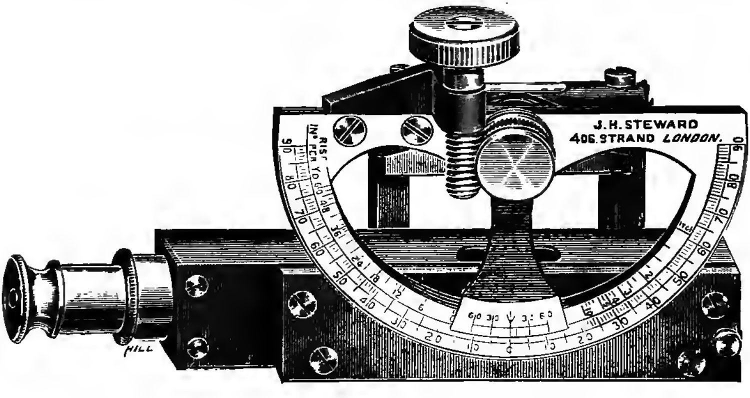


Fig. 57.

surveyors. A new form of it, introduced by Mr. Steward, is shown in the illustration (Fig. 57).

It was invented by Captain Abney, and consists of a hollow arm containing a telescope. Attached to this arm is a vertical arc, each quadrant of which is divided right and left into 60 degrees and subdivisions. The arm is of sufficiently stout metal to enable at its centre a horizontal spindle to be fixed, carrying a spirit-level, the case of which has a slot underneath, so as to expose the bubble, so that in whichever position the arm is held the bubble will be reflected on to the mirror. A vernier * fixed to the spindle and at right angles to the arm of the bubble indicates the relative angles of acclivity or declivity on the vertical arc. The instrument shown in the illustration is much more compact than the usual form, having a couple of telescopic tubes, which close up into the body of the instrument, and are drawn out when the level is to be used. Another new feature is the adjustment for moving the vernier arm and the bubble tube attached to it by means of a wormwheel fitted on the vernier arbor. This arrangement also gives room for a larger divided arc than usual.

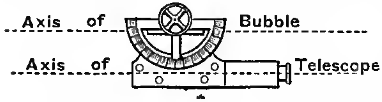


Fig. 58.

Referring to Fig. 58, it will be observed that the instrument in its entirety is in a truly horizontal position. Fig. 59 shows the instrument being used for the angle of acclivity (which in this case is 34 deg. 15 min.), and Fig. 60 that of declivity, or 19 deg. 30 min. with the horizon. Thus the level tube is always horizontal, and the arm of the vernier vertical, whilst the telescope assumes whatever angle it may be desired to observe, and the vertical arc consequently has its zero varying in position accordingly.

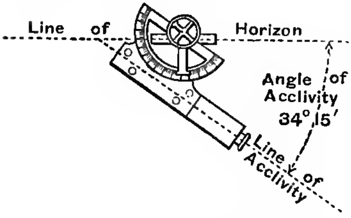


Fig. 59.

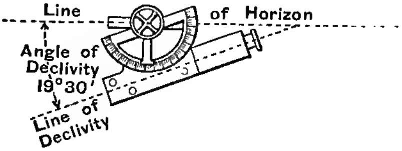


Fig. 60.

The Abney level may be made to fit on to a tripod with a ball-and-socket movement, whereby greater steadiness and consequently more accuracy may be attained.

---

* A vernier is fully described on p. 52.

Reflecting Clinometer Scale.—This (Fig. 61) is somewhat on the principle of the Abney level, and has the advantage of being half the cost. It consists of a telescope with a mirror half silvered, to reflect the bubble into the slot. The vertical arc to which the level-tube is attached rests in a triangular frame, and its outer edge is cogged, so as to be actuated by the pinion; and for some reasons this motion is preferable to that of the Abney level. The cost of this instrument is £1 1s.

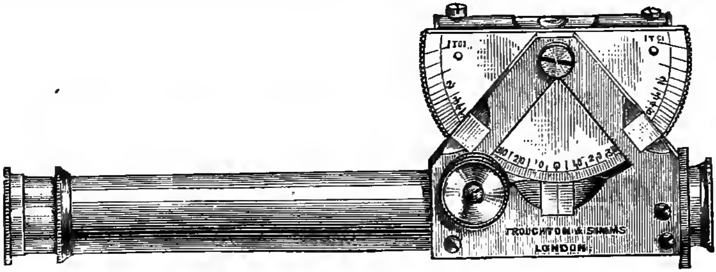


Fig. 61.

Combined Telescope Clinometer and Prismatic Compass.—This is an extremely valuable instrument, and for ordinary surveys will serve the same purpose as a theodolite. It has one great advantage, and that is, it can be attached to a light tripod, so that it may be accurately adjusted over a station.

The instrument consists of a bronze box C, about 5 in. in diameter, containing the clinometer disc F (Fig. 62) and compass card G, protected, as shown, by a pierced cover. When used as a clinometer it is fixed upon the stand in position, and by means of a clamp-screw may be arranged for observation either as compass or clinometer. In the latter case the box must be fixed perfectly vertical, so as to allow the clinometer end, which is weighted, to swing freely. The telescope B is now directed towards the object required, and having cross wires, it may be clamped at the exact point of intersection. This being done, the microscope will mark the rise or fall in inches per yard on the disc, which is so compensated that its zero shows a perfectly horizontal plane. The observation may also be taken with the slot in the prism and the wire of the vane I. The price of this instrument, packed in a mahogany case, with solid leather cover

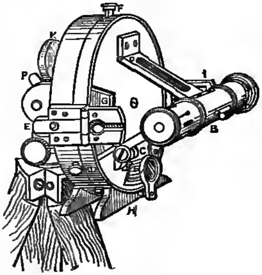


Fig. 62.

and sling, also leather-bound canvas case for tripod stand, is £14 14s.

Combined Clinometer and Prismatic Compass.—A

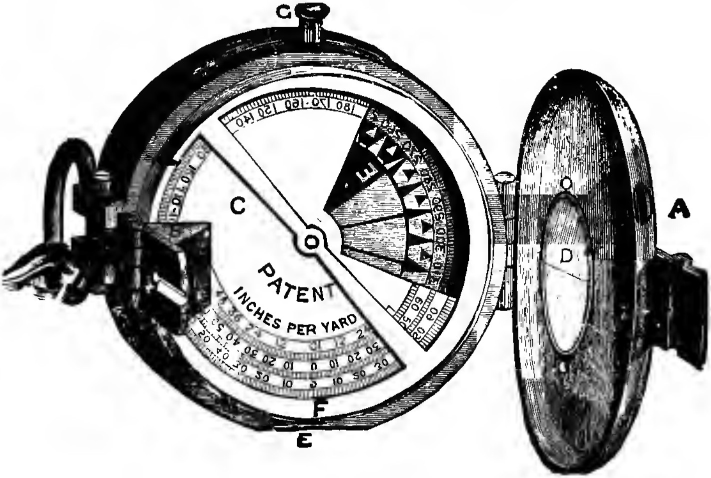


Fig. 63.

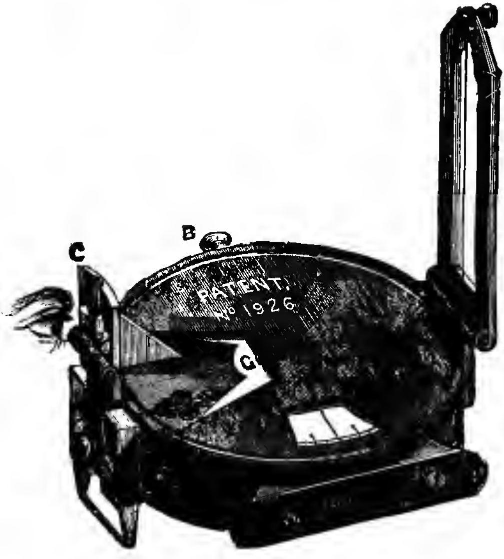


Fig. 64.

modification of the foregoing instrument, invented by Captain Barker, will be found extremely valuable for ordinary work in the field. Being of a pocket size, it is of course only a hand instrument. It has a compass card E (Fig. 63) and a clinometer disc C, with the slotted prism B and the vane D. It is illustrated in position for observing the angle of slope, but if held horizontally it can be used as a prismatic compass. When being used as a clinometer, as in

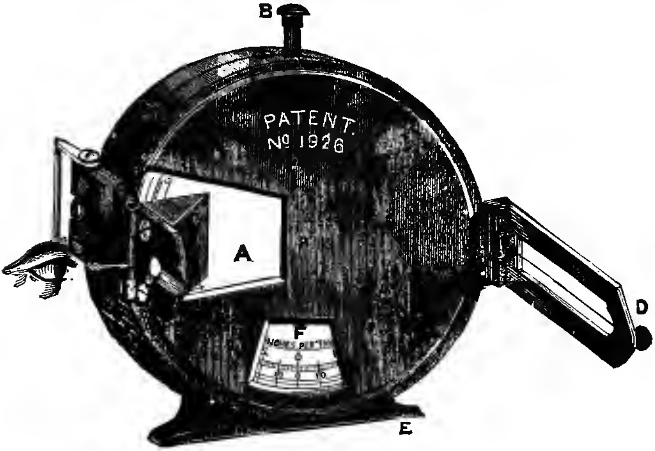


Fig. 65.

Fig. 65, the disc A records the angle of slope by means of the prism C, whilst F is a scale of rise or fall in inches per yard corresponding with the observed angle. The disc is balanced so that zero corresponds with the horizon. When it is desired to use this instrument as in Fig. 64, by pressing the knob B the disc revolves, so that the compass card will be revealed beneath the prism. The cost of this appliance is £4 4s.

There is another type of combination having the prismatic compass on one side and the clinometer disc on the other, in which case the sight-vane of the former folds over the compass. A shorter vane which folds back, being only the depth of the box, is used for vertical angles, and which, when not in use, also throws the clinometer disc out of gear. Fig. 66 illustrates this instrument when closed ready for being put into its case, where A is the prism folded

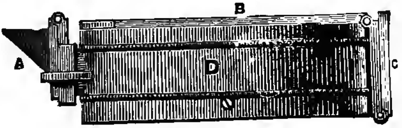


Fig. 66.

back, B the sight-vane for the compass, and folding over it, in doing which it presses a knob which throws the card off its centre,

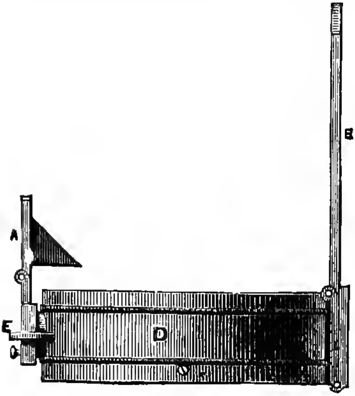


Fig. 67.

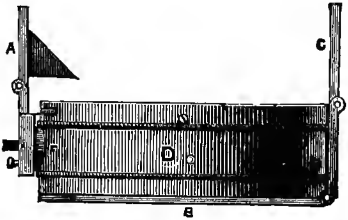


Fig. 68.

and C is the sight-vane for the clinometer, which also folds back. Fig. 67 shows the instrument open for use as a compass,

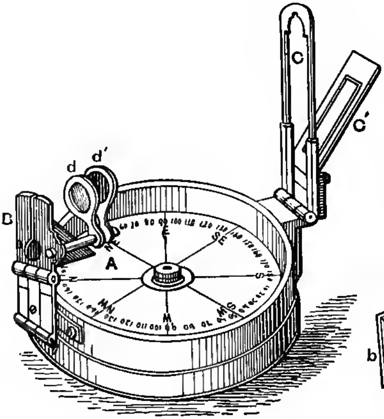


Fig. 69.

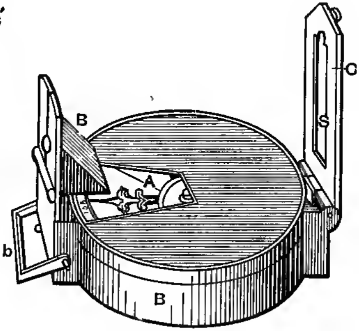


Fig. 70.

and in Fig. 68, by turning the box over and reversing the prism, it may be made to serve for clinometer observations with the arm C.

Prismatic Compass.—No surveyor should be without this instrument (Figs. 69 and 70), as apart from the fact that it is extremely useful for observing bearings, and even traversing, it is, in the absence of a theodolite, the only reliable means of determining the magnetic north in connection with a survey. It consists of a magnetic needle balanced on an agate centre or pivot, and carrying a card A, or metal ring, divided into 360 degrees and subdivisions of one-half or one-third of a degree, according to the size and manufacture of the compass. This is contained in a brass or bronze box, from 2½ inches diameter and upwards, at one end of which is a sight-vane C, and at the other is a magnifying prism B, enclosed in a metal case, having a slot for observation, so arranged that whilst the eye sights through the slot—towards the wire contained in the vane—the prism, by means of being silvered on its slope, reflects the reading on the card at the same time. When in use the prism is turned by a hinge over the card, and similarly the vane is fixed in a vertical position; but for portability, when not in use, the vane folds on to the glass of the compass, and in doing so it presses a knob, which throws the needle off the bearing to save undue wear. Whilst the prism is turned back on to the ring of the box, and is held in position by the movable strap *b*, the whole is covered with a lid (which may be attached to the bottom during use) to protect it from injury. The better kind of compasses have a permanent metal top, with a glazed aperture over the prism for taking observations. It should be stated that a knob is arranged in the ring under the vane to enable the operator to steady the needle, by pressing the card, to avoid undue swinging. The best kind of prismatic compasses are fitted with green and red glasses as at *d d'* for azimuth observations. The prismatic compass gives the bearing of a line, or in other words, the angles formed by that line and the magnetic meridian.

I have explained that the card or ring is divided into 360 deg., but whereas in ordinary cases this 360 deg. on north would point in the direction of the vane, in the case of the prismatic compass, for facility of reading the angle during observation, the order is reversed, so that the north on the card is marked 180 deg., south 360 deg., east 270 deg., and west 90 deg. By this means the 360 deg. is brought under the prism as at A in Fig. 71, so that in directing the vane towards the point from which the bearing is required, the operator is enabled to simultaneously read the angle and cut the point of observation with the vertical wire of the vane.

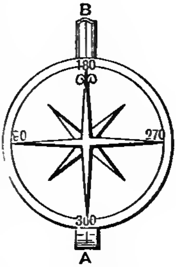


Fig. 71.

It should be observed that the prismatic compass cannot be used in places or under any circumstances where there is the slightest metallic attraction, as the needle is so sensitive that the least thing will cause a variation. Again, the compass must not be relied upon for extensive triangulation, as from local and other causes slight errors are certain to occur.

It has been argued that more accurate observation would be taken if, instead of holding the compass in the hand, it were attached to a tripod-stand; but I doubt whether the steadiness that is thereby assured compensates for the advantage of keeping the card or ring alive by the motion of the hand.

The Circumferentor.—This is an extremely valuable instrument (Fig. 72), being next to the theodolite in point of reliability, always excepting local or other attractions. Fixed upon a tripod-stand with a ball-and-socket arrangement, it may be placed over a station and so adjusted that the observation may be accurate.

It consists of a compass-box A, from 4 in. and upwards in diameter, which is divided into 360 deg., subdivided into minutes; over this is fixed the pivot carrying the needle (which, when not in use, should always be thrown off its point to preserve it), and attached to the outer case are the two sights C and B, with which the angle is observed. These sights, made of brass, have in each two slots and a circle. Alternating in each, the larger slot in the upper sight (B) contains a vertical wire, whilst the corresponding slot in C is only just wide enough to look through, the lower slot in C has the wire, and that in B is similar to that in C to look through. It will also be observed that at the top of B and bottom of C are what appear in the figure to be dots, which are really small holes, while at the bottom of B and top of C are circles with horizontal and vertical wires. These are, again alternately, for observing the intersection of the cross wires with an object back or front. When out of use these sights or arms, by hinges, fold over the glass, protecting the dial.

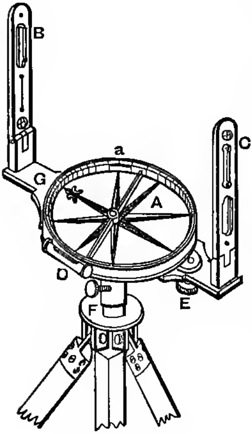


Fig. 72.

There is a spirit-level D attached to the lower part of the box for the purpose of fixing the dial perfectly horizontal, which is effected by a ball-and-socket arrangement. The dial has a socket, which fits on to the pivot of the tripod, and by means of a screw F the instrument can be clamped or fixed. The arms or sights and the rim of the circumferentor are turned round by the screw E, by which they may be brought into accurate line of sight. Thus, to take an observation, place the instrument directly over the station point by means of a plumb-bob attached to the apex of the tripod; fix the two plates by the pin underneath the compass-box, and bring the sights round, so that the needle and 360 deg. coincide. Having levelled the instrument, fix it by the screw F, then release the pin, and by the screw turn the sights until they cut the points set up; the vernier on the rim will give the angle taken from the meridian.

Preliminary Adjustment of the Theodolite.—The preliminary adjustment of the theodolite is to plant the instrument accurately over the station.

Before going into the merits of each it is necessary to explain those portions which are similar in all instruments. The stand, or tripod, is usually in the form of three legs, each of which being in the section of a V on two sides, and in the form of an arc on the other (as in Fig. 73), when not in use, they may be closed for convenience of transit. These legs are shod with an iron spike to facilitate their being firmly held in the ground, whilst at the top is a segmental plate carrying the legs which fit into two corresponding legs in the stock, thus forming a hinged joint. This joint is generally formed by a large surface-bearing pin, which is tapped at the end to receive a slot-headed tightening screw. In spite of excellence of manufacture, this joint is found to be a constant source of trouble from the fact that in process of time its screw gets loose, and the steadiness of the instrument is impaired; but by a very ingenious and valuable arrangement, Messrs. Troughton and Simms have substituted a double lug on the legs and one of extra thickness on the stock, and instead of the slot-headed tightening screw the joint is formed by means of a bolt and nut, to tighten which the cap-piece has its upper portion made in the form of a box-spanner. For ordinary purposes, equally as in levelling, this form of tripod-stand is the best and most convenient; while for special work there can be no doubt that the framed stand—similar to those used for cameras, but more elaborate—is the best, for reasons which will be explained presently.

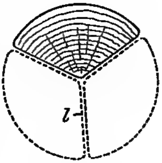


Fig. 73.

Adjusting the Vertical Axis.—This is done by means of a plumb-bob suspended from a small hook held in the centre of the stock-head. It requires the greatest care and patience to do this, as much depends upon the manner in which the tripod has been fixed. Approximately, bring the plumb-bob over the nail or cross-cut of the station, and gently pressing each leg into the ground, one by one, the point of the bob may be made coincident with the centre of the station, as is illustrated by Fig. 74. The cord carrying the plumb-bob should be strong, and as the distance from the stock-head to the ground is constantly varying according to

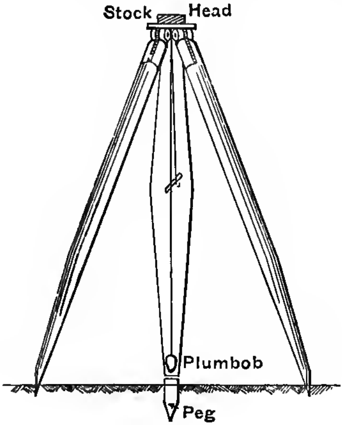


Fig. 74.

circumstances, it should be elongated or shortened by means of a runner.

Centering Plates.—There have been several improvements in the theodolite with a view to greater accuracy in adjusting the

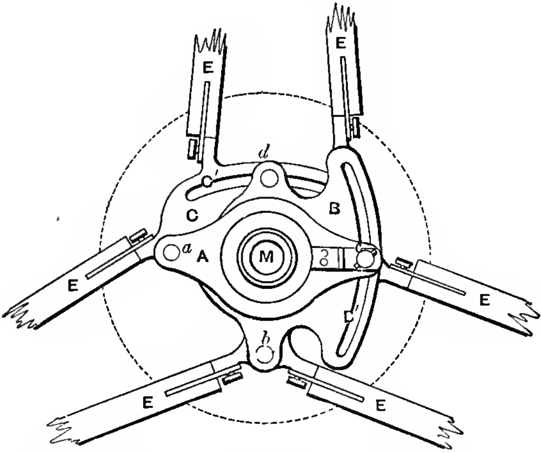


Fig. 75.

vertical axis of the instrument over a station, by means of centering-stands, whereby the plumb-bob may be adjusted to a nicety. One of these arrangements, by Messrs. Troughton and Simms, is illustrated in Figs. 75 and 76, and is only applicable to the framed stand. It consists of three plates, A, B, and C, which work one upon the other as follows: the plate A, to which the theodolite is attached, works upon a pivot at *a*, fixed to the underneath plate B, by which means the pin *c* (having a clamping arrangement) may work in the segmental slot B′. The middle plate B, to which A is pivoted, also works on a pivot in C at *b*, and traverses the segmental slot C′, and can therein be clamped by the thumb-screw *d* underneath the plate, so that the upper plate A can be made to traverse in all directions a space of about 3 in., thus admitting of the plumb-bob being adjusted to a hair's-breadth. Figs. 77 (A), 78 (B), 79 (C) are sketches of each plate separately in relative positions, the plumb-bob being held by a hook at M (Figs. 75 and 77), and attached to the plate A.

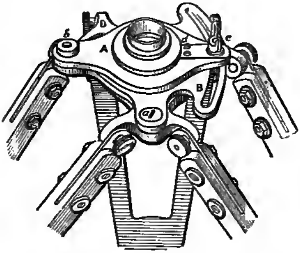


Fig. 76.

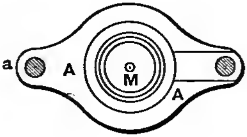


Fig. 77.

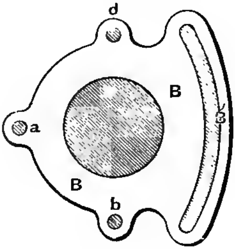


Fig. 78.

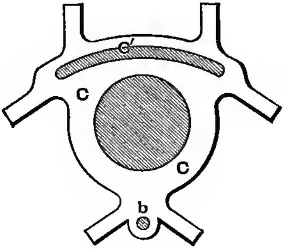


Fig. 79.

A modification of this arrangement has been devised by the same eminent firm, adapted to the ordinary tripod-stand, and is illustrated by the sketches in Figs. 80, 81, 82, whereby two plates, one upon the other, are made to take any lateral movement to the extent of about an inch by the ingenious method of making the upper plate move in a double slot J J (Fig. 81), at right angles to slots K K in the underneath plate, so that a loose pin L (Fig. 82) enables one plate to revolve upon the other nearly the length of the slot. The plate D (Fig. 80) is made to screw on to the box C, and is held firm by a screw *d*. The instrument may be thus moved in any position to adjust its vertical axis over a station, and when accomplished, the centering-plates may be firmly secured by means of the clamp-screw F, which is connected with the wedge E, and by drawing it inwards jambs the two plates so that they cannot move.

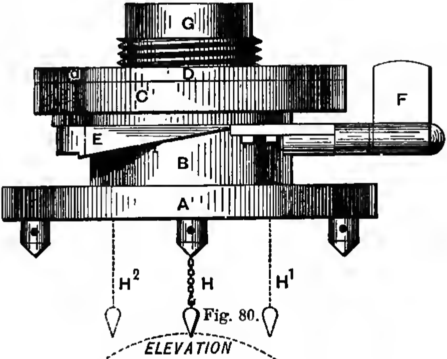


Fig. 80.

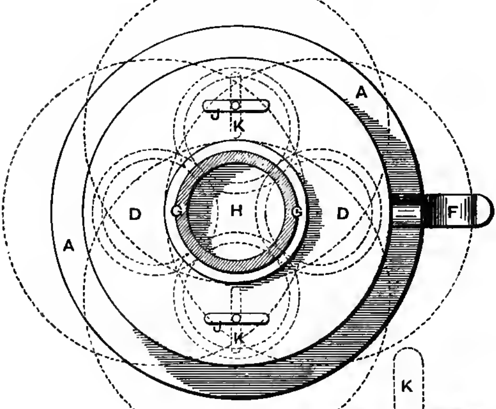


Fig. 81.

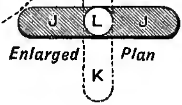


Fig. 82.

The possible movement of the centering-plate is illustrated by the dotted circles in Fig. 81, and the range of movement of the plumb-bob is shown by the dotted lines H^1 and H^2, Fig. 80. It should also be stated that the sketch shows an arrangement for utilising this contrivance as a wall-stand, the lower plate A (Fig. 80) having four spikes provided for that purpose, or having a female screw within, A and B. It can be screwed on to the tripod-head.

Above the joints of the stock-head is the screw upon which the instrument is held; and when not in use this screw is protected by a hat-shaped cap, but which has recently been improved by having a box-spanner attached. Fig. 83 shows the stock-head complete, Fig. 84 is the ordinary cap, and Fig. 85 is the improved cap and spanner.

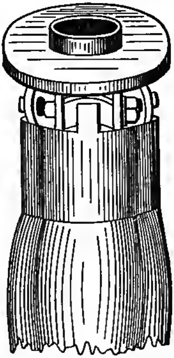


Fig. 83.

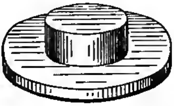


Fig. 84.

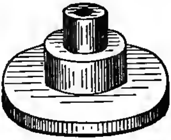


Fig. 85.

We now come to the instrument proper, which consists of three parts: the parallel plates, the horizontal and the vertical limbs.

Parallel Plates.—The parallel plates are illustrated in Figs. 86 and 87, and consist of two circular plates, kept a certain distance apart by a ball-and-socket (which forms part of the upper portion of the instrument), and four screws, B^1, B^2, B^3, B^4, placed at right

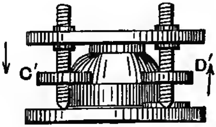


Fig. 86.

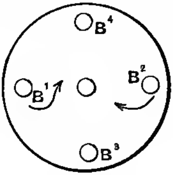


Fig. 87.

angles to each other, and called the parallel screws. The upper plate is pierced with four holes, which are tapped with a female screw, in which a screw having in its centre a milled head works, but whose lower extremity rests and works upon the lower plate; and in order to prevent the upper plate revolving there is a **U**-shaped guard round one of the screws.

Parallel Plate Screws.—The action of the parallel plates is regulated by screwing and unscrewing each pair of opposite screws. Thus, if the right end of the plate, as D^1, Fig. 86, is required to be raised, then the left end C^1 must be depressed, which is effected simultaneously by turning the screws B^2 and B^1, Fig. 87, inwards, whereby B^2 is elongated and B^1 shortened. If, on the other hand, it is desired to elevate at C' and depress at D', then these screws must be turned outwards, whereby B^1 is elongated and B^2 shortened. Similarly, B^3 and B^4 have to be dealt with. Fig. 88 illustrates how the screws are manipulated.

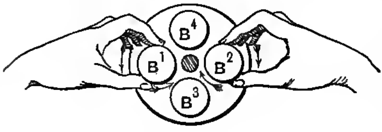


Fig. 88.

Ball-and-Socket Arrangement.—Referring to the ball-and-socket arrangement, it is necessary here to explain that it forms one of the most important parts of the theodolite. The lower parallel plate has a dome-shaped socket very accurately turned to receive the semi-spherical lower portion of the body-piece. The upper parallel plate has also a socket, upon which rests the shoulder of the body-piece; thus the four parallel screws serve to keep the upper and lower plates apart; and according to the elongation or shortening of each pair, so the ball-and-socket arrangement admits the elevation or depression of the upper plate as required. The object of this is to maintain the instrument in a truly horizontal position, as will be presently explained; but having by means of the four screws adjusted it level, it is necessary that they should all firmly bite the lower plate, *but not too much so*, otherwise the threads of the screws will be injured, and indentations will appear on the plate.

Now the body-piece before referred to is hollow, but its interior is in the form of an inverted cone, within which works a solid spindle of similar shape, both being so accurately ground to fit that the axes of the two cones may be parallel.

The Limb or Lower Plate.—The body-piece supports a circular plate (whose diameter distinguishes the particular size of the instrument), called the lower plate or limb. This plate is bevelled to an angle of about 60 deg., and has besides a graduated scale (called the primary scale) divided into 360 deg. and subdivisions, marked upon silver. This scale reads continuously from

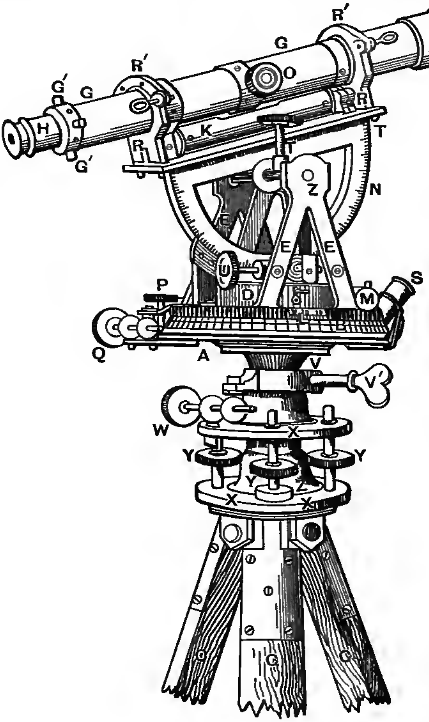


Fig. 89.

right to left. Fig. 89 is an elevation of an ordinary **Y** theodolite, and shows the limb A and also on the plan (Fig. 90).

The Upper or Vernier Plate.—Working upon the limb, supported by the internal cone or vertical axis, is another circular plate of less diameter (by the width of the chamfered edge), called the vernier plate, B, which, unlike the limb, is only bevelled in two places, 180 deg. apart. At these two points, L L, Fig. 90, are what are called the verniers, for the purpose of minutely reading the subdivisions of the primary scale. Now the vernier plate is so constructed that it will freely move upon the limb, being held thereto by a washer and set-screw passing through the lower portion of the body-piece into the vertical axis of the vernier plate, so that, unless prevented, both plates can work concentrically, one upon the other, and the lower one upon the socket of the upper parallel plate. But for the purpose of taking

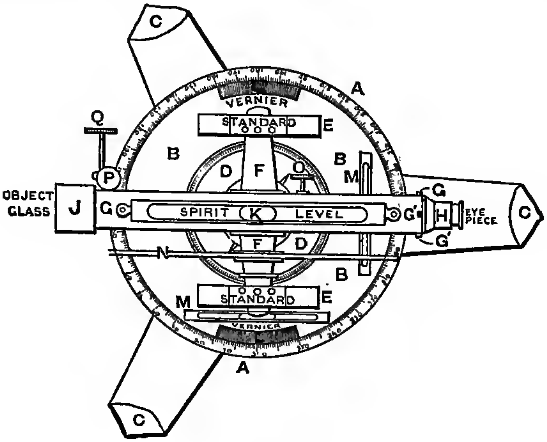


Fig. 90.

an angle, it is necessary to fasten each plate according to its relative position.

Clamps.—It will be noticed in Fig. 89 that, underneath the limb, there is a loose collar marked V, terminating in two lugs, through which a screw V′ passes, which being tightened (Fig. 91) clutches round the body-piece and holds it tightly. This is called the clamping arrangement, as V′ is the clamp-screw.

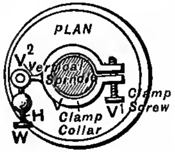


Fig. 91.

Tangent or Slow-motion Screw.—Now we have seen that this body-piece only rests upon and works in the socket of the upper parallel plate, and the mere fact of clamping it, unless it were prevented from moving on this plate, would be of no avail. Thus a bracket, also attached to the collar, is connected with a sphere moving on a pivot from the bracket, and pierced to receive a screw W (Fig. 89), which also pierces a sphere and works tangentially to the vertical axis. This last sphere revolves on a spindle attached to the upper parallel plate. By this means a gradual motion is imparted to the limb, which admits of the most accurate adjustment. This latter arrangement is called the tangent or slow-motion movement, and the screw W is the tangent-screw. For the purpose of fixing and regulating the vernier-plate there is a clamping arrangement P (Figs. 89 and 90) which clutches the upper plate to the limb, whilst a gradual motion of the upper upon the lower plate is effected by means of the slow-motion screw Q, which works tangentially through pierced spheres attached equally to the one and the other.

The foregoing is a brief description of the ordinary clamping and slow-motion process in nearly all theodolites.

Troughton's Clamp and Tangent Arrangement.—It is necessary, however, to mention that a very considerable improvement in this system has been introduced by Messrs. Troughton and Simms, and by their courtesy I am able to give an illustration in Fig. 92. The vertical axis A works within a collar B, having two arms. This collar has a segmental recess on the left side to secure the axis when pressed by the tumbling-piece C, which is actuated by the clamp-screw D. The slow motion is imparted by means of the spring F on the one side and the tangent-screw G on the other, and pressing against a lug E fastened to the plate below, so that as the screw is turned the spring recedes or follows it, and the most minute movement may thus be effected. It occurred to me that this spring might in time lose its power, but the makers assure me that with ordinary fair usage this is not likely to happen. Such being so, there can be no doubt that it is an improvement upon the other method.

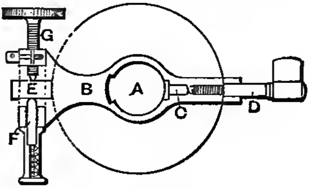


Fig. 92.

Levelling the Plates.—I have spoken about the necessity of having the instrument perfectly horizontal, to ensure which, there are placed upon the vernier plate, at right angles to each other, two spirit-levels (J and K, Fig. 93), each of which is parallel to one pair of screws, thus: J with B^1 and B^2, and K with B^3 and B^4; and by the manipulation of these screws the bubbles of J and K should be brought exactly to the centre of their run; and this being so, if the instrument is in good order and adjustment, then the vernier plate and limb are parallel with the horizon. This is one of the most important matters to be attended to, as unless the instrument is perfectly level the value of the observation is impaired.

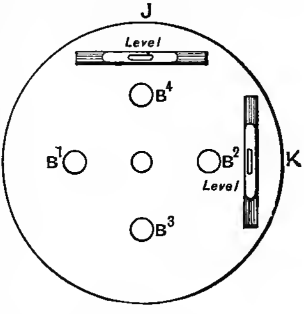


Fig. 93.

Vertical Arc.—Proceeding with the consideration of the component parts of the theodolite, we now come to the vertical arc or circle, which in the former case consists of a semicircle of metal, divided on one side each way from 0 deg. to 90 deg., and subdivisions, whilst the other side is divided to show the hypotenusal allowance in links per chain. Connected with this semicircle are trunnions, which work in journals in the head of the bearers or "A" frames E E (Figs. 89, 90, and 94); and on the top is a strong bar, carrying the supports or Y's, R′ R′, in which rests the telescope. It is from these supports, from their resemblance to the letter Y, that this particular type of instrument is so called. The telescope G is held in the Y's (Fig. 95) by clips, working upon a hinge on one side, and fastened on the other by a pin passing through eyes in the jaw of the Y and the clip. This arrangement has been superseded in the most modern theodolite by a spring, which prevents any possibility of the telescope falling out of the supports by neglecting to fasten the pin in the clips.

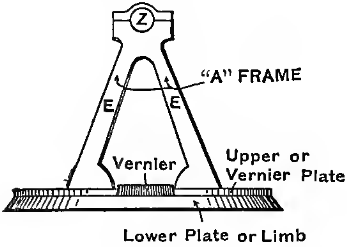


Fig. 94.

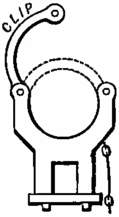


Fig. 95.

Compass.—Beneath the telescope and between the "A" frames a compass-box D is fixed upon the vernier plate, containing a silver ring, graduated from 0 deg. to 360 deg. The needle is supported upon an agate centre, and has a lever connected with it, by which, when not in use, it may be thrown off its centre, and thus save undue wear. Attached to the compass-box is a vernier for reading the minute subdivisions on the vertical arc, which is clamped by the screw T on the top of the "A" frame, whilst slow motion is effected by the tangent screw U (Fig. 89).

Telescope.—The telescope G consists of two tubes, one sliding within the other. The outer tube has at its further end J (Fig. 96) the "object-glass," which forms at its focus an inverted image of the object looked at. This is protected by a sliding cylinder J, which pulls out to shield the object-glass from the sun and weather, and at the end of it is a movable disc to close it up when not in use (see sketch, Fig. 97). The inner tube has at its nearer end

H a combination of glasses called the eye-piece, which magnifies the inverted image. By moving the inner tube inwards and outwards, with the rack and pinion contained within it, the milled-head O working a screw through a collar on the larger tube, the foci of the object-glass and eye-piece are adjusted till they coincide, which is known by the distinct and steady appearance of the image. Upon the outer tube are also collars *a a*, very accurately turned, to fit into the Y's of the "A" frame.

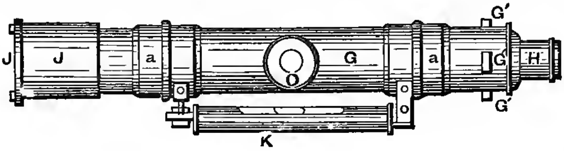


Fig. 96.

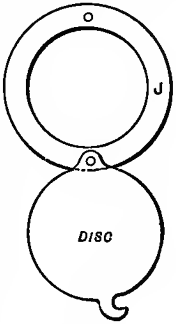


Fig. 97.

The Diaphragm.—At the common focus of the object-glass and eye-piece where the inverted image is seen, is the diaphragm or partition at G′ G′ G′. The diaphragm consists of a ring of metal (Fig. 98) held within the telescope by means of four capstan screws placed at right angles to each other. The screws work easily through holes in the telescope, but the threads actuate the diaphragm ring, so that it may be brought vertically or horizontally nearer the sides of the telescope, by screwing or unscrewing. Across this diaphragm or partition are usually three spider's webs, or equally very fine platinum wires (see Fig. 99), one horizontal, A B, and the other two, C D, E F, deviating slightly to opposite sides of a vertical plane. The point G, where these wires intersect, should be exactly in the axis or

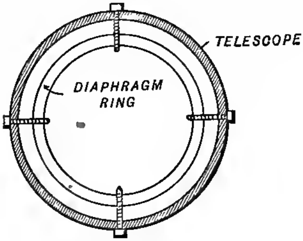


Fig. 98.

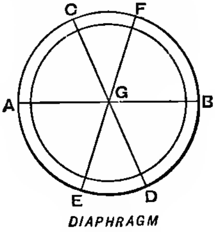


Fig. 99.

line of collimation of the telescope. In some theodolites there are only two wires, one vertical and the other horizontal.

Attached to the telescope underneath (in some cases above) is a spirit-level K (Fig. 96), which by screws may be set exactly parallel to the line of collimation; so that when the air bubble is in the centre of the level the telescope is horizontal.

The Vernier.—The vernier, in its ordinary sense, is a contrivance wherewith the intervals between the divisions on the primary scale may be accurately measured. It is a scale whose length is generally one less than a certain number on the primary scale, so that, supposing the lower plate is divided into degrees and half-degrees, if we take 29 of the subdivisions (or 14 deg. 30 min.*) and divide this length into one more or less parts than those of the primary scale, whose length regulates that of the vernier, we shall have a means of determining the actual number of minutes which intervene between the subdivisions.

It is customary to divide the vernier into thirty equal parts, so that it has thirty spaces to the twenty-nine subdivisions on the limb.

For greater minuteness of observation some modern theodolites are divided into thirds and fourths as well as into half-degrees, in which cases the verniers are divided into twenty and fifteen parts respectively, so as to accurately record the intervals between the subdivisions.

In consequence of the limb and vernier being circular in shape, it is found more easy to illustrate the relationship of the latter to the former by straight line, and Figs. 100, 101, 102 will serve to do so.

Fig. 100 shows a portion of the primary scale drawn straight from 45 deg. to 72 deg., and from 50 deg. to 64 deg. 30 min. I have marked the 29 half-degrees as the length of the vernier. Now, taking this length and dividing it afresh into thirty equal parts, it will be seen by Fig. 101 that, whereas the vernier scale commences at 50 deg. and terminates exactly at 64 deg. 30 min., so that the commencement and termination are coincident with the division 50 deg. and 64 deg. 30 min. on the lower scale, yet not one of the divisions of the vernier intermediate between its commencement and termination will cut any one of the points in the lower scale between 50 deg. and 64 deg. 30 min. If the student can once grasp this fact, then the difficulty of the vernier is simplified.

Now, if the foregoing argument be proven, it is easy to understand that once the vernier moves from 50 deg. it is possible for any *one* of its divisions to intersect any one of the divisions and subdivisions of the lower scale, but only *one* at a time.

---

* The degree is shown by a circle thus "°," minutes by one dash thus "′," and seconds by two dashes thus "″."

As an illustration, the first division of the vernier may be in line with 50 deg. 30 min., and such being the case, the other twenty-nine divisions would not be coincident. This then would show the angle to be 50 deg. 1 min. Again, the tenth division may be coincident with 55 deg. This shows that ten minutes more than the 50 deg. or commencement have been recorded, in other words, 50 deg. 10 min. Further, if the twentieth division on the upper scale is coincident with any division or subdivision on the lower one, it must of necessity be at 60 deg., consequently the reading of the vernier is 50 deg. 20 min. And lastly, if the thirtieth division or end of the upper scale is coincident with one

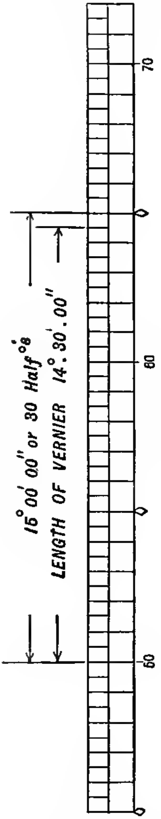


Fig. 100.

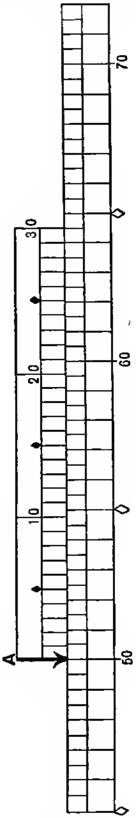


Fig. 101.

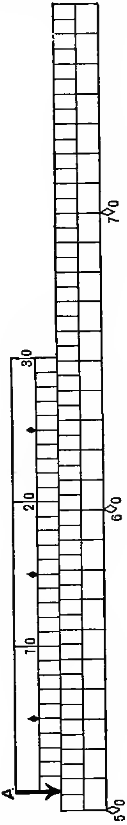


Fig. 102.

of the divisions or subdivisions of the lower one, it must be at 65 deg., and thus, thirty of the divisions in the upper scale having traversed from left to right, the arrow A (Fig. 101) will be coincident with the subdivisions between 50 deg. and 51 deg., or at 50 deg. 30 min. So we see, that even if each of the thirty divisions of the upper scale be consecutively coincident with any division or subdivision of the lower one, at the end we have only moved one half-degree in a direction towards the right.

Now, supposing it is discovered by aid of the microscope that the arrow A (Fig. 102) has passed 50 deg. 30 min., common sense will tell that the first half-degree in the lower plate has been passed, and it is desired to ascertain how many of the minutes in the second half-degree are recorded by the vernier.

In this case (Fig. 102) it will be seen that the seventh division of the upper scale is coincident with 54 deg., and seeing that the arrow A has passed the first half-degree beyond 50 deg., then the reading will be 50 deg. 30 min. + 7 min. = 50 deg. 37 min., and supposing the thirtieth division of the vernier was coincident with any in the lower scale, it must be that at 65 deg. 30 min. when the arrow A will have reached the full length of the first degree past 50 deg. or 51 deg.

At the risk of being thought verbose upon this subject, I have endeavoured to make the vernier appear as clear as possible. The foregoing remarks apply to those theodolites whose limbs are only divided into degrees and half-degrees; but in the larger instruments the degrees are divided into third parts of twenty minutes each. Suppose, for example, the limb is so divided, and that it is to be subdivided by a vernier to third parts of a degree or 20 min., each subdivision being one-sixtieth part of the primary division, the length of the vernier will be 60 − 1 = 59 divisions of the primary scale; and it will be divided into sixty equal parts, each equal to 59-60ths of a division of the primary scale. To make this more simple, if we take twenty of the divisions and subdivisions of the lower scale and deduct one from that length, then by dividing this length of nineteen parts of the lower scale into twenty we shall have a vernier which will exceed each single minute of the first third of a degree, as each one becomes coincident with one of the divisions or subdivisions of the primary scale.

The cost of "Y" theodolites is as follows: four-inch, £21 to £23 10s.; five-inch, £25 to £26; six-inch, £28 to £30.

Transit Theodolite.—This (Fig. 103) is unquestionably a more reliable instrument than the cradle (Fig 89), although objections against it have been raised, principally on account of the increased height of the standards. I can only say that I would never use any other where accuracy and facility of work are important. One of its greatest advantages is that by reason of the standards being made higher, the telescope is free to revolve on its axis, and we thus save the troublesome work of taking it out of the Y's, as is the case with the cradle, so that in prolonging a line from A to B (the instrument being at B) on to D it is only necessary to turn the telescope over, and, providing the instrument is in adjustment, you obtain a more accurate result in much less time. The transit, so far as the clamping and slow-motion screws of the lower and upper plates and the vertical circle (not an arc or semicircle), is similar to the cradle theodolite. The vertical circle, like the horizontal limb, is divided into 360 degrees and subdivisions, and has two microscopes in length of its diameter, which revolve round the horizontal axis. For special purposes it is fitted with a supplementary level X (Fig. 104) fixed by standards with jaws on to the horizontal spindle, this being an additional safeguard against the upper portion of the instrument being out of the horizontal. The larger instruments, such as are chiefly used in constructive works, are fitted with a small spirit lantern, resting on a bracket attached to one of the A frames. This is to throw rays of light into the telescope, through a crystal let into the arc, when observations are required to be

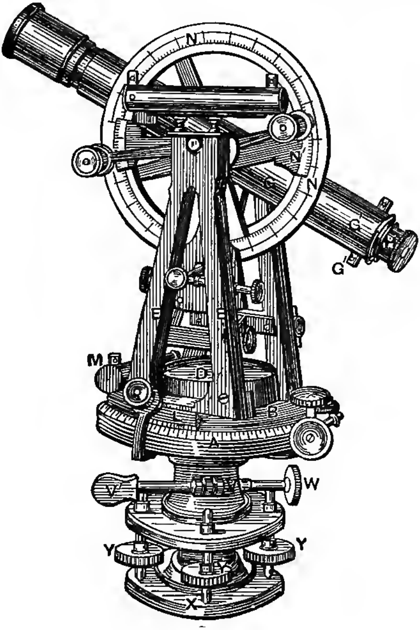


Fig. 103.

taken in the dark. My own experience has been that such appendages are more trouble than they are worth, and the extra money would be better spent upon strengthening the working parts of the instrument. If it is required to work in the dark you can have a man with a lamp against the instrument, which answers the same and a better purpose, for if you are taking angles you can also use this lamp to read the vernier.

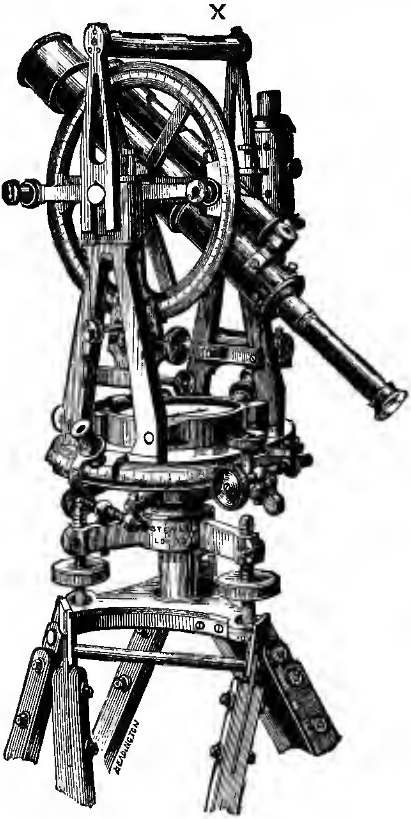


Fig. 104.

Triangular Plate.—Figs. 104 and 105 show the tripod to be surmounted by a triangular plate, in which only three levelling screws are used, the parallel plates being dispensed with. The lower part of the screw has a shoulder which fits into a pearshaped hollow slot. The advantage claimed for this method is that the instrument can be levelled with one hand only, whereas with four screws both hands are required. But I must confess I have a strong prejudice in favour of the old system, and most transit theodolites are made with parallel plates and four screws.

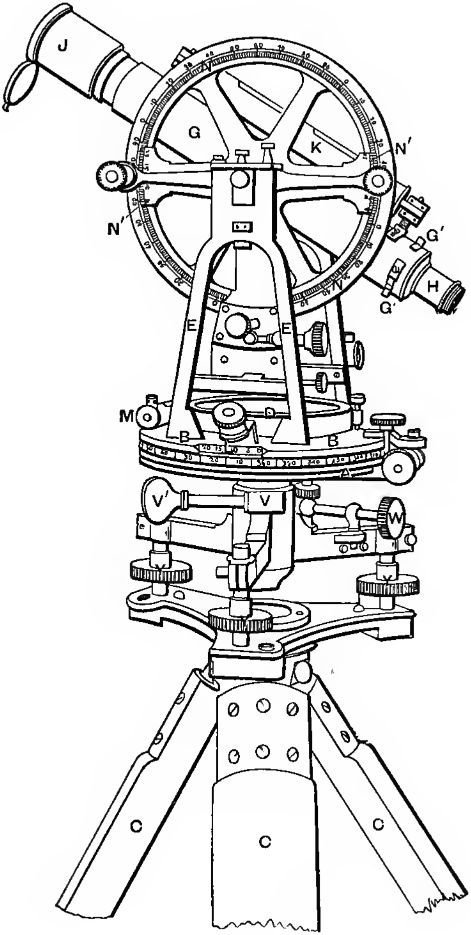


Fig. 105.

The cost of transit theodolites is as follows: three inch, £18 to

£25; four-inch, £21 to £30; five-inch, £26 10s. to £45; six inch, £28 to £50; seven-inch, £45 to £66 10s.; eight-inch £50 to £85.

Everest Theodolite.—So called after the name of its designer, the late Sir George Everest, of the Indian Trigonometrical Survey. The chief efficacy of this beautiful instrument (Figs. 106 and 107) is that the limb may be made of much greater diameter, and consequently there is scope for greater detail in the working parts and a more powerful telescope.

There is only one horizontal plate divided into 360 deg., and instead of an upper plate three arms radiate from the vertical axis, thus Y Y′ being the verniers and the arm which clamps this arrangement to the limb *a*, having also a slow-motion or tangent screw with which to manipulate it. The standard is much stronger than in other theodolites, and, whilst the telescope is mounted somewhat in the same manner as the transit, yet it cannot revolve upon its axis. Instead of having a vertical circle the Everest has two arcs, M M, with an arm fixed to the telescope, using the same axis and consequently travelling with it, at either end of which are the verniers.

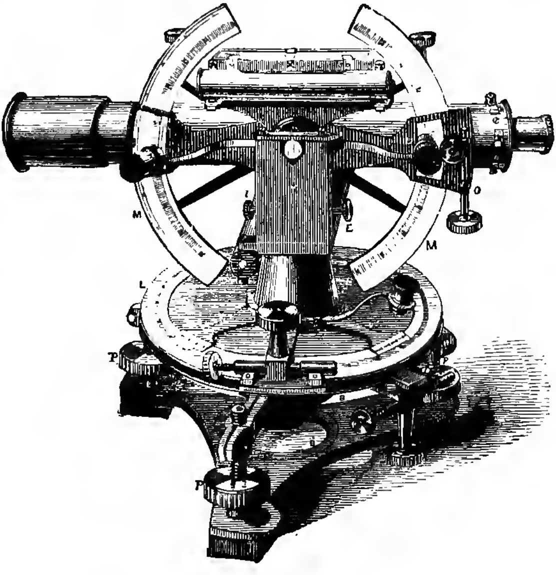


Fig. 106.

The Everest, so far as the arrangement for levelling is concerned, is similar to that already described, having only three screws, and no ball and socket, and when not required to be used upon a tripod is so constructed that it may rest upon a wall or any flat surface.

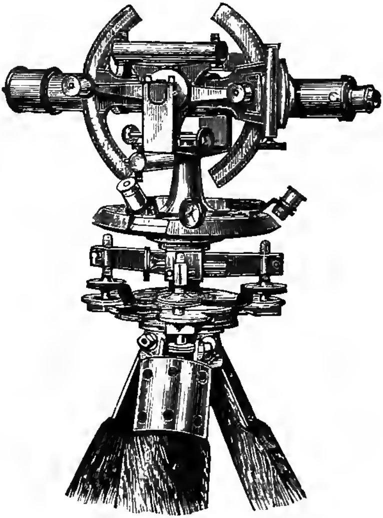


Fig. 107.

The cost of an Everest theodolite is as follows, including case and tripod-stand: 4-inch, £22; 5-inch, £24 to £30; 6-inch, £27 to £35; 7-inch, £36 to £42; 8-inch, £50; and 10-inch, £60.

Box Sextant.—This is an instrument, without which no surveyor should go into the field. It may be made to serve the purpose of an optical square. I have had an opportunity of testing its merits, as some years ago I had to make a survey of a large portion of the town of Sunderland for Parliamentary purposes; and for a length of seven miles, when a theodolite would have been impossible in consequence of the heavy traffic in the principal streets, I found this little instrument invaluable, and the results highly satisfactory.

The box sextant is about 3 in. in diameter and 1½ in. deep, and has a lid which completely covers it when not in use, but which can be screwed on to the bottom, and serves as a handle when taking observations. Fig. 108 is a view showing the chief features of the instrument, and also gives a fair idea of its internal arrangements. A graduated scale F from 0 deg. to 140 deg. with subdivisions, is engraved on a silver arc, and along this moves the vernier attached to the index arm E, to which is fixed a mirror C. This arm is moved by a milled-head screw acting upon a rack and pinion within the box. In the line of sight, but fixed, is another mirror called the horizon glass D, the upper part of which only is silvered, the lower and transparent portion being opposite the opening. This glass is fixed perpendicular to the plane of the instrument. These two mirrors, when the vernier is adjusted to zero, should be parallel.

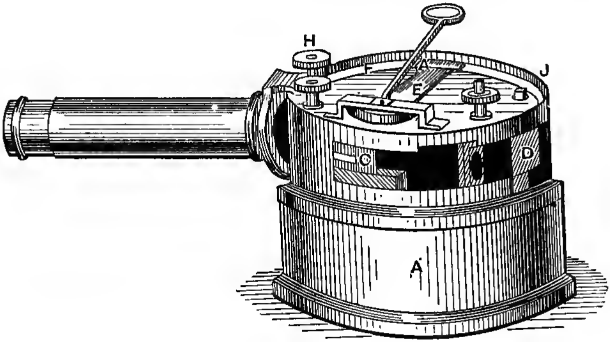


Fig. 108.

There are two levers connected with coloured glasses, which may be interposed when solar observations are taken, but when not required can be depressed into the box. Many sextants are provided with a telescope, which can either fit into a socket within the instrument, to be pulled out when wanted, or attached at the top by means of a screw as H in Fig. 108. But for general use the naked eye is quite sufficient, under which circumstances a segmental plate with a hole pierced in the direct line of vision is made to take the place of the telescope aperture by slide.

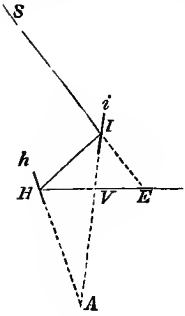


Fig. 109.

The principle upon which the sextant acts is as follows: "When a ray of light, proceeding in a plane at right angles to each of the two plane mirrors, which are inclined to each other at any angle whatever, is successively reflected at the plane surfaces of each of the mirrors, the total deviation of the ray is double the angle of inclination of the mirrors." For, let I *i* and H *h* (Fig. 109) represent sections of the two mirrors made by the plane of incidence at right angles to each of them, and let S I represent the course of the incident ray, then the ray S I is reflected into the direction I H, making with I *i* the angle H I A equal to the angle S I *i*, and is again reflected at H into the direction H E, making the angle E H A equal to the angle I H *h*. Now the angle A H V being equal to the exterior angle I H *h* is also equal to the two interior angles H I A and H A I; and because the angles A V H and I V E are equal, and that the three angles of every triangle are equal to two right angles, therefore the two angles V I E and S E H are together equal to the two angles A H V and H A I, and therefore to the angle H I A and twice the angle H A I (since A H V has been proved to be equal to H I A and H A I); but V I E, being equal to the vertical angle S I *i*, is also equal to the angle H I A, therefore taking away these equals the remainder of the angle S E H is equal to the remainder, twice the angle H A I.—Q.E.D.

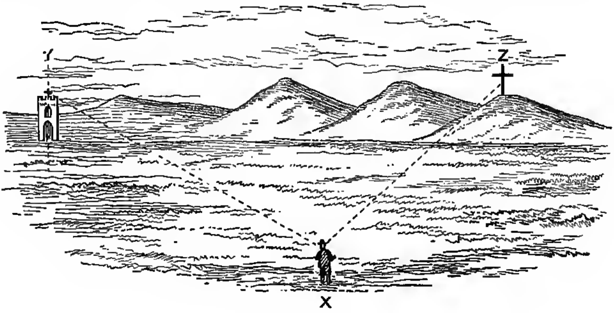


Fig. 110.

To use the sextant, it should be held up to the eye by the right hand, so that (Fig. 110) the line of sight is in the direction of the the tower, the operator standing exactly over the station X, and the vertical axis of the instrument is directly over its centre. With the left hand the milled-headed screw is manipulated so that the index mirror, being gradually turned in the direction of Z, shall reflect the image of the cross at this point, so that its centre is coincident with that of the tower Y, as in Figs. 111. Thus the vernier will record the number of degrees and subdivisions contained in the angle Y X Z.

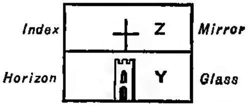


Fig. 111.

If the instrument, having been set at zero, does not show the object to which it may be directed to be exactly in the same vertical plane with the horizon and index glasses, it must be adjusted by a key being applied to the key-hole at J and turned right or left until the reflected images coincide exactly.

The necessary rules to be observed with the adjustment of the sextant are :—

1st. That the two mirrors are parallel to each other when the zero of the vernier coincides with that of the graduated arc.

2nd. That the horizon glass is perpendicular to the plane of the instrument.

To correct this latter (*i.e.* the perpendicularity of the horizon glass to the plane of the instrument) it is necessary to observe whether the reflected and the direct images of the distant horizon appear as *one*. If two horizons appear we apply the key at L and turn it until they agree. Figs. 112 and 113 illustrate the manner in which this instrument is held and manipulated when taking an observation.

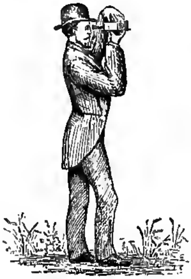


Fig. 112.

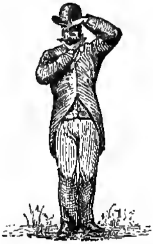


Fig. 113.

The cost of a box sextant, with telescope and large mirror and sunshades, complete in a sling case, is from £3 10s. to £6.

Hughes's Improved Double Sextant.—This is an instrument which for some reasons may be said to almost supersede the box-sextant, having the advantage of measuring angles nearly double the arc which can be measured by the ordinary sextant. It consists of a five-inch or six-inch circle, with two index glasses mounted in the centre on two index arms with verniers, one measuring the angle to the right and the other to the left. The horizon glass is silvered top and bottom, with a narrow slit in the centre. The centre object is observed with a telescope through this, and the other two objects, to the right and left respectively, reflected by the index glasses into the silvered portions of the horizon glass; the three objects being in contact, the observed angles are read off with the verniers. The adjustments of the index and horizon glasses are made in the regular manner, the upper index glass being adjusted last.

This instrument, as is shown by the accompanying sketch, has a handle by which it is held in the right hand, whilst the index arms are manipulated with the left. It may also be made to fit on to a tripod-stand, similar to that described for the clinometer.

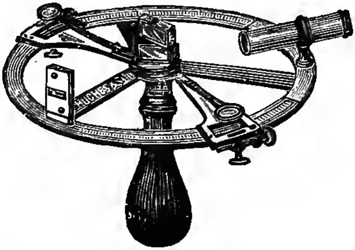


Fig. 113A.

The price of the five-inch sextant, complete in mahogany box, with two telescopes, is £5 10s., and the six-inch is £6 10s.

Plane Table.—This consists of a drawing-board A (Fig. 114), (usually framed, A′, with a movable panel), having a sheet of drawing paper strained on it, mounted on a portable three-legged stand B, and capable of turning about a vertical axis, and of being adjusted by screws to a horizontal position, as indicated by a spirit-level C being attached to the frame. The vertical axis has a clamp and tangent screw to adjust it to any required position. The index E is a flat, straight-edged ruler, having upright sights at its end. These sights have slots (F F G G) similar to those in a prismatic compass or circumferentor.

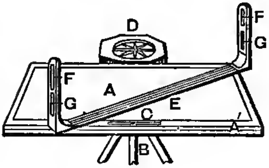


Fig. 114.

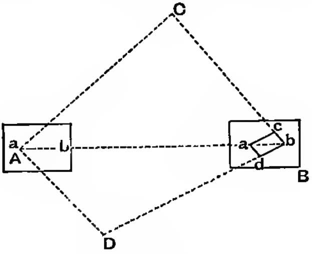


Fig. 115.

The use of the plane table resembles trigonometrical surveying on a small scale, except that the angles, instead of being read off on a horizontal circle and then plotted, are at once laid down on the paper in the field.

Fig. 115 is a simple illustration of the use of the plane table in the field. It is required to make a survey of the trapesium A C B D. Having set up rods at C, B, and D, the surveyor plants his table at A and brings the north point of his compass (360 deg.) directly under the needle when at rest, and makes a point (with a needle or pricker) on some convenient part of his plane table paper to represent his station in the field. To this point he brings the fiducial * edge of his brass rule, and directs it forwards until, through the slots F F or G G (Fig. 114) he intersects with the vertical wire the rod at B. The rule lying in this direction, he draws a point pencil line.† He now proceeds to chain the line A B, and having determined its length, with a scale he proceeds to work off the exact length on the paper. The rule is now directed towards C and D, and in like manner the distances are measured on the ground and plotted on the paper, so that we have the relative positions and length of the lines, A B, A C, and A D. The table is now moved to B and adjusted as before, care being taken to check the line A B in reference to the position of the table at B by directing the index-rule back upon A. Now proceed to direct the rule towards C and D, and measure the lines B C, B D upon the ground, and having plotted them with the scale we have now completed the survey, and the accuracy of the work is proved if the lengths B C, B D are found by measurement to exactly coincide with the points C and D formed by measuring the lines A B, A C, and A D.

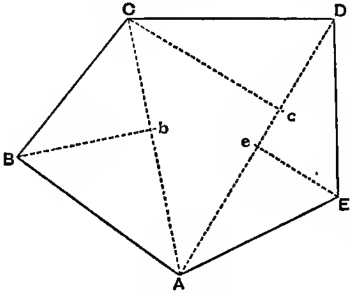


Fig. 116.

Whilst upon this subject, it may he well to mention that the plane table will be found to be very useful for ascertaining the area of the ground one is measuring. For example, suppose we have the irregular figure A B C D E (Fig. 116), and it is required to find its superficial contents. Plant the plane table at A and direct the index-rule to B, C, D, and E, measure on the ground and plot on the paper the length A B = 665, A C = 885, A D = 1030, and A E = 580, and make a correct plan of the ground. Now, if you erect perpendiculars B b = 424, C c = 595, and E e = 285, there will be by the well-known rule

$$\frac{\text{A C} \times b\,\text{B}}{2} = \frac{885 \times 424}{2} = \qquad 187{,}620 \text{ sq. links}$$

and

$$\frac{\text{A D} \times (\text{C}\,c + e\,\text{E})}{2} = \frac{1030 \times (595 + 285)}{2} = \frac{453200}{640820} \text{ sq. links}$$

= 6 acres, 1 rood, and 25·3 perches, the contents of the field.

---

* The "fiducial" is the reliable or accurate edge of the rule.

† An H H H pencil is best for this purpose.

The price of a plane table, with compass, level-sighted straightedge, and stand complete, is about £8 8s. to £10.

Telemeter.—This is an exceedingly clever little instrument, invented by Labbez, and is designed to give, without any calculation whatever, the distance of objects from 250 to 3000 yards. It is most simple in construction, easily understood, very accurate, and not likely to get out of order. The chief merits claimed for it are, that it does not require much training to use it, nor is it necessary that an absolute right angle be laid out, and it is not dependent on seeing a definite-sized object (such as a man standing erect). Fig. 117 is a full-sized illustration of this little instrument, and Fig. 118 is a sketch showing the method of using it. The following are the directions for using the instrument:—

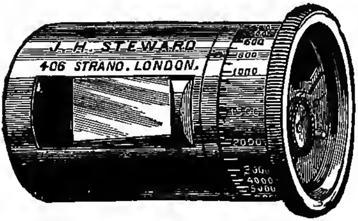


Fig. 117.

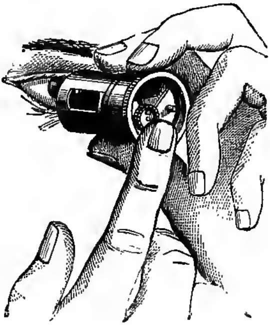


Fig. 118.

1. Open the slide at end of cylinder.

2. Set the small-toothed wheel so that the zero is opposite zero-line, also set the revolving part of the cylinder so that the zero on it is exactly on the zero-line of fixed portion of cylinder.

3. To find the distance of A (Fig. 119), stand at D, face to the left, E, and notice an object (say B) of a prominent nature (known hereafter as the mark) as near as possible at right angles to the object A of which the range is required. Hold the instrument with the thumb and finger of the left hand, as shown in Fig. 118 (in such a way that the oblong opening is quite free), and place it to the eye; look through the hole at the small end of the instrument at mark B, and with the forefinger of the right hand turn the small-toothed wheel until coincidence between the range object A and the mark B is obtained—in other words, A is reflected on B.

4. Fasten the end of the line into the ground at D by passing one of the arrows through the loop and walk to the other end (C) of the 30-yard line in the direction of B. Let some one standing

exactly over D dress the observer exactly with B (by calling out quarter or half pace, &c., right or left, until the right side of head of observer covers the mark B).

5. The observer, facing the same way as in previous operation, now looks at B, revolving the end of cylinder until the object A is reflected on B.

Directly this is done the line opposite the fixed zero will represent the distance of the object A in yards.

Should it so happen (and the occurrence would be very rare) that no natural or other prominent object is to be found somewhere near at right angles to A to use as a mark, then a man can run out with a lance, rifle, &c., and place himself at any position near the right angle at any distance over 60 yards.

The observation may be made the reverse of above if no suitable object is found to the left (see Fig. 120). The only thing is to turn the instrument over and follow the same instructions as above, reading "right" for "left."

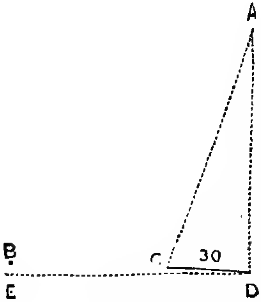


Fig. 119.

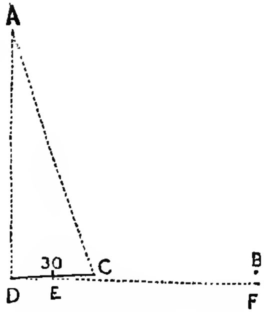


Fig. 120.

The base may be paced, intead of measured, when approximate accuracy is sufficient, and time is of consequence.

The length of base may be half (viz. 15 yards) or double (60 yards), and the results will be half and double respectively of the distances shown on the drum of instrument.

Observations may be checked by stopping at 15 when using the 30 base, and taking an observation there.

If no second person is at hand to dress the observer with the mark B, and greater accuracy is required than can be obtained by walking straight by the eyesight in the direction of B, the following plan may be resorted to:—

For Laying out the Base.—Note the object you intend taking as a mark about at right angles to the object, then walk about

33 paces in the direction of it, see that nothing obscures the object or mark, and place a sword or picket through loop of line; now return, unwinding the 30 yards of line as you go, and when at the end move right or left until the picket is aligned on mark. Then proceed as in previous directions, paragraphs 1, 2, 3, at this point (D), and on going to C turn cylinder until object and mark coincide. Read off the distance in yards opposite zero.

When time is of consequence very good results can be obtained by walking the equivalent number of paces to 30 yards in the direction of mark, placing the picket or sword, and pacing the same number back, taking mean of error in doing so, and aligning picket as before.

Another plan for use by one person only:—

(This illustration, Fig. 120, is shown the reverse way to that generally adopted; that is, looking right instead of left.)

1. Use a line 15 yards long, and place an arrow or picket through the loop of it at E. Set the instrument to zero as before, and looking through it in direction of B, see what object will coincide nearest with the reflection of A.

2. Walk to end of line at D, and, moving right or left, stop when E is aligned with B, and place a mark or picket in that position, then look through the instrument and turn small-toothed wheel until A is exactly reflected on mark B.

3. Now take the line, and walking past E stop at extremity of it (C), when E and D are aligned, right about turn very exactly so as not to shift the position.

4. Face B, and on looking through the telemeter rotate the end of cylinder until A is reflected on B. The range can now be read off opposite the zero.

In rotating the cylinder it is better to stop exactly at the point where the object aligns the mark; and if it goes beyond, then it should be turned back and gradually brought up to the mark again. This precaution, although not absolutely necessary, ensures greater accuracy.

If time permits, a second observation can be taken, and the mean of the two readings taken as the distance of object A.

The Use of the Labbez Telemeter as a Surveying Instrument.—The instrument will determine the distance apart of two inaccessible objects by laying out a triangle as follows:—

Let A and B (Fig. 121) be two points inaccessible from point C. Having measured with the telemeter the distance C A and C B, carry on in these two directions proportional lengths C B′ and C A′.

The triangle A′ B′ C being similar to the triangle A B C, one has

$$\mathrm{A\,B} = \mathrm{A'\,B'} \times \frac{\mathrm{C\,B}}{\mathrm{C\,B'}}.$$

A′ B′ is known, as it can be measured directly, and the product $\frac{\text{C B}}{\text{C B}'}$, is an equality of it.

The following is a simple method of carrying out the above instructions :—

Find by the telemeter, by method previously described, the distances of A and B from C. Place a man or picket at one-tenth of the distance of C B, *i.e.* at B′, in a line with B. Set the telemeter to its zeros and walk in the direction of A from C, keeping in dressing with A until B′ is reflected on A. The distance between A and B will be ten times that of A′ and B′.

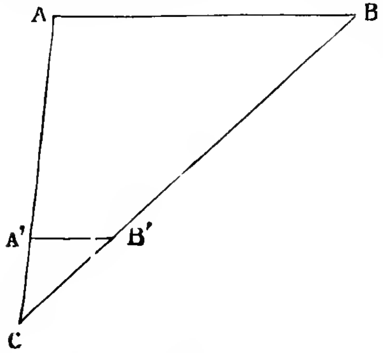


Fig. 121.

Any other proportion can be taken, but if the point at one-tenth is accessible, that is the most convenient, being decimal.

In Fig. 122 is shown the latest development of the Labbez telemeter, made to give finer readings calculated for a shorter base, viz. 20 instead of 30 units. The telescopic power is supplied to add to the accuracy of observation, and enable a more correct alignment to be made.

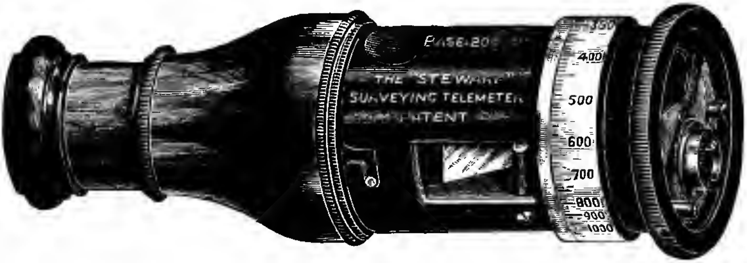


Fig. 122.

To measure the Width of a River.—Make an imaginary line, C L, by pickets, parallel to breadth of river at A B (Fig. 123), by forming two equal angles at B and C, then draw a line through O at the half of line C B till it touches line C L (at D); the distance from D to C is the same as the breadth of the river.

The telemeter also enables the observer to—

1. Raise a perpendicular from a point.
2. To draw a line from a point perpendicular to a straight line.
3. To draw a straight line parallel to a given straight line.
4. To prolong a straight line beyond an obstacle.
5. To measure a surface of land.
6. In making of plans.
7. In making military reconnaissance of a zone of land or of a road in time of peace or war.

The price of the telemeter in pocket bag is from £2 15s. to £3 3s.

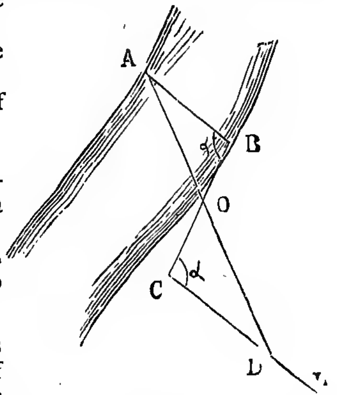


Fig. 123.

The Level.—There are various types of levels, the peculiarities of which are identified with their maker, but the chief with which the surveyor has to do are the "Y" and the "dumpy," the former being illustrated in Figs. 124 and 126, and the latter in Figs. 125 and 127. In almost every case these instruments are supported upon tripod-stands, similar to those described for the theodolite, varying in their size and strength with the nature of the level. Some makers have substituted for the parallel plates and four screws the tribrach or three-screw arrangement (Fig. 126), whereby a three-arm-plate forming a part of the body-piece is substituted for the upper parallel plate; and indeed there are levels with parallel plates but having only three screws. Such arrangements may give greater delicacy in levelling, but for good practical use there is nothing to equal the parallel plates and four-screw system.

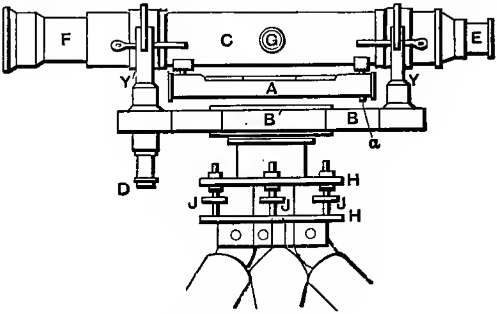


Fig. 124 (Y Level).

The vertical axis of the level, made conical in shape and solid, forms part of the upper parallel plate, upon which turns easily the body-piece supporting the instrument, which is connected with the vertical axis by means of a washer and fixed screw. This body-piece is turned most accurately to the same shape as the vertical axis, which is connected with a half ball-and-socket fitted into the domed socket of the lower plate and securely screwed. The upper and lower parallel plates are kept apart by the four milled-headed screws, one of which is held in position by a stop-block attached to the lower plate.

Thus it may be taken that up to this point all levels are similar in construction, but beyond this the difference between the "Y" and the "dumpy" should now be described.

Screwed upon the outer jacket or body-piece forming the vertical axis is the stage or horizontal bar B (Fig. 124), so formed that a compass-box B′ may be inserted, as shown in Fig. 127. At one end of the stage is a joint carrying one of the supports, Y, whilst at the other end is an adjusting screw D, carrying the other support, Y′, and fitted with a socket.

The "Y" Level.—This instrument, illustrated by Figs. 124 and 126, has its supports Y and Y′ made exactly similar to those of

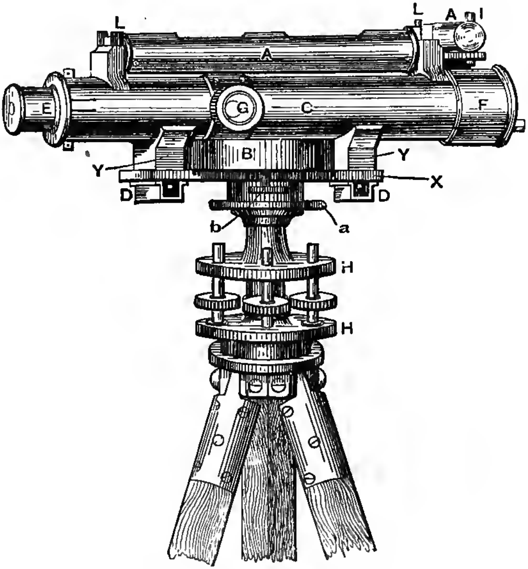


Fig. 125 (Dumpy Level).

the **Y** theodolite described on page 47, so that the telescope C may be removed if necessary by raising the clips. In some cases, as in Fig. 124, there is attached to the telescope, underneath, a spirit-level fixed at one end by a joint and at the other by a capstan-headed screw, for the purpose of adjustment. This method of carrying the level is only adopted in cases where the compass-box B′ is inserted in the stage. Fig. 126 shows how in the absence of the compass the level A is attached to the stage B. Some **Y** levels are provided with a clamp and slow-motion arrangement, similar to that described in the theodolite, for the purpose of taking magnetic bearings.

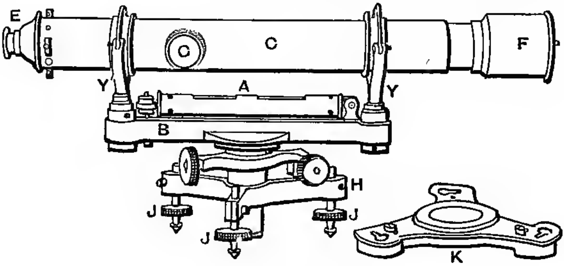


Fig. 126 (**Y** Level).

The Dumpy Level.—This is the most approved instrument, the telescope being permanently fixed to the stage in the supports, the method of so fixing it varying with the peculiarity of make, as illustrated by Figs. 125, 127, and 128. In Fig. 125 it will be noticed that the telescope C simply rests in the jaws of the supports Y Y, whilst in some dumpy levels the telescope is contained within two collars, to one end of which, by a hinged joint, the spirit-level is attached, whilst upon the other is fixed a capstan-screw,

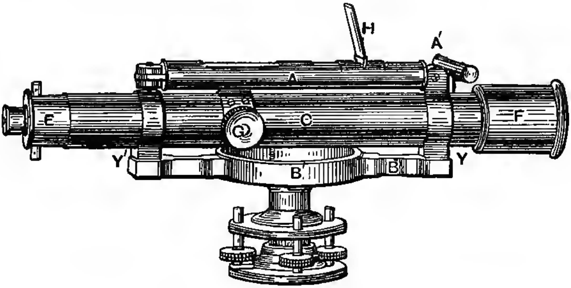


Fig. 127 (Dumpy Level).

for the purpose of adjusting the axis of the telescope horizontal to the spirit-level when in the centre of its run. This is an unquestionable advantage, for the makers of the better class of instruments adjust this axis to that of the level so accurately that except under extraordinary circumstances, such as substituting a new bubble-tube, it should very seldom if ever require attention. Troughton's improved level (Fig. 129) is arranged so that one support is connected with the stage by a hinged joint, whilst the other has an adjusting screw working through a spring fixed on the stage and through the stage itself, and actuated by a capstan-head, which, when not required to be used, is protected by a screw-cap.

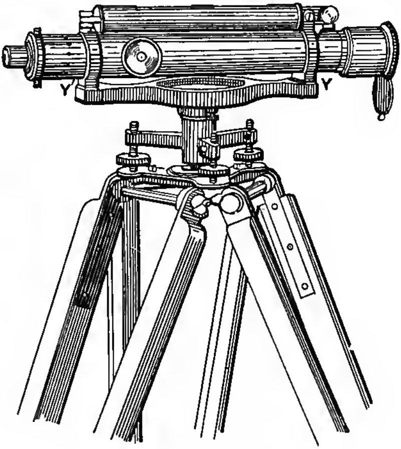


Fig. 128 (Dumpy Level).

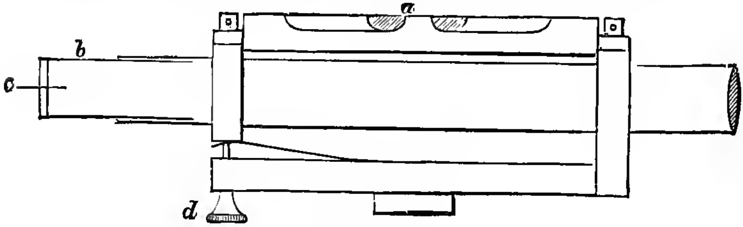


Fig. 129.

Telescope.—The telescope, which is held within the supports, consists of two tubes, the outer containing the object-glass F and its shield, and the inner tube, not quite so long, carrying the diaphragm and eye-piece.

Object-glass.—The object-glass consists of what is known as a compound lens, composed of a plano-concave and double convex glass, forming together plano-convex lenses; and the plane sides are presented towards the object.

Eye-piece.—The eye-piece, invented by Ramsden, consists of two lenses of equal focal lengths, one plano-convex and the other convexo-plano, so that the convex sides are turned towards one another, the interval between them being two-thirds of the focal length of either.

The Diaphragm.—The diaphragm (Fig. 130) consists of a brass ring A of smaller dimensions than the inner tube, to which it is fitted by four capstan-headed screws, C, C^1, C^2, C^3, for the purpose of adjusting the ring. Attached to this ring are what are termed the cross-wires, consisting of two vertical, *a* and *b*, and one horizontal hair, *d*, and which usually are the web of a common garden spider wound round a forked piece of stick (Fig. 131), so that the lines may be accurately attached to the ring with a little gum or wax.

"In looking through a telescope a considerable field of view is embraced; but the measurements, indicated by any instrument, of which the telescope may form a part, will only have reference to one particular point, which particular point is considered as the centre of this field of view. We must therefore place some fixed point in the field of view, at the focus of the eye-piece, and the point to which the measurement will have reference will be that point of the object viewed which appears to be coincident with this fixed point, or which, as the technical phrase is, is bisected by the fixed point." The intersection of the two vertical and the horizontal wires of the diaphragm furnishes us with this fixed point.

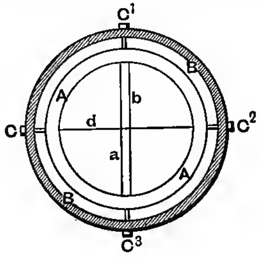


Fig. 130.

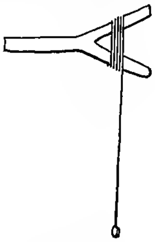


Fig. 131.

"When the instrument is in adjustment [*i.e.* the diaphragm has been truly adjusted so that the cross-wires are coincident with the centre of the field of view], the axis of the tube of the telescope is set truly horizontal by means of the level beneath it, and the line of observation ought consequently to be parallel to this axis. Let A (Fig. 132) represent the proper position of the intersection of the cross-wires, and O A the direction of the axis of a pencil of light passing through the object-glass and coming to its focus at A. Then, the axis of the tube of the telescope being set truly horizontal, the line A O is also truly horizontal, and every point bisected by the intersection of the cross-wires will be situated on the prolongation of the horizontal line A O.

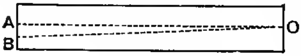


Fig. 132.

"Suppose now the position of the diaphragm carrying the cross-wires to have become deranged, so that the point of intersection is moved to B, then every point bisected by the intersection of the cross-wires will be on the prolongation of the line B O, and will, consequently, be below the true level point on the line A O." *

Line of Collimation.—"The error from misplacement of the spider-lines has a technical denomination. The line O A, or O B, from O to the point of intersection of the cross-wires, is called the *line of collimation*, and the error arising from their derangement ∴ is called the *error of collimation*." †

Adjustment of the Level.—The following is a copy of the instructions issued by Messrs. Troughton and Simms with all their levels :—

"Art. 1.—When the bubble *a* (Fig. 129) is in the centre of its run, the direction of the movement of the draw *b* should be horizontal.

"Consequently there can be but one position for this level, which, having been determined by the maker of the instrument, may be regarded as constant.

"The insertion of a new level will alone disturb the relation which exists between the level and the telescope; in such a case a re-determination of its position will be necessary.

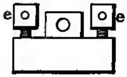


Fig. 133.

"The screws *e*, *e* (Fig. 133) are supplied for the purpose of bringing the bubble into the position indicated in Art. 1. Unless the level be broken, they should not be touched, and after the new level has been inserted, if once adjusted, no correction will subsequently be needed.

Adjustment for Collimation.—"Art. 2.—The line joining the optical centre of the object-glass and a point in the line of the horizontal wire should be horizontal when the bubble *a* (Fig. 129) is in the centre of its opening. ‡

---

* "Surveying and Astronomical Instruments," p. 13, by J. F. Heather, M.A. Crosby Lockwood & Son, London.

† Ibid., p. 15.

‡ This is technically known as "being in the centre of its run."

"This adjustment is effected by the collimation screws at the eye end of the instrument, which serve to raise or depress the wire-frame or diaphragm. Whenever the object-glass has been removed, a re-determination of this adjustment, known as the adjustment of the line of collimation, will be necessary.

"Art. 3.—The horizontality of the line of collimation should be maintained during a complete revolution of the instrument upon its axis. In order to effect this adjustment the bubble *a* is brought into the centre by means of the parallel plate screws. The telescope is then turned 180 deg. upon its axis; should the position of the bubble have changed, half the difference between this new place and its old position has to be effected by the parallel plate screws, and the remaining half by the screw *d*, which is placed inside a cap under the stage. This adjustment is known as the adjustment for reversion; it is comparatively of little importance, and can, in our instrument, be accomplished at any time in a few seconds.

To adjust for Collimation (Fig. 134).—"Place the instrument exactly halfway between the two staves held at any convenient distance from each other, bring the bubble *a* to the centre of its run, and read both staves; the points thus obtained will be equidistant from the earth's centre, and consequently level. The instrument may now be set up nearly in line with the staves, but not between them; the bubble *a* must be brought to the middle of the opening, and, by means of the collimating screws, the horizontal wire *c* may be made to bisect the two level points or others equidistant therefrom.

"If the distance between the more distant staff and the level be considerable, allowance is to be made for the curvature of the earth. An adjusted level (used as a collimator) will supply a ready means for the accurate adjustment of other instruments. In order to effect this purpose, the wires must be placed in the principal focus of the object-glass, the eye-piece should be removed from its socket, and, as a temporary protection for the webs, a piece of plain glass may be inserted. The level should now be erected, and the bubble brought accurately into the centre.

"Let A be the adjusted level; B the instrument, the adjustment of which is to be checked. A pencil of rays proceeding from a point in the plane of the horizontal wire *c* will, after passing through the object-glass, be parallel, and, should the axis of the pencil be horizontal, the parallel rays will after emergence be also horizontal. It will be perfectly clear that if the level A be in adjustment, and the bubble *a* in the centre of its opening, the horizontality of this parallel pencil is ensured.

"The level under adjustment, B, is to be placed with its object-glass in front of the object-glass of A. As the diagram indicates, a little difference in the height of the instruments will not affect the result; but it is desirable that there should be no great difference. The parallel rays from A falling upon the object-glass of B will be converged to a point within the telescope, and an image *c′* of the wires *c* will be formed.

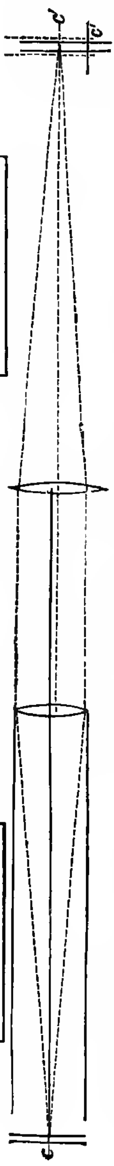


Fig. 134.

"If the bubble *a′* of the level B be brought into the centre by the parallel plate screws, and the level B be in adjustment, the wires in B will lie upon the image of the wires in A. Should this not be the case, let them occupy a position as *c″*; the operator will now have to release the lower collimation screw and to tighten the upper, thereby raising the wire frame (diaphragm) until he has brought *c″* upon *c′*; when this has been done (provided the bubbles in both instruments retain their central position) the adjustment for collimation has been accomplished."

Adjustment for Parallax. — There is another adjustment of the level of great importance, and that is what is termed parallax, or when the image of the object viewed formed by the object-glass falls either short of or beyond the place of the cross-wire. The existence of parallax is determined by moving the eye about when looking through the telescope and observing whether the cross-wires change their position and are flittering and undefined.

To correct this error, first adjust the eye-piece by means of the movable eye-piece tube, till you can perceive the cross-wire clearly defined and sharply marked. Then by moving the milled-headed screw G, which by a rack and small pinion wheel within the telescope enables the internal tube to be thrust out or drawn in to focus the object, according to distance, and you are able to see, at the same time, the object clearly and the intersection of the wires clearly and sharply defined before it. The existence of parallax is very inconvenient, and when disregarded has frequently been productive of serious error. It will not always be found sufficient to set the eye-piece first and the object-glass afterwards. The setting of the object-glass by introducing more distant rays of light will affect the focus of the eye-piece and produce parallax or indistinctness of the wire where there was none before. The eye-piece must in this case be adjusted again.

Generally, when once set for the day, there is no occasion for altering the eye-piece, but the object-glass will of course have to be altered at every change of distance of the object. The nearer the object is to the instrument the greater length will the inner tube have to be drawn out; but there is a limit even to this, as the figures on the staff are indiscernible within a few yards, and the greater the distance there is between the instrument and the object, so much shorter must the telescope be. The best 14-in. levels will only read figures clearly at a distance of about 150 yards.

It should be noted that the correction for parallax should be made previous to that of collimation.

Reflecting Mirror.—There are several appendages to the level, such as a reflecting mirror, which by means of a saddle-shaped clip may be fixed upon the bubble-tube, and having a hinged joint may be placed at such an angle as to enable the operator to see if the bubble is in the centre of its run at the same time he is looking through the telescope. There is also provided a magnifying glass, which, fitting into a socket attached to the side of the compass-box, is used to facilitate reading the magnetic bearings. The needle, when not required, may be thrown off the agate centre by means of either a screw or a lever fixed in the side of the compass-box. The cost of a good level varies from £12 for a 10-in. to £18 and £20 for a 16-in. instrument.

Reflecting Level.—A most useful instrument for taking preliminary and sketch levels, which it determines with commendable correctness, is a little pocket level, the invention of Colonel Burel (Fig. 135). The glass, silvered on its lower half, is fixed in a frame suspended from a gimbal, and adjusted to hang in a vertical plane by filing the pin passing through the lower part of the frame. The observer's thumb is passed through the ring and held at arm's length; and the hand, steadied by a staff, is slided up or down until the pupil of the eye is bisected at the junction of the plain and the silvered glass. The object in front that cuts this line is then level with the eye of the observer.

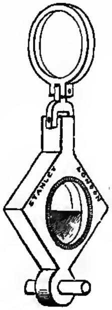
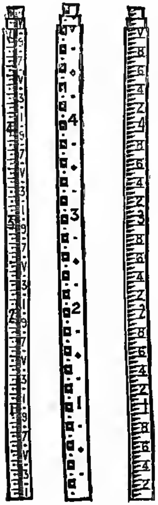


Fig. 135.—Reflecting Level.

Levelling Staff.—There are various types of levelling staves; the chief with which the surveyor has to do, however, being what is called the telescope staff. This consists of a mahogany box 5 ft. long, 4 in. broad, and 2½ in. deep, within which is another hollow box 4 ft. 6 in. long, made so that it will slide easily within the other, and a third joint, of solid mahogany, works within the inner one, so that when pulled out to their full length, having springs or clips to secure them, it represents a staff 14 ft. long, which when not required can, by sliding the divisions one within the other, be made compact for transit, as represented by Figs. 136, 137, and 138, which illustrate the usual method of graduation.



Figs. 136, 137, 138.

The face of this staff is either covered with printed papers or, preferably, is painted so as to represent each foot from 1 ft. to 14 ft., which are again subdivided into hundredths of feet, the feet being represented by red figures and the subdivisions by black.

It must be understood that the tops of the red figures represent the value in feet upon the staff; but in the subdivisions, whilst the tops of the black figures, which are always odd numbers, represent their value in hundredths of feet, the bottoms of these same figures represent the intermediate even numbers. Thus by reference to Figs. 136 and 139 it will be seen that there are in each foot five figures respectively, 1, 3, 5, 7, and 9, representing $\frac{10}{100}$, $\frac{30}{100}$, $\frac{50}{100}$, $\frac{70}{100}$, and $\frac{90}{100}$. Consequently the bottoms of 3, 5, 7, and 9 of the black figures and the top of each red figure represent $\frac{20}{100}$, $\frac{40}{100}$, $\frac{60}{100}$, $\frac{80}{100}$, and $\frac{100}{100}$. Fig. 140 shows a tenth of a foot and $\frac{6}{10}$ of another, and each $\frac{10}{100}$ is delineated by a black stroke across the left side, the lowest white stroke representing $\frac{1}{100}$, the next black stroke $\frac{2}{100}$, the next white stroke $\frac{3}{100}$, the next black $\frac{4}{100}$, the next white $\frac{5}{100}$; at five is a longer stroke, and consecutively each white and black stroke make up the sum of $\frac{10}{100}$ to the top of the 1 of the black subdivisions. By a similar process is made up the $\frac{10}{100}$ of the next subdivision to the bottom of the 3, and so on seriatim.

In using the staff the observer must notice carefully where the horizontal wire of the diaphragm cuts the staff, counting the red figures from the bottom and the nearest subdivision—in other words, supposing the wire were to cut the staff between the red figures 5 and 6 and above the black figure 3 of the subdivision and coincide with the top of the second of the black strokes, it would represent that the line of collimation cut the staff at 5·34 ft.

The greatest care should be observed in holding the staff so that it is perfectly vertical; and some old-fashioned ones had a plumb-bob suspended in the lower joint, as a guide to the staff-holder. It is now usually recommended that the staff be gently waved backwards and forwards (see Figs. 266 and 267, p. 216), and the lowest reading taken. Unless, however, the staff-holder is well accustomed to his work, it frequently happens that he waves the staff sideways as well as backwards and forwards. The most reliable guide for him is a disk-shaped bubble-glass fixed on a brass bracket to slip into a socket on the back of the staff; and a very little practice is found to enable anybody to keep the bubble in the centre of its glass.

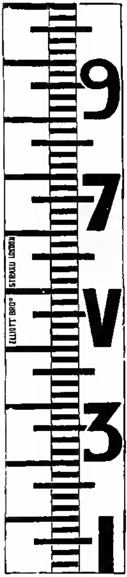


Fig. 139.

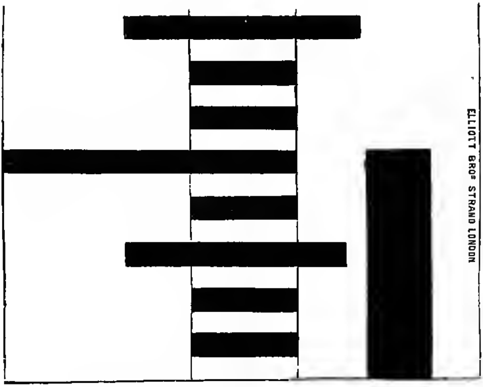


Fig. 140.

The preceding description is that of a 14-ft. or the usual kind of levelling staff, but they can be obtained 16 or 17 ft. long and made proportionately, the figuring of course being the same kind. There is, no doubt, a considerable advantage in the use of such staves, especially in hilly country, but in winter weather it is often difficult for a man to maintain a vertical position with so long a staff.

The cost of a 14-ft. levelling staff is from £1 12s. to £2 5s.

The Aneroid Barometer.—This invaluable instrument, the invention of which is attributed to M. Vidi of Paris, is greatly used by surveyors for ascertaining great altitudes, where the ordinary operation of levelling is impossible, or for approximation. The action of this barometer, "for ascertaining the variations of the atmosphere, depends on the effect produced by the pressure of the atmosphere on a metallic box, hermetically sealed, from which the air has been previously exhausted." * This box (*a* in Fig. 141 and D D in Fig. 142) is corrugated upon its upper and lower surfaces, and by means of an elaborate system of levers and springs the index, or hand, is made to traverse the dial which surmounts the frame, being actuated by increase or diminution of the atmospheric pressure upon the metallic box. The aneroid thus "records the changes in the weight or *pressure* of the atmosphere on a *given surface*, suppose a square inch; and it would, therefore, have greatly facilitated the comprehension of the action of the instrument had the dial been graduated to show the difference of the atmospheric pressure in absolute weight or pounds;" but seeing that "the density of the atmosphere would decrease in geometrical progression for altitudes in arithmetical progression, and since this density also varies directly as the pressure to which it is subjected, and which is measured by the height of the barometric column, it follows that, if at different altitudes these columns decrease in geometric progression, the altitudes will increase in arithmetical progression, and will therefore be proportional to the logarithms of the barometric columns. Hence, if the temperature remained constant,

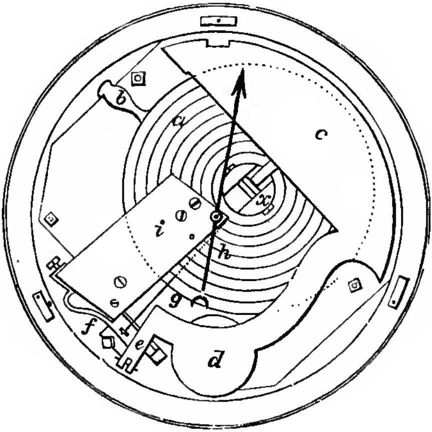


Fig. 141.

---

* "Surveying and Astronomical Instruments," p. 104, by J. F. Heather, M.A. Crosby Lockwood & Son, London.

the difference of two altitudes would vary as the difference of the logarithms of the barometric columns at those altitudes; so that if h be taken to represent the height of a higher station above a lower one, and if B be the height of the barometer at the *lower* station, and b the height at the *higher* station, we should have

$$h = k \log. \frac{B}{b};$$

where k is a constant quantity, to be determined by experiment." I cannot do better than refer the student to the very exhaustive consideration of the theory of the aneroid in the work by Mr. Heather, from which the foregoing is extracted.

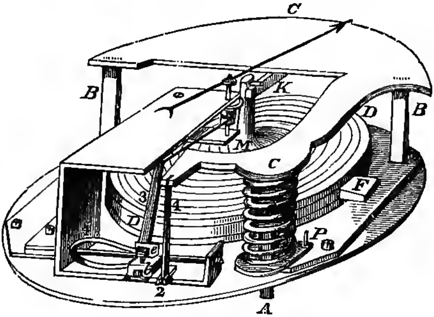


Fig. 142.

It should be stated that the variation of temperature greatly affects the results of observation with the aneroid. To guard against any error a thermometer is attached so that the difference of temperature may be noted at the various stations.

Fig. 141 is a plan of the aneroid with the dial removed, and Fig. 142 is an isometrical view of the same. The instrument is from 4 to 5 in. in diameter (some even larger) and about 1$\frac{3}{4}$ in. thick. "The pressure of the atmosphere is indicated by the hand h (Fig. 141) pointing to a scale, which is graduated to correspond with the common barometer." There is also a scale compensated to agree with the altitudes in feet, which is attached by a movable rim so that its zero may be regulated as necessary. Referring to Fig. 142, A is the screw adjusting the hand, B B the fulcrum, C C the principal lever, D D the vacuum vase, 1 vertical rod connecting lever C C with levers 2 and 3, *e b* adjusting screws for leverage, S spiral spring, M socket in vacuum vase, K pin attached to socket.

The cost of an aneroid barometer varies from £3 3s. to £8 8s.

The Stadiometer.—This is one of those instruments for expeditiously measuring distances (illustrated in Fig. 143). It consists of a telescope M B, fitted to a vertical arc O, which works in the frame P G which is attached to the body-piece of the instrument, and above which a round disc or table A rotates. "There is a scale D D′ fastened to the frame, the centre of which corresponds with the centre of the instrument, and which is graduated to the scale to which the surveyor wishes to plot his work. The telescope M B is fitted with a diaphragm, with two horizontal hairs, distant from one another a hundredth part of a foot of the focal length of the object glass. From this proportion it follows that, when any ordinary levelling-staff is held on any distant point and the telescope brought to bear upon it, if the readings, in feet and decimals of a foot, of the intersections of the hairs on the staff be observed, their difference multiplied by a hundred gives the true distance in feet from the instrument."

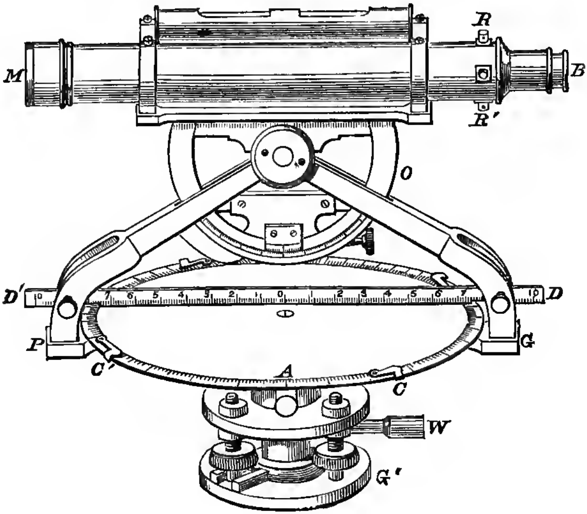


Fig. 143.

The Omnimeter.—"This instrument (Fig. 144), like the stadiometer, is intended to measure base-lines and distances without chaining, and also the differences of altitudes and angles." It consists of a graduated limb (1) reading by means of a vernier to ten seconds for the measurement of horizontal angles; a good telescope (2) revolving in a plane perpendicular to the limb; a powerful microscope (3) closely united to the telescope; a highly sensitive level (4) lying upon the rule (5), which has a fixed length (of twenty centimetres, for example); a scale (6) fixed vertically at the extremity of the said rule, in the plane of the optical axis of the microscope, and divided into half-millimetres, the millimetres only being indicated by figures from 1 to 100; a micrometrical screw (7) attached to the base of the scale, and giving a correct reading of the scale to the $\frac{2}{10000}$ of a millimetre, denoted on the graduated circle of the screw; an extremely sensitive level (8) capable of being applied to the telescope, and for determining in case of necessity its horizontality.

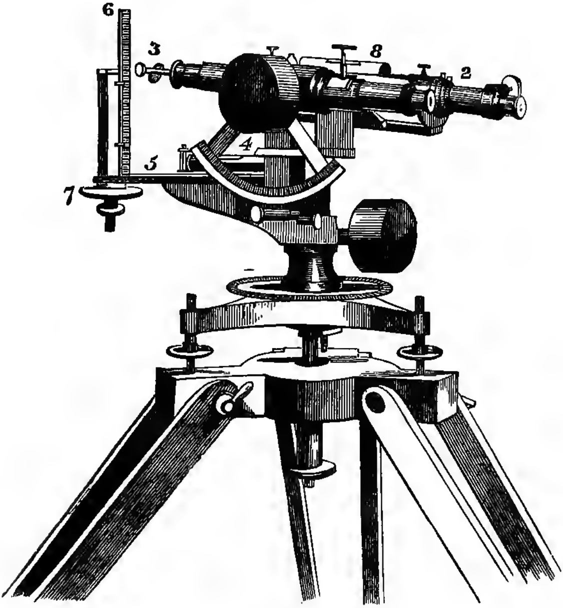


Fig. 144.

Accompanying the instrument is a staff of a fixed length—say three metres—having a white mark upon a black ground at either end. This staff is held at the point of which the distance from the instrument is required, and the telescope having been directed to the staff, which must be held perfectly plumb, the inclination is read off by means of the microscope from the scale. This being done, the distance may be calculated.

The Tacheometer.—Whilst for military or approximate purposes appliances such as the foregoing may be very useful and expeditious, I do not hesitate to confess a predilection for ascertaining the lengths of my base and other lines by actual measurement. There is, however, another instrument—the tacheometer—

for determining distances otherwise than by measurement, which during recent years has come into more frequent use by English engineers, but it can only be mentioned here.*

Dredge-Steward Omni-telemeter.—This excellent instrument, originally intended to be only a range-finder, has upon the advice of Colonel Fraser, R.E., been improved to an extent which proves it to be "a very accurate instrument for telemetric surveying."

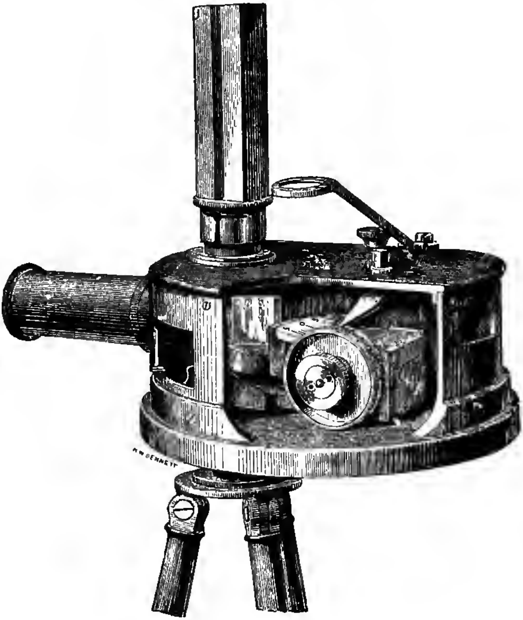


Fig. 145.

The general view of the instrument is shown in Figs. 145 and 146, whilst Fig. 147 shows a section through it. From these it will be seen that the instrument is a modification of the ordinary box sextant, but in place of the two mirrors of the instrument being parallel to each other, when the graduation reads o, the mirrors of the omni-telemeter make an angle of 45° with each other in this position, so that on looking through the instrument under these conditions the eye of the observer occupies the apex of a right angle, the sides of which are formed by the direct line of sight, and that seen by reflection in the two mirrors.

---

* In 1890, a paper on "The Tacheometer" was read before the Institution of Civil Engineers by Mr. Neil Kennedy, M.Inst.C.E.; and a work by the same author—entitled "Surveying with the Tacheometer"—was issued in 1900 by the publishers of the present work (see page 13 of Catalogue at end of this volume).

"Tacheometry" was also the subject of papers read before the same Institution by Mr. Bennett H. Brough, in 1887, and by Mr. R. E. Middleton, M.Inst.C.E., in 1894.

There is still another point in which the instrument differs from the box sextant. In that instrument, as is well known, one of the mirrors, viz., that known as the horizon glass, is fixed in position, whilst the fully-silvered mirror only can be adjusted. In the Dredge-Steward instrument, however, both these mirrors are adjustable. Referring to Fig. 147, D is the mirror which is fixed in the sextant, but which in the present instance can be adjusted through a limited range by means of the micrometer screw E. This mirror is mounted on an arm B, which, turning round the pivot F at one end, is kept in contact with the screw E already mentioned by the spring O, and it is by moving this arm that the mirror D is adjusted. At the free end of this arm

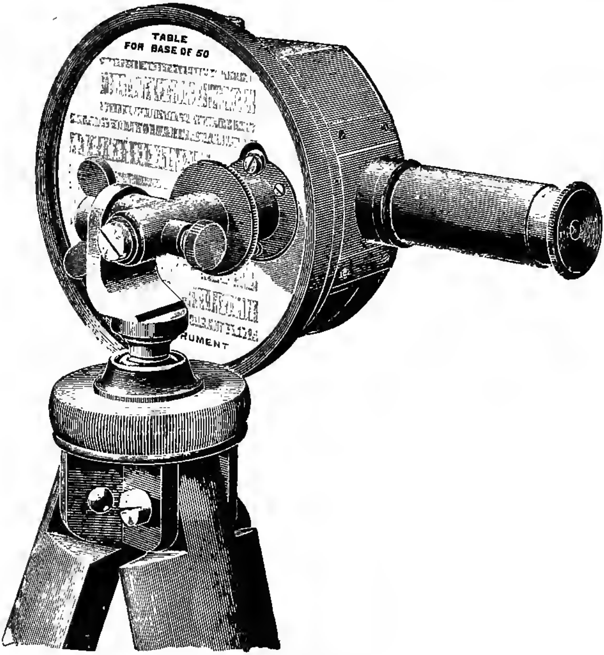


Fig. 146.

is a nut C, carrying a micrometer screw with a graduated head G, as shown. The other end of this screw abuts against the arm L, on which the fully-silvered mirror P is mounted, and which can, therefore, be rotated through a small angle by turning the milled head H of the micrometer screw.

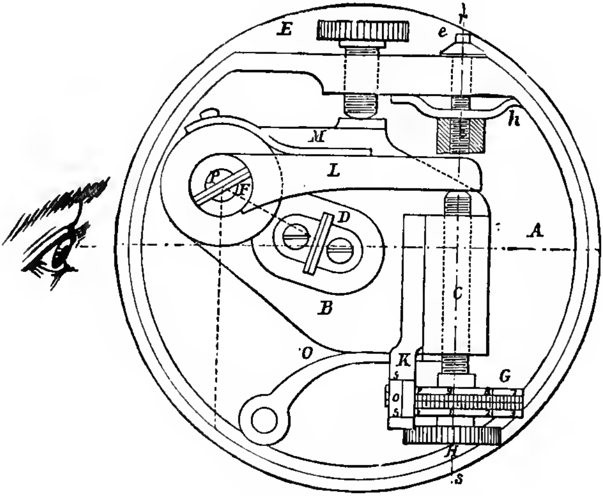


Fig. 147.

In using the instrument to take a range, as A B (Fig. 148), the observer being at B, and facing so that the mark lies on his right hand, he views it by reflection in the instrument. The line of sight from the object reaching first the mirror P, is reflected on to the mirror D (Fig. 147), and from it to his eye. The observer now, looking through the unsilvered part of the mirror, tries to find some prominent object C (Fig. 148), which he can superimpose on the reflected image of A, without, be it understood, touching the micrometer screw H. When this is done the angle A B C is a right angle. Now putting a mark at B, he paces in the direction of C B produced a distance B D = 50 yards, and looking again through the instrument as before, he turns the milled head H until the image of the mark A is again superimposed on C. Then reading the index on the micrometer screw, and referring to

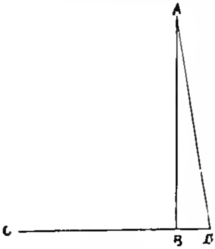


Fig. 148.

a short table attached to the instrument, he reads off instantly the range A B.

Usually it is impossible to get a mark C, such that the angle A B C is a right angle, without several trials, and it is here where the advantage of the omni-telemeter comes in, since it is not necessary with this instrument that the angle A B C shall be a right angle. Hence, if a point cannot be found fulfilling this condition, the observer chooses some point that nearly does so. Looking at this object and the mark through the instrument, when the latter is adjusted in its zero position, the image of the mark will be to one side or the other of it. In this case the screw E is turned round, moving the mirror until coincidence is secured, and the second observation is taken at D (Fig. 148) as before. Owing to this adjusting screw E, much less trouble is required to find a suitable mark on which to project the object whose range is sought.

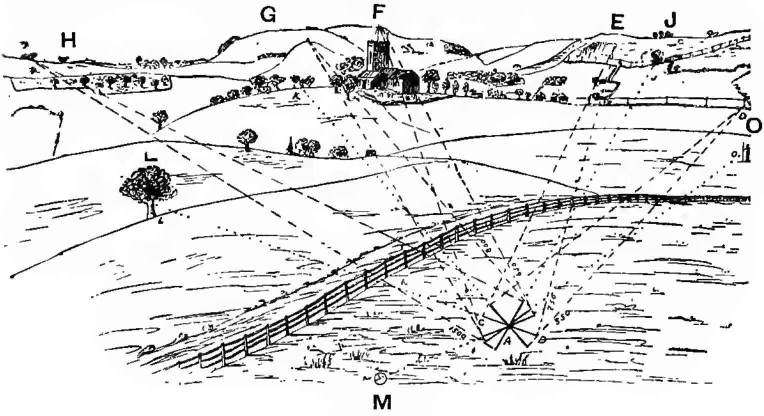


Fig. 149.

In the case of instruments having a fixed base angle, the observer may have to shift his position considerably before the desired coincidence is obtained. The adjustment allows base angles varying 8 deg. on either side of 90 deg. to be used, and it therefore is only in very exceptionally monotonous countries that a suitable mark cannot be obtained with comparative ease.

The best plan of using this instrument is indicated by Fig. 149. A stick is placed at the point A, and distances of 25 yards measured on each side of it. Then to get any range such as E, a suitable object such as L is taken, and the observations taken at the points B and C. To take the distance of the point G a man is sent out with a picket to O, and used as the auxiliary mark.

For the range of the steeple F the mark M would be used, and J when taking H. In this way a whole round of objects can have their distances ascertained very rapidly. With moving objects two instruments are used, held by observers 50 yards apart, the line between them making an angle of nearly 90 deg. with the object. Both of the instruments having been first set to zero, each observer reflects the other on the object by using the micrometer screw H (Fig. 147). The range is then got by adding or subtracting the readings of the two instruments, the sum being taken when the zero line of the two is on the same side of the zero line of the scale K, and the difference when they are on opposite sides. An optical square and one instrument can be used.

For telemetric surveying the following method of using the instrument has been worked out by Colonel Fraser, and is illustrated in Fig. 150. The instrument is mounted on its stand in the position shown in Fig. 146, and a bright object—Colonel Fraser uses a gilt prism—is placed on the ground vertically below it by means of a plumb bob. The instrument is then set to zero. An assistant is sent out to the object, whose distance is required, with a staff having targets 5 ft. apart, the lower one being 2 ft. 6 in. from the ground. This target is aligned with the image of the ground mark by means of the compensating screw E, and afterwards the upper target is aligned by means of the index screw, the reading of which then gives, by reference to a table, the distance of the staff. This method can be employed in all cases when the slope does not exceed 8 deg., that being the limit of adjustment by the compensating screw. At the same time the instrument can also be used to ascertain the heights of objects, the distance of which is known. For this purpose the instrument is mounted on its stand as already described, and the base of the object reflected by the compensating screw on to the plumb bob, and then its summit by the other screw. The height of the object is then obtained from a table of rise in inches per yard.

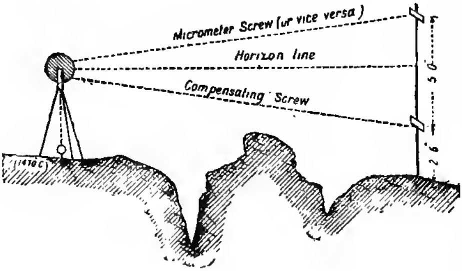


Fig. 150.

It is obvious that the instrument can also be conveniently used as an optical square, and hence (having regard to what has been already stated) it would appear that it can be used almost as a universal instrument for preliminary survey work.

The Francis Surveying Compass and Clinometer.—This is a new surveying instrument, invented by Mr. W. R. Francis, of Swansea, and designed especially for work in mines. It is capable, however, of performing all kinds of work that do not require an exceptional degree of accuracy. The main part of the instrument (Fig. 151), apart from the stand, will go into the side pocket of a coat, when the compass is set in plane of the frame, and may be thus carried with safety. The total weight of the instrument, with case, fittings, stand, and plane table, is about 17 lbs. only. The whole apparatus, being a light one, could be easily transported by one person across a difficult ground, and is thus very suitable for explorers in new countries.

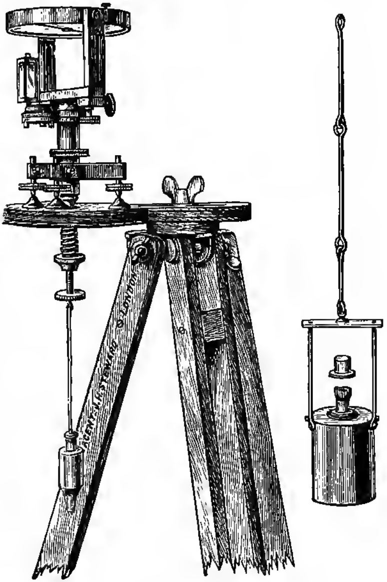

AGENT J. H. STEWARD LONDON

Fig. 151.

The graduated circle of the compass is 3½ in. in diameter, and can be adjusted to correct the deviation of the needle. This latter is blunt-ended; it is silvered near its tip, and a fine line is drawn along its centre to read the deflections by. This adds considerably to the facility of using it, and angles can be read within a ¼ deg. As is usual, there is a device by which the needle can be lifted off the central pin to reduce wear in travelling, and also to aid in bringing it to rest. Sights are taken by means of two vertical slots in the frame; these are formed with bevel-edged plates, and give a very clear and exact vision of the object under view. The frame itself can be rotated on a vertical pivot on the three-armed stand, while fine adjustments are made by a screw with a milled head. A second screw

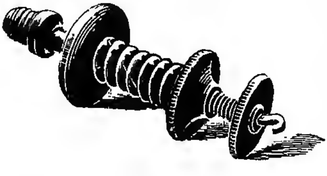


Fig. 152.

serves to clamp the frame in position. The compass is levelled by a single bubble, the levelling screws having ball-and-socket heads, enabling them to take a firm bearing in all positions, when the instrument is fixed by a central screw (Fig. 152). This is a new feature, at least in this country.

The measurement of vertical angles is effected by unshipping the rectangular frame, and mounting it horizontally by means of the right-angled fitting shown at Fig 153 in the lower view. This brings the compass into a vertical plane, and allows the clinometer needle to hang freely. Any object can then be sighted either above or below the spectator, and its vertical angle read off. The two slots are exactly alike, so that sights can be taken both backwards and forwards without reversing the instrument.

As shown in Fig. 151, the instrument is set upon an extension bracket fixed to the top of the tripod stand. This bracket is made of three layers of wood glued together with the grain crossing to prevent warping. It has a central hole through which a plumb-line may be dropped when it is desired to work from a certain point. By means of this line the centre of the compass can be set exactly in position. Ordinarily, however, this is not requisite, and then the instrument is fixed directly on the head of the tripod stand by means of the thumb-nut shown. It may also be held in the hand, without any stand at all, or suspended on a slack string passing through the V cuts at the tops of the sighting slits. This latter plan is often very useful in getting round awkward positions in underground surveying, where there is no place for a tripod to stand. The weight of the instrument keeps it steady and plumb, while the V cuts align it directly between the points of suspension of the string.

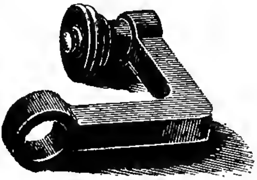


Fig. 153.

Fig. 154.

Fig. 155 shows a straight edge for use with the plane table. It carries a stool on which the compass can be fixed by two screws, and when this is in position a survey can be laid down directly by the use of the same instrument by which it was made. The plane table, 16 in. by 16 in., can be mounted on the stand and fixed by clamp screw.

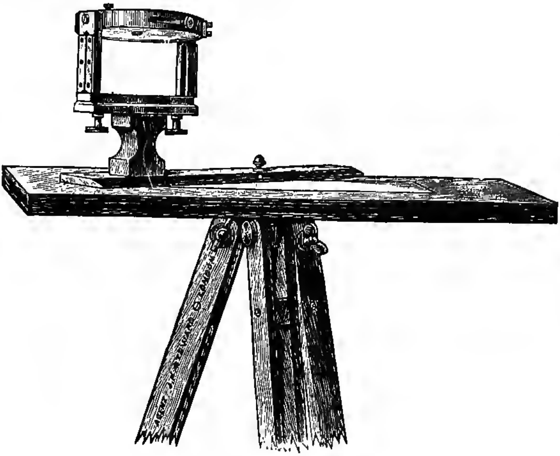


Fig. 155.

The legs of the tripod are telescopic, so as to pack into convenient length. At the foot each leg carries a step (Fig. 154), an ingenious device by which the leg can be pressed firmly into the soil.

Stanley's Patent Model Transit Theodolite.—This is unquestionably a great improvement upon the general construction of transit theodolites, for it meets the objection I make on page 56, where, in speaking of the various adjuncts to the instrument, such as lanterns, &c., I say: "My own experience has been that such appendages are more trouble than they are worth, and the extra money would be better spent upon strengthening the working parts of the instrument."

This instrument (Fig. 156) is constructed (as will be seen by the illustration) upon the solid system as compared with the building up by separate parts, whereby greater rigidity must of necessity be obtained. And the central axis, being one-and-a-half the diameter, is about double as strong as that of an ordinary transit.

"The plate has not to support the superstructure as in an ordinary theodolite, but has only to hold the two axis bubbles, which are thereby brought distinctly in view, and the clamp and tangent motion, which is only placed conveniently for use upon this plate, in a position where there is less risk of accident than when it is placed upon the outer edge of the limb."

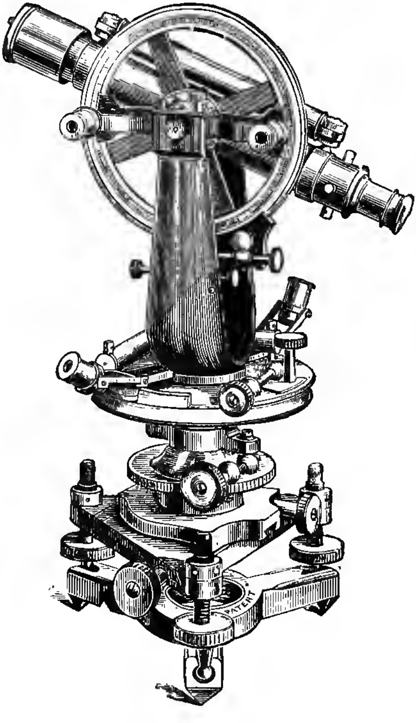


Fig. 156.

Trotter's Curve-Ranger.—This little instrument, which is shown in perspective in Fig. 157, is intended to perform all the operations of curve-ranging without, in most cases, requiring any calculation whatever to be made by the operator, as the chord, arc, versin, and many other quantities can be read direct from the scales with which the instrument is provided. The principle is that of Euclid III., prop. 21, where it is proved that the angle in the segment of a circle is constant.

The mirror moves with the upper scale. The figure of the curved edge of the scale is a polar curve whose equation is

$$r = a + b \sin 2\theta$$

where a is the distance from the zero graduation to the axis of the mirror, and b is the length of the scale from zero to 2, and θ is the inclination of the mirror. On the left of the woodcut is shown an eyepiece containing a half-silvered mirror, set at such an angle that when the instrument is closed, and reads 90° on the graduated limb, it may be used as an optical square.

If three points, A B C, on the curve are given, and all are accessible, set up rods at A and C, place the instrument over B, and adjust the mirror by means of the tangent-screw, so that the rod at C is seen direct through the eyepiece, and the rod at A is seen by reflection in the mirror. Then if any intermediate position on the curve be taken up, both A and C can be seen simultaneously through the eyepiece of the instrument, one by reflection, the other by direct vision, superimposed. If the two rods are not seen superimposed, the operator must move to the right or to the left until this is the case. The instrument will then be over a point on the curve. In this way any number of points in the curve can be fixed as the observer moves from A to C, and on arriving at C, the tangent to the curve may be obtained, for when the rod at A is observed by reflection, the direction of the line of sight through the eyepiece is the tangent to the curve, and a ranging rod may be set up at any convenient distance to mark it. Similarly the tangent at the other end of the curve may be found.

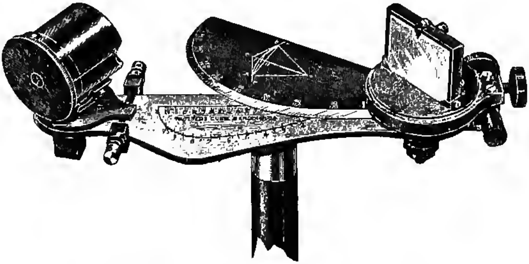


Fig. 157.

On the back of the movable plate, a scale showing the ratio of the length of the arc to the length of the radius is read at the point where the body of the instrument cuts the graduations. An engraved figure on the instrument shows also all the elements of a curve that can be obtained by direct reading from the scales of the instrument, or by simple arithmetical calculation.

Although the instrument is not intended to replace the theodolite in very accurate surveying, one advantage is claimed for it over the older instrument, in that errors made by it are not cumulative, and no assistants, chain, or tables are required, as is the case when curves are set out by the method of tangential angles. The only adjustment required is the setting of the eyepiece. This is adjusted as an optical square when the graduated limb reads 90°. Price, with telescope in leather sling case, £10.

Dalrymple-Hay's Curve-Ranger.—This instrument (Fig. 158), which has been invented by Mr. H. H. Dalrymple-Hay, an assistant engineer on the London and South-Western Railway, is designed to facilitate operations in the field when ranging curves by the method of tangential angles. It can be fitted to

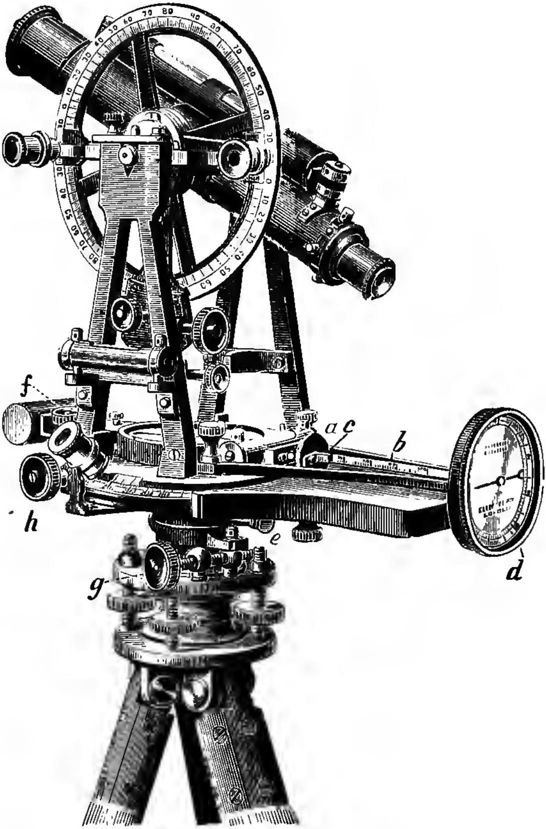


Fig. 158.

theodolites of any pattern, by simply clamping the horizontal plate to the divided limb and the arm carrying the dial to the vernier limb, by milled screws. This attachment can be readily removed from the theodolite when not required, and will fit into the case of the theodolite.

By simple mechanical movements it at once gives the tangential angle for each chain point in all curves in general use on railways, and thus obviates the necessity of referring to a table of tangential angles, and then reading the vernier as with the theodolite. When ranging new tangents no calculations are required.

The following are the directions given for use of the curve-ranger:—Having set up the instrument at the commencement of the curve A (Fig. 159), move the disc *a* (Fig. 158) along the graduated scale *b*, until the setting-index *c* fixed to the disc coincides with the division on the scale corresponding to the number of chains radius in the curve.

Thus if the curve to be set out is of 60 chains radius, move the disc *a* along the scale *b* until the edge of the setting-index *c* is coincident with division marked 60.

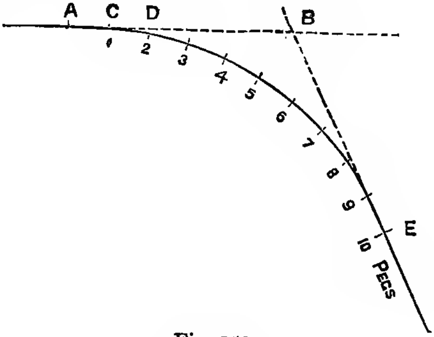


Fig. 159.

Now direct the telescope along the tangent to the curve A B, and bring the needle to the zero of the dial *d*, clamping both the clamps *e* and *f*. Perfect the bisection of the cross hairs in the diaphragm of the telescope with the pole at B by turning the tangent screw *g*. If the curve bends to the right as in the figure, turn the tangent screw *h* until the needle indicates 99 on the dial. In this position the telescope will point along the chord line A C, along which measure a chord of 1 chain. C will then be the first peg in a curve of 60 chains radius.

To obtain the second tangential angle, again turn the tangent screw *h* in the same direction until the needle points to division 98 on the dial. The telescope will then have described the second tangential angle, and will point along the chord A D. To obtain D, measure the chord line C D intersecting the line of sight, A D in D. This determines the second point, and so on a considerable number of points can be easily set out.

If the curve bends to the left, the needle, instead of pointing to 99, 98, 97, &c., should be made to point to 1, 2, 3, &c., divisions on the dial, by turning the tangent screw *h* the reverse way.

The operation of ranging a new tangent is equally simple. Suppose it is required to set out a new tangent at peg 10, as in the above figure. The needle, when the telescope was pointing to peg 10, from the position A, indicated 90 on the dial, or 10 divisions from the zero, the dial face being divided into 100 equal parts.

Remove the instrument from A to E, peg 10, and there set it up, still pointing in the direction that the curve is being set out.

Transit the telescope, directing it to A, and bring the needle to division 10 on the dial, clamping the two clamps *e* and *f*. Turn the tangent screw *h* until the needle, passing over 10 divisions, indicates zero on the dial.

Now transit the telescope into its former position, when it will point along the tangent to the curve, and the curve may be continued in either direction, to the right or left, as described above.

The limits of the graduation on scale *b* are from 50 to 100 chains radius, but curves of smaller or larger radius may be set out by the following simple method:—

Say radius of curve 25 chains. Set index *c* to division 50 on scale; and for tangential angles read 2, 4, 6, &c., on dial.

For a curve of larger radius, say 200 chains, set index *c* to division 100 on scale *b*, and read the dot between zero and division 1 for the first tangential angle, and division 1 for the second tangential angle, and so on, the dots being odd numbers, and the divisions even.

The price of the instrument, fitted to new or old theodolites, ranges from £7 7s. to £10 10s.

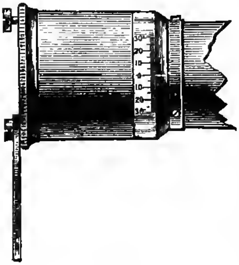


Fig. 160.

Stanley's Ray-Shade.—This (Fig. 160) is an ingenious combination. The accepted necessity for a ray-shade in levels is made the opportunity for utilising it for other and useful purposes, especially for cross-section work in hilly country. The ordinary ray-shade has two narrow slits opposite each other at 180°. A zero line is carried from one slit to a line on the ray-shade, fitting when the slits are quite horizontal. Sight through the slits at zero enables an approximate cross-level to be taken. The edge of the tube of the ray-shade is divided 20° on each side of the zero line to 2°, so as to take approximate lateral inclines of the surface of the land in levelling.

This plan of cross-sighting was originally proposed by Gravatt.

Stanley's Improved Plane Table.—On page 63 (Figs. 114 and 115), I have described the *modus operandi* of the plane table, an instrument too little called into requisition, especially in contouring, and I here introduce a very complete instrument, adapted for all purposes. The drawing-surface of this table (Fig. 161) consists of a loose panel which stretches the sheet of paper by pressing it into its frame, where it is afterwards held by a pair of ledges, which fit at their ends into long slots. The board is mounted upon a firmly-braced tripod-stand, as illustrated in Fig. 162. The head of the tripod-stand is secured to the board with a central screw (not shown), which permits the board to be set in any direction, it being the rule that the edge w (Fig. 161) should always take a north to south direction. Three screws, *s s s* (Fig. 162), at the corner of the triangular head, can be raised or lowered by milled heads from the under side. These screws permit about 15 deg. of adjustment to the table without any unsteadiness, as the centre-screw clamps the table finally hard down upon them when all adjustments are made. A small trough form of magnetic compass *a* is placed upon the rule to strike the magnetic to south line, to which all angles are referred in transposing the work of the plane table. The diaphragm of the telescope is provided with a platino-iridium point fixed vertically at the mutual focus of the object glass and the eye-piece.

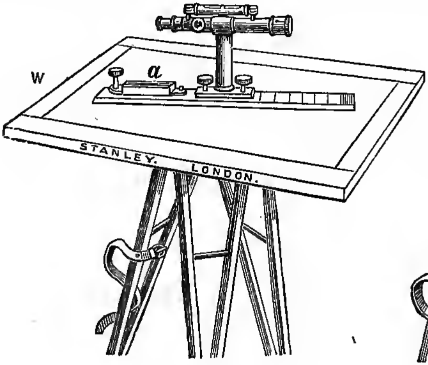


Fig. 161.

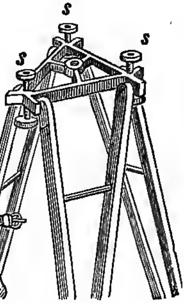


Fig. 162.

Johnson's Improved Tripod-Head.—The illustrations (Figs. 163 and 164) will sufficiently explain the principle of construction, which admits of an instrument being quickly adjusted in an approximately horizontal position, and then firmly clamped.

The instrument, attached to the screw on *b*, is set approximately level by moving the tripod in the usual manner; release the clamping nut *d*, and the sockets *b c* will then be free to move in any direction, and can be fixed in any position by again screwing up nut *d*. The upper socket *b* may be turned in azimuth by releasing the nut *e*, which leaves the hemispherical surface *b* free to move around the concave ring *a* of the tripod-head.

This form of head, mounted on a framed stand of special construction, may be used to great advantage with an ordinary dumpy level, particularly on embankments, where it is difficult to find a level space for the legs of the tripod, as the movement of the sockets compensates the difference of level of the legs and so prevents undue straining of the parallel plate-screws; and these remarks apply equally with regard to theodolites, especially those used for mountain work. For plane tables, also, this form of head is a valuable improvement.

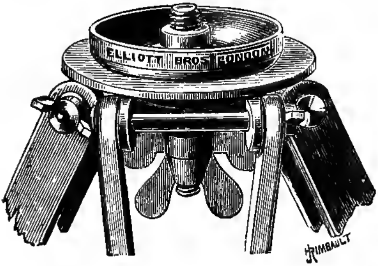


Fig. 163.

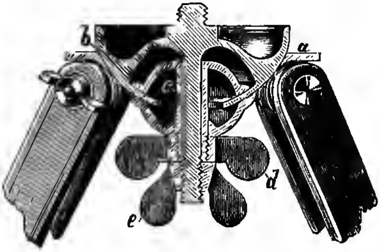


Fig. 164.

Price, complete with special framed tripod-stand, for plane tables and small levels, £5 5s.

Erskine's Altazimuth Theodolite.—Notwithstanding the high appreciation in which I hold the various instruments referred to in the foregoing pages, after a quarter of a century's experience with them under varied circumstances, I am bound to admit that Erskine's altazimuth, an instrument recently brought out, supplies in my judgment a long-felt want. It is at once a theodolite, a level, and a clinometer, and is comprised in so small a space as to render it available anywhere and under any circumstances.

Fig. 165 fairly illustrates the instrument, which consists of an ordinary vernier and limb, properly encased, upon which two substantial brackets carry the horizontal axis, also encased in (what may be termed) an enclosed collar. On the left is the vertical circle ($\frac{7}{16}$ inch wide), also encased, with the divisions marked upon the edge; whilst on the right-hand side is the telescope, duly adjusted to the vertical circle. Over and above these is the compass, with its complement of right-angle levels for adjustment.

The weight of this beautiful little instrument is only 4 lbs., or with the tripod-stand 8 lbs. The latter is made in the form known as the "open-lath" (similar to a camera stand), but by an ingenious arrangement the joints, instead of being folded are clipped with a screw adjustment, so that the lower leg may slide within the upper frame and be clamped at pleasure, especially in side-lying ground.

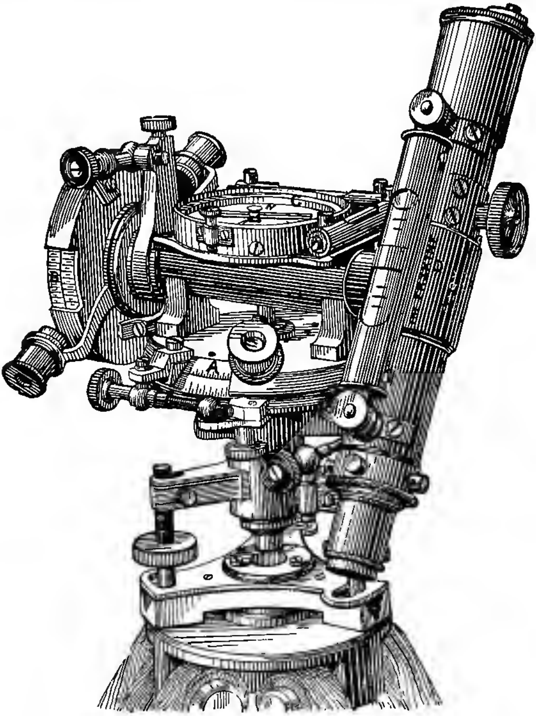


Fig. 165.

Another feature in this instrument is the provision of a reflector, to be attached to the object end of the telescope, for use at night (or especially in mining), consisting of a bulb of silver so arranged by an arm that it can be brought into the centre of the field, and thus reflect rays of light into the telescope.

I have recommended Messrs. Cary, Porter & Co. to improve the instrument by a special extra clamping arrangement, whereby its efficacy for levelling operations may be secured.

The price of the instrument complete is about £23.

Cary's Rack Adjustment to Eye-piece.—This ingenious arrangement—shown in Fig. 166—entirely obviates the necessity of pulling out the eye-piece. In focussing the instrument, by a rack and pinion motion, the webs may be clearly defined with greater facility and without any risk to the diaphragm. This arrangement is applicable to every kind of instrument.

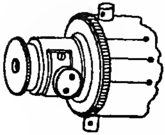


Fig. 166.

To replace a Level-tube: Spare Parts.—Accident to the glass tubes of spirit-levels will occasionally happen in travelling, or in the course of work in a rough country. These tubes as applied to instruments "are generally mounted in brass covering tubes. Small level tubes under 2½ inches are conveniently mounted in such tubes with a fixing of slaked plaster of Paris inserted at each end of the brass tube. Larger level-tubes should be bound round with thin paper pasted round the ends, which is allowed to get quite dry, to be afterwards fitted to the brass tube with a file. . . . It is convenient in mounting level-tubes to place white glazed paper under the bubble to reflect the light that passes through it to ensure better observation. In fixing undivided level-tubes, or replacing such tubes in instruments, it is important to observe that the *test mark*, which is a small ground facet, should be placed on the *upper side*." *

A surveyor's equipment for colonial work should always include one or more spare tubes, together with some duplicates of screws and other small parts of his instruments, the lack of which may interfere with or wholly prevent the prosecution of his work. A list of the most needful of these will be readily furnished by the instrument-maker.

NOTE.—In the instruments above described, improvements, minor and otherwise, are from time to time introduced by various makers, the principles of construction, action, and use in the field remaining in most if not all cases unaffected. To describe and illustrate every new invention, would swell this volume to encyclopædic dimensions and beget in the inquirer a belief that proved value justified their insertion. Such a belief would assuredly perplex him, and might possibly be in some cases misleading, since lengthened experience alone can decide the question of preference to be given to new forms over those long tried in practice and at present in everyday use, and the necessary supersession of the latter in favour of the former. The student is therefore cautioned not to regard any form here given as being an ultimately perfected one; but to make himself acquainted with new ones as they from time to time appear, by examination of circulars and price-lists (which makers will probably be glad to supply on application), and to ascertain from the professional man under whom he studies, or with whom he is acquainted, whether their intrinsic merits are in practice found to answer the expectations naturally entertained by their inventors and introducers.

---

* Stanley, "Surveying and Levelling Instruments."

CHAPTER IV.

TRIGONOMETRY REQUIRED IN SURVEYING.

It is not intended in this chapter to do more than explain the general principles of trigonometry required in surveying.* Trigonometry is a science of great scope and interest, involving a vast amount of patient study if its higher branches are required; but for "Practical Surveying" it is quite possible, in such a chapter as the present—presupposing that the student has acquired from proper text books a moderate amount of elementary mathematics—to give a sufficiently general outline of trigonometry to enable the student to apply it thereto himself.

We do not pretend here to apply the science to every position required in surveying, but rather to enumerate the different definitions and theorems which the student should study and learn to apply where necessary.

Trigonometry has for its object the solution of triangles, and its application to surveying is the "art of measuring and computing the sides of plane triangles,† or of such whose sides are straight lines." Triangles consist of six parts, viz. three sides and three angles; and in every case in trigonometry three parts must be given in order to find the other three; and of those three given parts one must be a side, because with the same angles the sides may be greater or less in proportion.

We will commence with a few of the principal definitions of Euclid's geometry which bear upon trigonometry.

1. **Plane Surface.**—A plane surface, or plane, is a surface in which if any two points be taken, the straight line between them lies wholly in that surface.

2. **Plane Angle.**—A plane angle is the inclination of two lines to each other in a plane, which meet together, but are not in the same direction.

Note.—This definition includes angles formed by two curved lines, or by a curve and a straight line, as well as angles formed by two straight lines.

---

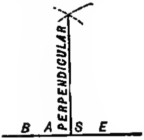
* The word trigonometry is derived from two Greek words, τρίγωνον (trigo-non), *a triangle*, and μετρέω (met-re-o), *to measure*.

† "Plane Trigonometry" is the science which deals with straight lines, as compared with "Spherical Trigonometry," which involves the consideration of curved figures.

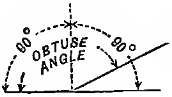
3. Plane Rectilineal Angle.—A plane rectilineal angle is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.

Note.—When an angle is simply spoken of, a plane rectilineal angle is always meant.

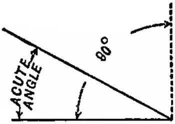
4. Perpendicular.—When a straight line standing on another straight line makes the adjacent angles equal to one another, each of these angles is called a right angle, and the straight lines are said to be perpendicular to each other.



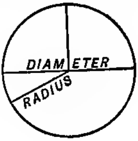
4



5



6



9

5. Obtuse Angle.—An obtuse angle is greater than a right angle.

6. Acute Angle.—An acute angle is less than a right angle.

7. Circle.—A circle is a plane figure contained by one line, which is called the circumference, and is such that all lines drawn from a certain point within the figure to the circumference are equal to one another.

8. Centre of Circle.—And this point is called the centre of the circle.

9. Diameter of Circle.—The diameter of a circle is a straight line drawn through the centre, and terminated both ways by the circumference.

Note.—The radius of a circle is a straight line drawn from the centre to the circumference.

10. Semi-circle.—A semi-circle is a figure contained by a diameter and by the part of the circumference cut off by the diameter.

11. Segment of Circle.—A segment of a circle is a figure contained by any straight line and a part of the circumference which it cuts off.

12. Rectilineal Figures.—Rectilineal figures are those which are contained by straight lines.

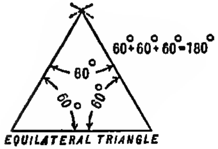
13. Trilateral Figures.—Trilateral figures or triangles by three straight lines.

14. Quadrilateral Figures.—Quadrilateral figures by four straight lines.

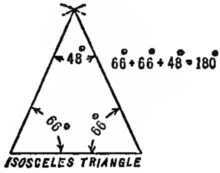
15. Multilateral Figures.—Multilateral figures, or polygons, by more than four straight lines.

16. Equilateral Triangle.—Of three-sided figures, an equilateral triangle has three equal sides.

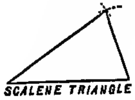
17. Isosceles Triangle.—An isosceles triangle is a triangle which has two sides equal.



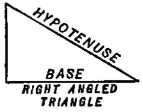
16



17



18



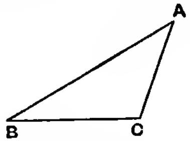
19

18. Scalene Triangle.—A scalene triangle has three unequal sides.

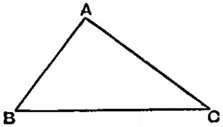
19. Right-angled Triangle.—A right-angled triangle is a triangle which has a right angle.

Note.—The side which subtends, that is, is opposite to the right angle, is called the hypotenuse.

20. Obtuse-angled Triangle.—An obtuse-angled triangle is a triangle which has an obtuse angle, which by Def. 5 is greater than a right angle.



20



21

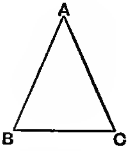
21. Acute-angled Triangle.—An acute-angled triangle is a triangle which has three acute angles.

Theorems.—1. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise the angle contained by those sides equal to one another, they shall likewise have their bases or third sides equal, and the two triangles shall be equal, and their angles shall be equal each to each, namely those to which the equal sides are opposite.

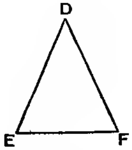
2. The angles at the base of an isosceles triangle, A B C and

A C B, are equal to one another; and if the equal sides be produced the angles on the other side of the base, D B C and B C E, shall be equal to one another.

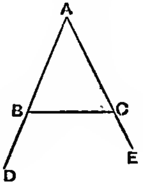
3. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal; the angle which is contained by the two sides of the one shall be equal to the angle which is contained by the two sides equal to them of the other.



1



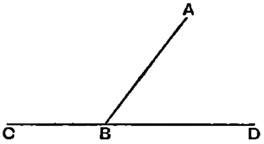
1



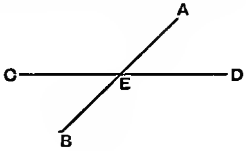
2

4. The angles which one straight line makes with another straight line on one side of it either are two right angles or are together equal to two right angles.

5. If at a point in a straight line, A B, two other straight lines, C B and B D, upon the opposite sides of it, make the adjacent angles together equal to two right angles, these two straight lines, C B and B D, shall be in one and the same line.



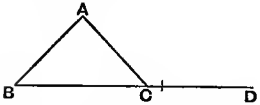
5



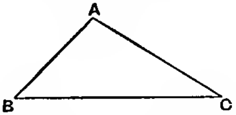
6

6. If two straight lines cut one another, the vertically opposite angles shall be equal.

7. If one side of a triangle, B C, be produced to D, the exterior angle, A C D, is greater than either of the interior opposite angles, C A B and A B C.



7



9

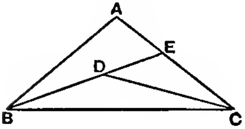
8. Any two angles of a triangle are together less than two right angles.

9. If one side of a triangle, A C, be greater than a second, A B, the angle, A B C, opposite the first must be greater than that opposite the second, A C B.

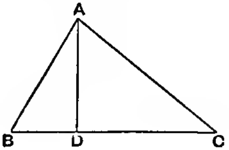
10. If one angle of a triangle be greater than a second, the side opposite the first must be greater than that opposite the second.

11. Any two sides of a triangle are together greater than the third side.

12. If, from the ends of the side of a triangle, C and B, there be drawn two straight lines, B E and C D, to a point D, within the triangle, then B D and C D will be together less than the other sides, B A and A C, of the triangle, but will contain a greater angle, B D C.



12

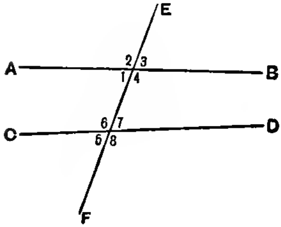


13

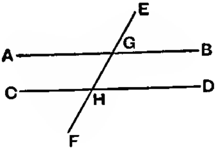
13. Every straight line, A D, drawn from the vertex of a triangle to a point D within the base, is less than the greater of the two sides, A C, or than either, if they be equal.

Theory of Parallel Lines.—Hamblin Smith has very properly detached the propositions, in which Euclid treats of parallel lines, from those which precede and follow them in the first book, in order that the student may have a clearer notion of the difficulties attending this division of the subject. It is necessary here to explain some of the technical terms used.

14. If the straight line E F cut two other straight lines A B, C D, it makes with those lines eight angles, to which particular names are given. Thus the angles numbered 1, 4, 6, 7 are called the *interior* angles; and 2, 3, 5, 8 are called the *exterior* angles; 1 and 7, and 4 and 6, are called *alternate* angles; and the pairs of



14



15

angles, 1 and 5, 2 and 6, 4 and 8, 3 and 7 are called the *corresponding* angles.

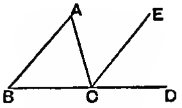
The angles 1, 4, 6, and 7 are equal to four right angles.

15. If a straight line, E F, falling upon two other straight lines, A B and C D, make the alternate angles equal to one another, then the two straight lines must be parallel.

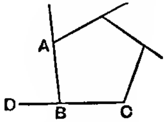
16. If a straight line fall upon two parallel straight lines, it makes the two interior angles upon the same side together equal to two right angles, and also the alternate angles equal to one another, and also the exterior angle equal to the interior and opposite upon the same side.

17. Straight lines which are parallel to the same straight line are parallel to one another.

18. If a side of any triangle B C be produced to D, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of every triangle are together equal to two right angles.



18



19

19. The exterior angles of any convex rectilinear figure, made by producing each of its sides in succession, are together equal to four right angles.

Now one of the most essential things to be understood with regard to angular measurement is the circle and its various divisions. A circle is divided into 360 equal parts or degrees, each degree into 60 minutes, and each minute into 60 seconds. The following symbols are used to denote these divisions and sub-divisions: degrees (°), minutes (′), and seconds (″), so that 85 degrees, 27 minutes, and 13 seconds would be shown thus: 85° 27′ 13″.

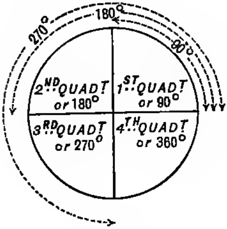


Fig. 167.

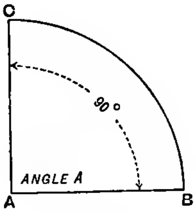


Fig. 168.

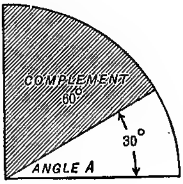


Fig. 169.

The circle (Fig. 167) is divided into four quadrants of 90 degrees each, and by Definition 4 (p. 103) each of these is a right angle.

In trigonometry it is usual to consider the radius of a quadrant as unity; and, as a line identical with the horizontal arm of the quadrant moves in an upward direction towards the vertical arm A C, Fig. 168, so the angle formed by this line produces certain functions which, for simplicity, are considered in the terms of the angle so formed, usually called the angle A. Thus Fig. 169 shows the angle A equal to 30 deg.; Fig 170, the angle A equal to 45 deg.; Fig. 171, the angle A equal 60 deg.; and so a diagram may be constructed to represent an angle which is any fractional part of 90 deg.

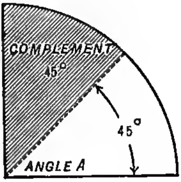


Fig. 170.

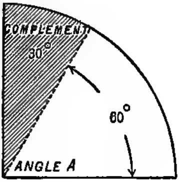


Fig. 171.

It may be well here to introduce and explain the trigonometrical canon or diagram (Fig. 172), which shows the different trigonometrical functions in terms of the angle A to the radius = 1.

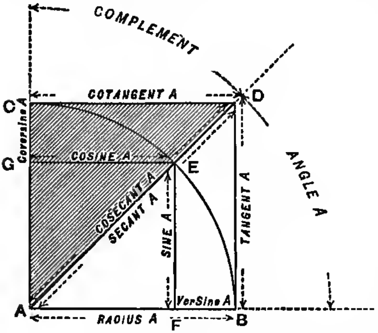


Fig. 172.

Now here, for simple illustration, I have taken the angle A as 45 deg.

The trigonometrical functions of the angle A are as follows: The SINE, CO-SINE, TANGENT, CO-TANGENT, SECANT, and CO-SECANT, with the VERSINE and CO-VERSINE, but the two latter do not enter largely into the consideration of the solution of triangles.

Now Fig. 173, illustrating the functions of an angle of 30 deg., shows by the strong lines certain positive functions of that angle, such as the sine, secant, and tangent; whilst the extended dotted lines, and dotted lines, show the complementary functions of the same angle, as the co-sine, co-secant, and co-tangent.

Here I should explain that the complement* of an angle is equal to its difference from 90 deg., so that 60 deg. is the complement of 30 deg.

The supplement of an angle is equal to its difference from 180 deg., so that the supplement of 30 deg. is 150 deg.

By referring to Figs. 173 and 174 it will be seen that in the former case the sine, secant, and tangent are much less than the co-sine, co-secant, and co-tangent (which are shown by dotted lines) by reason of the angle being small; whilst in Fig. 174 it will be seen that the sine, secant, and tangent are greater than are the co-sine, co-secant, and co-tangent; and going back to Fig. 172, we have the sine equal to the co-sine, the tangent equal to the co-tangent, and the secant equal to the co-secant, of an angle of 45°.

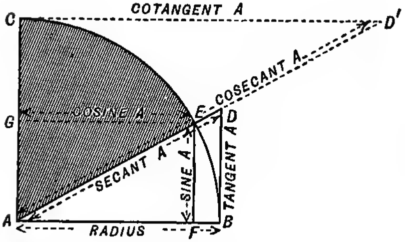


Fig. 173.

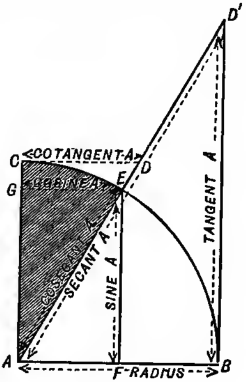


Fig. 174.

From the foregoing it will be seen that:—

Trigonometrical Ratios or Functions.—1. *Sine.*—The sine of an arc is a perpendicular let fall from the extremity of one radius to the other, as E F (Figs. 172, 173, and 174).

2. *Tangent.*—The tangent is a perpendicular line drawn from the extremity of the radius to meet the other produced, as B D (B D' in Fig. 174).

3. *Secant.*—The secant is that radius which forms the angle, produced until it meets the tangent, as A D (A D' in Fig. 174).

4. *Cosine.*—The cosine is a line parallel and equal to that part of the radius which lies between the foot of the sine and the centre, as G E.

5. *Cotangent.*—The cotangent is a horizontal line, commencing at the termination of the quadrant, and terminating on the

---

* The difference between an acute angle and a right angle is called its complement (*i.e.* the angle lacking to complete or fill up the right angle).

radius A E produced, in D (Fig. 172), D′ (Fig. 173), and D (Fig. 174).

6. *Cosecant.*—The cosecant is one of the radii produced until it intersects the cotangent in D (Fig. 172), and D′ (Figs. 173 and 174).

7. *Versed Sine.*—The versed sine is the portion of the radius between the foot of the sine and the arc, as F B.

8. *Coversed Sine.*—The coversed sine is the portion of the perpendicular between the cosine and the arc, as G C.

9. *Chord.*—The chord of an arc is a line joining the extremities of the arc.

I should like here to explain what may appear to be an anomaly, viz. why the lines G E (cos A), C D′ (cot A), and A D′ (cosec A) (Fig. 173), should be complementary to the functions of the angle A. But I hope the following will elucidate the matter. We have found (p. 109) that the complement of an angle is the angle lacking to complete or fill up the right angle; and by reference to Fig. 173 it will be seen that the line G E bears the same relation to the angle E A C as E F does to the angle A or E A B, consequently G E must be the sine of the angle E A C. Thus what is the sine of an angle (less than 90 deg.) is the cosine of the remaining angle or complement, and *vice versâ*. The line C D′ bears the same relation to the angle E A C as D B bears to the angle E A B, therefore what is the cotangent of the angle E A B is the tangent of the angle E A C; and the same equally applies to the secant and cosecant.

These trigonometrical functions are abbreviated as follows:—

Sin A = The sine of the angle A.
Cos A = The cosine do.
Tan A = The tangent do.
Cot A = The cotangent do.
Sec A = The secant do.
Cosec A = The cosecant do.
Vers A = The versed sine do.
Covers A = The coversed sine do.
Cho A = The chord do.

Relation of Hypotenuse to the other Sides of Right-angled Triangle.—Perhaps it may be better to refer to the 47th proposition of Euclid, which states the theorem: "In any right-angled triangle, the square which is described on the side subtending the right angle is equal to the sum of the squares described on the sides which contain the right angle" (Fig. 175).

By this proposition the sum of the squares on the sides A and B is equal to that on the side C; in other words, taking another form of a right-angled triangle, as Fig. 176—

Let A B = Hypotenuse.
A C = Base.
B C = Perpendicular.

Then

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{\text{Base}^2 + \text{Perp.}^2}. \\ \text{Base} &= \sqrt{\text{Hyp.}^2 - \text{Perp.}^2}. \\ \text{Perp.} &= \sqrt{\text{Hyp.}^2 - \text{Base}^2}. \end{aligned}$$

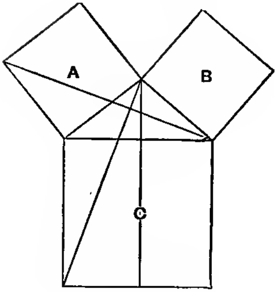


Fig. 175.

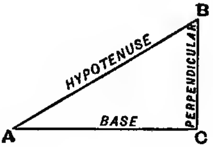


Fig. 176.

Now in the preceding descriptions of the various trigonometrical functions, I have shown that they all have reference to the angle A of the triangle B A C, a portion of the first quadrant (see Fig. 177),

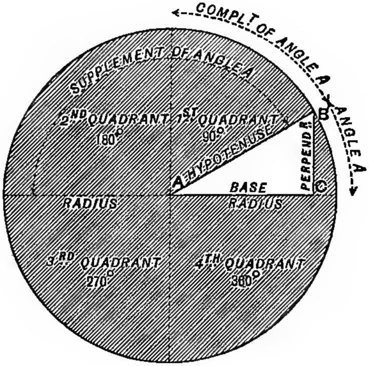


Fig. 177.

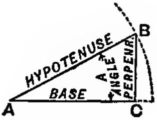


Fig. 178.

which is placed in the centre of the circle called the circle of reference.

We will now consider the functions of the angle A (B A C) in terms of the sides of the triangle A C B. We have seen (Figs. 173, 174) that the functions are the ratios borne by certain lines to the radius; and as a ratio or proportion may always be expressed in the form of a fraction, the functions may be obtained by dividing these lines by the radius. Now, so long as the angles of a triangle remain unchanged, the ratios of the sides of that triangle remain unchanged; hence, comparing Fig. 178 with Fig. 173 or Fig. 174, we are able to express the functions of the angles A in terms of the sides A B, B C, C A.

Thus

$$\text{Sin A} = \frac{\text{PERP}}{\text{HYP}} = \frac{\text{B C}}{\text{A B}}. \qquad \text{Cos A} = \frac{\text{BASE}}{\text{HYP}} = \frac{\text{A C}}{\text{A B}}.$$

$$\text{Tan A} = \frac{\text{PERP}}{\text{BASE}} = \frac{\text{B C}}{\text{A C}}. \qquad \text{Cot A} = \frac{\text{BASE}}{\text{PERP}} = \frac{\text{A C}}{\text{B C}}.$$

$$\text{Sec A} = \frac{\text{HYP}}{\text{BASE}} = \frac{\text{A B}}{\text{A C}}. \qquad \text{Cosec A} = \frac{\text{HYP}}{\text{PERP}} = \frac{\text{A B}}{\text{B C}}.$$

$$\text{Vers A} = \frac{\text{HYP} - \text{BASE}}{\text{HYP}} = \frac{\text{A B} - \text{A C}}{\text{A B}}.$$

$$\text{Covers A} = \frac{\text{HYP} - \text{PERP}}{\text{HYP}} = \frac{\text{A B} - \text{B C}}{\text{A B}}.$$

$$\text{B C} = \text{A B cos B}; \ \text{A C} = \text{A B sine B}; \ \text{A B} = \text{B C sec B}.$$
$$\text{B} = \text{complement of A} = 90 - \text{A}.$$
$$\text{A} + \text{B} + \text{C} = 180^\circ.$$

I may explain, by reference to Fig. 172, that the tangent, cotangent, secant, and cosecant appear therein much longer than the lines E F, A F, and E A, which correspond with the lines B C, A C, and A B in Figs. 177 and 178; and my reason for referring to it is to show that, as these lines are simply ratios to the radius, so what in Fig. 172 is the tangent of A, viz. $\frac{\text{B D}}{\text{A B}}$ is exactly the same ratio as $\frac{\text{B C}}{\text{A C}}$ in Figs. 177 and 178, or as follows :—

	Fig. 172.		Figs. 177 and 178.
Sin A	$= \dfrac{EF}{AE}$	$=$	$\dfrac{BC}{AB}.$
Cos A	$= \dfrac{GE}{AE} = \dfrac{AF}{AE}$	$=$	$\dfrac{AC}{AB}.$
Tan A	$= \dfrac{BD}{AB}$	$=$	$\dfrac{BC}{AC}.$
Cot A	$= \dfrac{CD}{AC}$	$=$	$\dfrac{AC}{BC}.$
Sec A	$= \dfrac{AD}{AB}$	$=$	$\dfrac{AB}{AC}.$
Cosec A	$= \dfrac{AD}{AC}$	$=$	$\dfrac{AB}{BC}.$

A little reflection will serve to impress upon the mind the equality of these ratios under the two circumstances I have illustrated.

Cotangent of Greater or Less Angles.—Here the cotangent and cosecant in Fig. 173 appear extravagantly out of proportion with the condition of those in Figs. 177 and 178, but seeing that we are dealing with ratios of lines one towards another, and not the actual lengths of the lines themselves, there will I think be no difficulty in comprehending this fact.

I have thus in some detail endeavoured to clear up a difficulty that appears to have presented itself to many students with regard to the relations of these functions, and having done so, I now proceed to consider the practical application of these ratios to the solution of triangles, for which purpose I shall abandon the more complicated reference letters, and, as illustrated in Fig. 179, shall refer to each side as a, b, or c, and the angles as A, B, or C. C being the right angle, c is the hypotenuse, and b is the side adjacent to the angle considered. The angle B is the complement of A, since two acute angles in a right-angled triangle must be always equal to one right angle (for all the angles of *every* triangle equal *two* right angles).

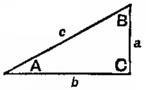


Fig. 179.

Hence, with the altered lettering, we have a new list of functions:—

$$\text{Sin A} = \frac{a}{c}. \qquad \text{Cos A} = \frac{b}{c}.$$

$$\text{Tan A} = \frac{a}{b}. \qquad \text{Cot A} = \frac{b}{a}.$$

$$\text{Sec A} = \frac{c}{b}. \qquad \text{Cosec A} = \frac{c}{a}.$$

If we know the numerical value of any one of these ratios we can find A. In other words, if the ratio between any two sides of a right-angled triangle is given we can define all the angles.

Now the relations of trigonometrical ratios to one another (since the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the two sides) are as follows :—

$$\text{Since } a^2 + b^2 = c^2,$$

$$\text{dividing by } c^2, \ \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2} = 1;$$

$$\text{or } \sin^2 \text{A} + \cos^2 \text{A} = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Dividing the first equation by b^2, we get $\left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2$; or reversing the order, $\sec^2 \text{A} = 1 + \tan^2 \text{A}$. . . . . (2)

Dividing the same by a^2, we get $1 + \left(\frac{b}{a}\right)^2 = \left(\frac{c}{a}\right)^2$; or reversing the order as before, $\text{cosec}^2 \text{A} = 1 + \cot^2 \text{A}$. . . . . . . (3)

Since $\frac{a}{b} \times \frac{b}{a} = 1$, $\tan \text{A} \cot \text{A} = 1$. . . . . (4)

$$\text{Again } \tan \text{A} = \frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}}, \ \therefore \tan \text{A} = \frac{\sin \text{A}}{\cos \text{A}}. \quad . \quad . \quad . \quad (6)$$

$$\text{Again } \cot \text{A} = \frac{b}{a} = \frac{1}{\frac{a}{b}}, \ \therefore \cot \text{A} = \frac{1}{\tan \text{A}}. \quad . \quad . \quad . \quad (7)$$

$$\text{Again } \cot \text{A} = \frac{b}{a} = \frac{\frac{b}{c}}{\frac{a}{c}}, \ \therefore \cot \text{A} = \frac{\cos \text{A}}{\sin \text{A}}. \quad . \quad . \quad . \quad (8)$$

$$\text{Again } \sec \text{A} = \frac{c}{b} = \frac{1}{\frac{b}{c}}, \ \therefore \sec \text{A} = \frac{1}{\cos \text{A}}. \quad . \quad . \quad . \quad (9)$$

$$\text{Again cosec A} = \frac{c}{a} = \frac{1}{\frac{a}{c}}, \ \therefore \text{cosec A} = \frac{1}{\sin \text{A}}. \quad . \quad . \quad . \quad (10)$$

$$\text{Vers A} = 1 - \cos \text{A}, \text{ and covers A} = 1 - \sin \text{A}. \quad . \quad . \quad (11)$$

The foregoing equations enable us to find the value of any function in terms of any other functions, thus :—

Sin A in Terms of Cos A.—Let it be required to express sin A in terms of cos A and *vice versâ.* By equation (1) we have seen that

$$\text{Sin}^2 A + \cos^2 A = 1. \quad \text{Consequently}$$

$$\text{Sin } A = \sqrt{1 - \cos^2 A} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\text{Cos } A = \sqrt{1 - \sin^2 A} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Tan A in Terms of Sin A.—Let it be required to express tan A in terms of sin A.

$\text{Tan } A = \dfrac{\sin A}{\cos A}$ (6), and in (13) we have seen $\cos A = \sqrt{1 - \sin^2 A}$,

$$\therefore \text{Tan } A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Tan A in Terms of Cos A.—Let it be required to express tan A in terms of cos A. Since by (6), $\tan A = \dfrac{\sin A}{\cos A}$; and, by (12), $\sin A = \sqrt{1 - \cos^2 A}$,

$$\therefore \text{Tan } A = \frac{\sqrt{1 - \cos^2 A}}{\cos A}. \quad . \quad . \quad . \quad . \quad (15)$$

Cos A in Terms of Tan A.—Let it be required to express cos A in terms of tan A.

$$\text{By equation (9) } \cos A = \frac{1}{\sec A}. \quad . \quad . \quad . \quad (15^*)$$

$$\text{But by equation (2) } \sec^2 A = 1 + \tan^2 A,$$

$$\therefore \sec A = \sqrt{1 + \tan^2 A},$$

$$\text{and therefore } \cos A = \frac{1}{\sqrt{1 + \tan^2 A}}. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Sin A in Terms of Tan A.—Let it be required to express sin A in terms of tan A. Now $\sin A = \cos A \times \tan A$, therefore by preceding article

$$\text{Sin } A = \frac{\tan A}{\sqrt{1 + \tan^2 A}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Sin A in Terms of Sec A.—Let it be required to express sin A in terms of sec A.

Since $\sin^2 A = 1 - \cos^2 A$; substituting (by (15*)) for $\cos A$, $\sin^2 A = 1 - \frac{1}{\sec^2 A}$; $\therefore$ reducing to a common denominator and taking the square root we have $\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$. . (18)

Cos A in Terms of Cosec A.—To express $\cos A$ in terms of $\operatorname{cosec} A$.

By (8) $\cot A = \frac{\cos A}{\sin A}$; $\therefore \cos A = \cot A \sin A$,

and $\therefore \cos A = \frac{\sqrt{\operatorname{cosec}^2 A - 1}}{\operatorname{cosec} A}$ . . (19). See (10) and (3)

Cot A in Terms of Sec A.—To express $\cot A$ in terms of $\sec A$.

$$\operatorname{Cot} A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}. \quad . \; . \; . \; . \; . \; . \; . \; . \; (20)$$

To express cosec A in terms of sec A.

$$\operatorname{Cosec} A = \frac{1}{\sin A} \text{ (see (10))} = \frac{1}{\sqrt{1 - \cos^2 A}} \text{ (see (12))}$$

$$= \frac{1}{\sqrt{1 - \frac{1}{\sec^2 A}}} = \frac{1}{\sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}},$$

and therefore $\operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$. . . . . . . (21)

To express sin A in terms of tan A.—Since $\sin A = \tan A$, $\cos A = \tan A \frac{1}{\sqrt{1 + \tan^2 A}}$; $\therefore \sin A = \frac{\tan A}{\sqrt{1 + \tan^2 A}}$. (22).

Following on, we arrive at these results :—

$\operatorname{Tan} A = \sqrt{\sec^2 A - 1}$ . . . . . (23). See (2)

$\operatorname{Sec} A = \sqrt{1 + \tan^2 A}$ . . . . . (24)

$\operatorname{Cot} A = \sqrt{\operatorname{cosec}^2 A - 1}$ . . . . . (25). See (3)

$\operatorname{Cosec} A = \sqrt{1 + \cot^2 A}$ . . . . . (26)

It is very desirable to learn to express every function in terms of every other function, as by means of working these out in detail the mind is impressed, and the relations of one function to another will become familiar.

Complemental Angles.—It has been shown that the complement of an angle (*i.e.* of an acute angle) is the difference between it and a right angle, or commonly called its "defect." Thus if the angle A be 30 deg. the complement will be 90 deg. − 30 deg. = 60 deg. Again, if the angle A = 56 deg. 16 min. then its complement will be 90 deg. − 56 deg. 16 min. = 33 deg. 44 min.

Now, I have endeavoured to explain by the trigonometrical canon the various functions, which are as follows:—To the lines which are the trigonometrical functions of the arc correspond certain ratios which are the trigonometrical functions of the angles which the arc subtends.

In Fig. 180 I have shown the angle A = 30 deg., the sine of this angle is B C, whilst the cosine is B D, and the angle B A E is its complement. Now the sine is that line lying within the arc which is perpendicular to the base, which in the angle B A C is B C. But if B D is perpendicular to E A, and since A D is the cosine of the angle B A E, and A D = B C, therefore the cosine of the angle B A E or the complement of A equals the sine of A.

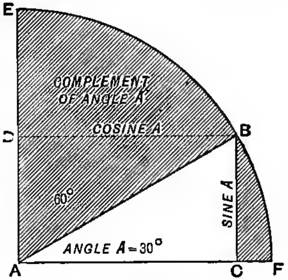


Fig. 180.

Thus we may deduce the following facts:—

The cosine of an angle is equal to the sine of its complement.

The cotangent of an angle is equal to the tangent of its complement.

The cosecant of an angle is equal to the secant of its complement, &c.

So far so good, referring to the diagram in Fig. 180; but I want to impress on the student that in trigonometry we have in practice to do without the canon and consider only the triangle.

Now, as a simple illustration, we will take the case of a right-angled triangle as Fig. 180, the angle B A C of which is 30 deg. We know B C A to be 90 deg., thereupon the angle A B C will be 90 deg. − 30 deg. = 60 deg., which is the complement.

If, as we have seen, sin A (Fig. 176) is $\frac{\text{PERP}}{\text{HYP}}$ or $\frac{\text{B C}}{\text{A B}}$, and cos A is $\frac{\text{BASE}}{\text{HYP}}$ or $\frac{\text{A C}}{\text{A B}}$; from the foregoing it will not be difficult to realise that in a triangle the functions of the angle and its complement are in the inverse ratio. To better illustrate this, somewhat anticipating the practical application of the foregoing, I may say that the value of the

Nat sin 30 deg. = 0·50000. Nat sin 60 deg. = 0·86603.
Nat cos 30 deg. = 0·86603. Nat cos 60 deg. = 0·50000.

Supplemental Angles.—*The supplement of an angle is the difference between it and two right angles.*

Thus two right angles are equal to 180 deg., consequently if the angle A $= 30^\circ$ the supplement will be $180^\circ - 30^\circ = 150^\circ$; or, if the angle A is $29^\circ\ 16'$, then the supplement will be $180^\circ - 29^\circ\ 16' = 150^\circ\ 44'$.

The sine of an angle is equal to the sine of its supplement.

In Fig. 181, C′ A B′ is the supplement of the angle F A B′, and is equal to F A B, and also C B is equal to C′ B′, and therefore

$\frac{C'B'}{AB'} = \frac{CB}{AB}$, but $\frac{CB}{AB}$ is the sine of the angle A, and $\frac{C'B'}{AB'}$, is the sine of the supplement, therefore they are equal.

The cosine of an angle is equal to the cosine of its supplement, but of opposite sign.

Use of the + and − Signs.—Before proceeding to reason this out it is necessary to speak of the conventional signs, plus and minus, used in trigonometry. As in Fig. 182, we may divide a circle into four quadrants, commencing with the first right-hand one above the horizontal or datum line F′ A F. With A as centre or origin, if a line revolving from the initial line A F forms any angle less than 90 deg., it is treated, as has been explained, as the angle A proper; but if this revolving line has passed through 90 deg. and makes therefore an angle greater than 90 deg. with the initial line, the supplement of this angle is less than 90 deg., and is the angle to be considered.

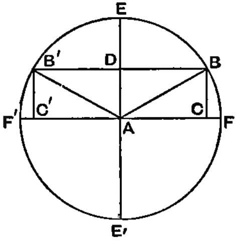


Fig. 181.

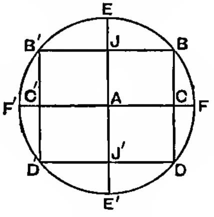


Fig. 182.

The definitions of trigonometrical functions are perfectly general, and therefore applicable to arcs of any magnitude. If an arc be greater than a quadrant some of the lines which have been defined as the trigonometrical functions lie to the left of the vertical diameter E A E′, and some below the horizontal diameter F′ A F. In order to take account of these variations of position, mathematicians have been led to adopt the following conventional signs as to the plus and minus, which enable us at the same time to represent the position as well as the magnitude of the line in question. Referring to Fig. 182, the lines F′ F and E E′ represent the horizontal and vertical diameter working around the centre or origin A. Now all horizontal lines, provided they are to the right of E E′, are positive or +, and those to the left are negative or −. Similarly, every vertical line, if it lie above F′ A F is positive, and negative if below that line. Thus A C is +, because it lies to the right of E E′; B C is + because it lies above A; and upon the same principle A C′ and C′ D′ are both −; B′ C′ is +, and C D is −.

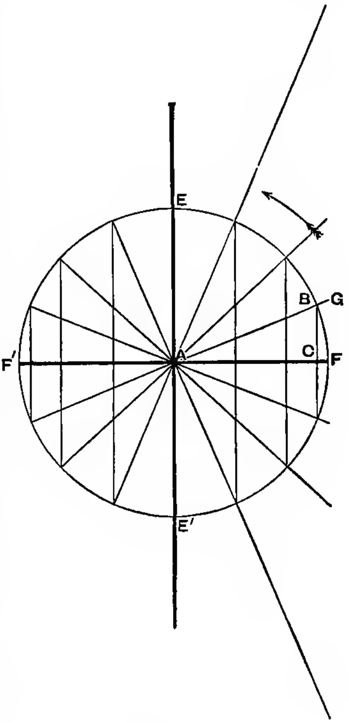


Fig. 183.

Referring to Fig. 183, if we trace the value of the sine in its progress round the circle from right to left, in direction of the arrow, we shall find that as the revolving line progresses through the four quadrants, in the first and second the sine is positive, whilst in the third and fourth it is negative. Now it has been established that—

1st. Any line drawn parallel to F′ A F to the right of E E′ is to be positive, and consequently any line drawn parallel to F A F′ to the left of E E′ is to be negative.

2nd. Any line drawn parallel to E′ A E above F′ F is positive, and consequently any line drawn parallel to E E′ below F′ F is negative.

3rd. The revolving line A B (Figs. 184, 185, 186, 187) is always positive.

We have previously seen that the following are some of the ratios.

$$\sin \mathrm{BAC} = \frac{\mathrm{CB}}{\mathrm{AB}}; \cos \mathrm{BAC} = \frac{\mathrm{AC}}{\mathrm{AB}}; \tan \mathrm{BAC} = \frac{\mathrm{BC}}{\mathrm{AC}}.$$

Therefore, keeping in mind that in the first quadrant C B is positive, being above G D; A C is positive because it is drawn to the right of E F, and A B is always positive.

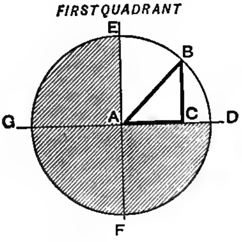


Fig. 184.

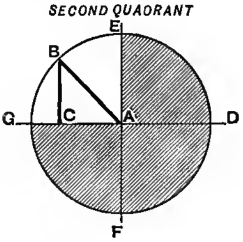


Fig. 185.

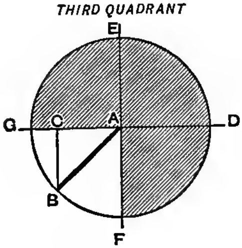


Fig. 186.

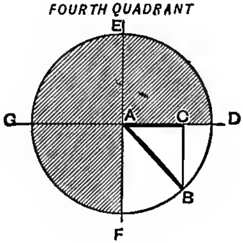


Fig. 187.

(1.) Thus if the angle A be anywhere within the first quadrant (Fig. 184)

Sin A $= \frac{\text{C B}}{\text{A B}}$ is positive; cos A $= \frac{\text{A C}}{\text{A B}}$ is positive; and tan A $= \frac{\text{B C}}{\text{A C}}$ is positive.

When the angle A lies in the second quadrant (Fig. 185) C B is positive, because above G D; A C is negative, because to the left of E F, and A B is positive.

(2.) Thus for second quadrant

Sin A $= \frac{\text{C B}}{\text{A B}}$ is positive; cos A $= \frac{\text{A C}}{\text{A B}}$ is negative; and tan A $= \frac{\text{C B}}{\text{A C}}$ is negative.

(3.) In the third quadrant (Fig. 186) C B is negative, A C is negative, and A B is positive, consequently

Sin A $= \frac{BC}{AB}$ is negative; cos A $= \frac{AC}{AB}$ is negative; and tan A $= \frac{BC}{AC}$ is positive.

(4.) In the fourth quadrant (Fig. 187) B C is negative, A C is positive, A B is positive. Thus—

Sin A $= \frac{BC}{AB}$ is negative; cos A $= \frac{AC}{AB}$ is positive; and tan A $= \frac{BC}{AC}$ is negative.

From the foregoing we can now tabulate the results as follows:—

TABLE I.

	First Quadrant.	Second Quadrant.	Third Quadrant.	Fourth Quadrant.
Sine . .	+	+	–	–
Cosine . .	+	–	–	+
Tangent . .	+	–	+	–

NOTE.—The secant, cosecant, and cotangent of the angle A have the same sign as the sine, cosine, and tangent of the angle A.

Now to prove that "the cosine of an angle is equal to the cosine of its supplement, but of opposite sign." Referring to Fig. 181, the lines A C and A C′ are equal, but being in different quadrants, A C lies in a different direction to A C′, and thus they have different signs.

Therefore, having regard to sign, $\frac{AC}{AB} = -\frac{AC'}{AB'}$;

Now $\frac{AC}{AB}$ = cos A, and $\frac{AC'}{AB'}$ = cos of the supplement of A (viz. C A B′)

$$\text{Cos } A = -\cos(180° - A). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Relations of Lines to Functions of the Angle of Reference.—Before proceeding any further in the practical application of the foregoing formulæ, I will speak of the relation the lines (or functions of the arc) bear to certain ratios, which are the trigonometrical functions of the angles which the arc subtends. They are as follows :—

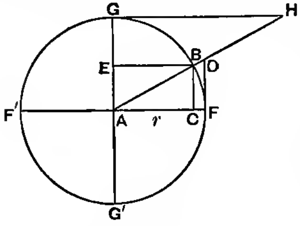


Fig. 188.

Definition—The sine, cosine, tangent, &c., of an angle at the centre of a circle is equal to the ratio of the sine, cosine, tangent, &c., of the corresponding arc to the radius of the circle.

The radius A F (Fig. 188) is denoted by r, and the angle H A F is denoted by A.

Then

$$\text{Sin A} = \frac{\text{B C}}{r};\ \cos \text{A} = \frac{\text{A C}}{r};\ \tan \text{A} = \frac{\text{D F}}{r};\ \sec \text{A} = \frac{\text{A D}}{r};$$

$$\cot \text{A} = \frac{\text{G H}}{r};\ \text{cosec A} = \frac{\text{A H}}{r};\ \text{vers A} = \frac{\text{C F}}{r};\ \text{and covers A} = \frac{\text{E G}}{r}.$$

Radius Unity.—In trigonometrical tables the radius is commonly taken as representing unity, and for practical purposes, if the radius is divided into the length of any one of the lines representing functions, it will give the value of that function.

Basis of Formulæ for Tables of Sines, &c.—It is necessary now to briefly consider how the foregoing equations may be worked out, so as to be of practical value. This has been done by many eminent mathematicians in the form of tables of natural sines, cosines, &c. With such available, it would be a waste of time to undertake calculations for ourselves, and a set of such tables sufficient for the purpose of this work will be found in the Appendix. To illustrate the basis upon which such tables are prepared, I will select a few examples, as follows, for angles of 18 deg., 30 deg., 45 deg., and 60 deg. I will take that of 45 deg. first.

By the equation (1), $\text{Sin}^2 \text{A} + \cos^2 \text{A} = 1.$

$$\therefore \text{Sin}^2 45^\circ + \cos^2 45 = 1.$$

But since the complement of 45° is $90^\circ - 45^\circ = 45^\circ$

$$\therefore \sin 45^\circ = \cos 45^\circ, \text{ and } \sin^2 45^\circ = \cos^2 45^\circ.$$

$$\therefore 2 \sin^2 45^\circ = 1;\ \text{and } 2 \cos^2 45^\circ = 1.$$

$$\therefore \sin^2 45^\circ = \tfrac{1}{2}, \text{ and } \sin 45^\circ = \frac{1}{\sqrt{2}} = 0{\cdot}70711;$$

$$\text{Similarly, } \cos 45^\circ = 0{\cdot}70711.$$

$$\text{Again, by (6), } \tan A = \frac{\sin A}{\cos A},$$

$$\therefore \tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{0{\cdot}70711}{0{\cdot}70711} = 1.$$

$$\text{Then by (7), } \cot A = \frac{1}{\tan A},$$

$$\therefore \cot 45^\circ = \frac{1}{\tan 45^\circ} = 1.$$

$$\text{Similarly, by (9), } \sec A = \frac{1}{\cos A},$$

$$\therefore \sec 45^\circ = \frac{1}{\cos 45^\circ} = \frac{1}{0{\cdot}70711} = 1{\cdot}41421.$$

$$\text{And finally, by (10), } \operatorname{cosec} A = \frac{\sin A}{1},$$

$$\therefore \operatorname{cosec} 45^\circ = \frac{1}{\sin 45^\circ} = \frac{1}{0{\cdot}70711} = 1{\cdot}41421.$$

Sines, &c., for 45 Degrees.—The following is the result of the preceding investigations :—

Sin 45° = 0·70711.
Cos 45° = 0·70711.
Tan 45° = 1·00000.
Cot 45° = 1·00000.
Sec 45° = 1·41421.
Cosec 45° = 1·41421.

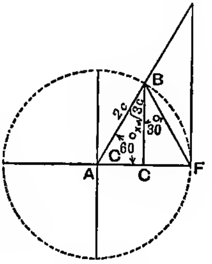


Fig. 189.

In the case of the angle of 60 deg., the revolving line forms a portion of an equilateral triangle, whereof A B, A F, and F B (Fig. 179), are equal sides, consequently the line B C, or sine, bisects the triangle ; now the angle B A C = 60 deg. and the angle A B C = 30 deg., therefore as the length of the base A F is equal to that of the two other sides, then A C is half A F.

Sines, &c., for 60 Degrees.—Let B C be represented by x, A C by c, A B by $2\ c$.

$$\text{Then } x^2 = (2\ c)^2 - c^2 = 4\ c^2 - c^2 = 3\ c^2$$
$$\therefore x = \sqrt{3} \times c$$

$$\text{And since } \sin 60^\circ = \sin \text{BAC} = \frac{\text{BC}}{\text{AB}} = \frac{\sqrt{3} \times c}{2\ c} = \frac{\sqrt{3}}{2} = 0{\cdot}86603$$

$$\text{Again, } \cos 60^\circ = \frac{\text{AC}}{\text{AB}} = \frac{c}{2\ c} = \qquad \tfrac{1}{2} = 0{\cdot}50000$$

$$\text{And } \tan 60^\circ = \tan \text{BAC} = \frac{\text{BC}}{\text{AC}} = \frac{\sqrt{3} \times c}{c} = \frac{\sqrt{3}}{1} = \sqrt{3} = 1{\cdot}7321$$

$$\text{Cot } 60^\circ = \quad \cot \text{BAC} = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}} \qquad = 0{\cdot}57735$$

$$\text{Sec } 60^\circ = \quad \sec \text{BAC} = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2 \qquad = 2{\cdot}0000$$

$$\text{Cosec } 60^\circ = \quad \text{cosec BAC} = \frac{1}{\sin 60^\circ} = \frac{2}{\sqrt{3}} \qquad = 1{\cdot}15470$$

Again, take the angle of 30 deg., when, because A C is half A F (Fig. 189), and the angle A B F, which is 60 deg., is bisected by B C, then A B C = F B C = ½ the angle A B F = 30 deg.

Thus—

$$\text{Sin } 30^\circ = \sin \text{ABC} = \frac{\text{CA}}{\text{BA}} = \frac{c}{2\ c} = \tfrac{1}{2} \qquad = 0{\cdot}50000$$

Sines, &c., for 30 Degrees.—

$$\text{Cos } 30^\circ = \cos \text{ABC} = \frac{\text{BC}}{\text{BA}} = \frac{\sqrt{3} \times c}{2\ c} = \frac{\sqrt{3}}{2} = 0{\cdot}86603$$

$$\text{Tan } 30^\circ = \tan \text{ABC} = \frac{\text{CA}}{\text{CB}} = \frac{c}{c\sqrt{3} \times c} = \frac{1}{\sqrt{3}} = 0{\cdot}57735$$

$$\text{Cot } 30^\circ = \cot \text{ABC} = \frac{\text{BC}}{\text{CA}} = \frac{c\sqrt{3} \times c}{c} = \sqrt{3} = 1{\cdot}7321$$

$$\text{Sec } 30^\circ = \sec \text{ABC} = \frac{\text{BA}}{\text{BC}} = \frac{2c}{c\sqrt{3} \times c} = \frac{2}{\sqrt{3}} = 1{\cdot}15470$$

$$\text{Cosec } 30^\circ = \text{cosec ABC} \quad = \frac{\text{BA}}{\text{AC}} = \frac{2\ c}{c} = 2 \quad = 2{\cdot}0000$$

Sines, &c., for 60 and 30 Degrees.—From the foregoing results we may tabulate the natural sines, &c., of the angles 60 and 30 degrees respectively, viz.:—

Sine of 60° = 0·86603.	Sine 30° = 0·50000.
Cos of 60° = 0·50000.	Cos 30° = 0·86603.
Tan of 60° = 1·73210.	Tan 30° = 0·57735.
Cotan of 60° = 0·57735.	Cotan 30° = 1·73210.
Sec of 60° = 2·00000.	Sec 30° = 1·15470.
Cosec of 60° = 1·15470.	Cosec 30° = 2·00000.

Thus it will be seen that the value of the sine of 60 deg. = cos 30 deg.; tan 60 deg. = cot 30 deg.; and sec 60 deg. = cosec 30 deg., and *vice versâ.*

Now, take the angle 18 deg. as another example, of which it is required to find the sine, cosine, and tangent, &c.

Sines, &c., for 18 Degrees.—Let the angle B A C (Fig. 190) = 18 deg., drop the perpendicular B C, which produce to meet the circumference in B′, then it is evident that the angle B A B′ is twice the angle B A C, or 36 deg. B B′ is therefore one side of a decagon, inscribed in the circle; and therefore B B′ is equal to the greater segment of the radius cut in extreme and mean ratio (Euclid IV. 11, and II. 11), and therefore

$$BB'^2 = AF\,(AF - BB')$$

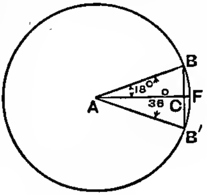


Fig. 190.

Solving this as an ordinary quadratic equation

$$\text{we get } BB' = AF \times \frac{\sqrt{5} - 1}{2}$$

But B C = ½ B B′, therefore

$$\text{Sin } 18^\circ = \frac{BB'}{2\,AF} = \frac{\sqrt{5} - 1}{4} \qquad = 0{\cdot}30902$$

$$\text{Cos } 18^\circ = \sqrt{1 - \sin^2 18^\circ} = \sqrt{1 - {\cdot}30902^2} \qquad = 0{\cdot}95106$$

By (6), page 114

$$\text{Tan } 18^\circ = \frac{\sin\ 18^\circ}{\cos\ 18^\circ} = \frac{{\cdot}30902}{{\cdot}95106} \qquad = 0{\cdot}32492$$

and by (7)

$$\text{Cot } 18^\circ = \frac{1}{\tan\ 18^\circ} = \frac{1}{{\cdot}32492} \qquad = 3{\cdot}07768$$

and by (9)

$$\text{Sec } 18^\circ = \frac{1}{\cos 18^\circ} = \frac{1}{\cdot 95106} \qquad = 1\cdot 05146$$

and by (10)

$$\text{Cosec } 18^\circ = \frac{1}{\sin 18^\circ} = \frac{1}{\cdot 30902} \qquad = 3\cdot 23607$$

From the foregoing we can now tabulate the following:—

Sin	18°	=	0·30902
Cos	18°	=	0·95106
Tan	18°	=	0·32492
Cot	18°	=	3·07768
Sec	18°	=	1·05146
Cosec	18°	=	3·23607

As far as we have gone we have considered only angles less than 90 deg., but it is necessary to briefly investigate what happens when the revolving line A B (Figs. 184, 185, 186, 187) passes the first quadrant. We will take 120 deg., or 90 deg. + 30 deg. as the angle B A D. Now we are dealing with two right angles, consequently the angle B A D if deducted from 180 deg. will give us the value of B A G or 180 deg. − 120 deg. = 60 deg. = B A G.

Sines, &c., for 120 Degrees.—Therefore, sine 120 deg. $= \frac{BC}{AB}$ which, referring to page 118, is equal to the sine of 60 deg., its supplement.

Therefore, sin 120 deg. = sin. 60 deg., and being in the second quadrant as we have seen in Table I. (page 121), it is positive, whilst the cosine and tangent are negative.

Thus

$$\text{Sin } 120^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Cos } 120^\circ = -\ \tfrac{1}{2}$$

$$\text{Tan } 120^\circ = -\ \sqrt{3}.$$

Sines, &c., for 225 Degrees.—Passing into the third quadrant, suppose it be required to find the sine, cosine, tangent, &c., of 225 deg.

Then 225 deg. − 180 deg. = 45 deg. = B A G (Fig. 186), and in the third quadrant from the Table I. we have seen that the sine and cosine are negative whilst the tangent is positive.

Consequently

$$\text{Sin } 225^\circ = -\frac{1}{\sqrt{2}} \quad \text{(page 123)}$$

$$\text{Cos } 225^\circ = -\frac{1}{\sqrt{2}} \quad (\ \ ,, \ \ )$$

$$\text{Tan } 225^\circ = \quad 1 \quad (\ \ ,, \ \ )$$

From the foregoing remarks we have seen the various functions of right-angled triangles, and have been able to deduce certain formulæ which enable us to arrive at the numerical value of each. These values are what are termed natural sines, cosines, &c., and they are based upon the understanding that the radius is always unity, in other words they are relatively circumstanced to unity. Thus sin 45 deg. = 0·70711, but the tan 45 deg. and the cotan 45 deg. = 1 = radius. To illustrate my meaning:—

Ratio of Radius.—Suppose the radius of a circle to be 40 ft., and a right-angled triangle formed by the base, perpendicular and hypotenuse of an angle of 45 deg. as in Fig. 191. A F = A B = 40 ft., and it is required to know the length of B C; referring to the trigonometrical canon (Fig. 172), we find E F (which is the same as B C in Fig. 191) is the sine.

Therefore as we have seen that sin 45 deg. = 0·70711, then if we multiply 0·70711 by 40 we shall get the length B C = 28·28440 ft., so that 28·28440 represents the ratio of B C to the radius 40 ft. just exactly as 0·70711 is its ratio to the radius of unity.

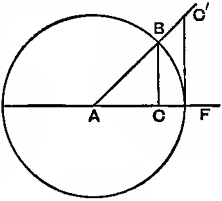


Fig. 191.

Again, if we want the length A C′ we know by our canon that A C′ is the secant (and also the cosecant of 45 deg.). Now our tables tell us that sec 45 deg. = 1·41421, therefore this multiplied by the radius of 40 ft. gives us

1·41421 × 40 ft. = 56·56840 ft. = the length A C′.

Now B A C = 45°, ∴ A C = B C = 28·28440 ft.

At the risk of being considered irregular, if not too elementary, I have elected to illustrate the foregoing examples in a somewhat rule-of-thumb style, for this work does not profess to do more than seek, by as graphic a manner as possible, to bridge over many of the difficulties which the student has to encounter.

Solution of Right-angled Triangles.—All triangles consist of six parts, viz., three sides and three angles; and it is possible with three of these, one part at least being a side, to find the others. Referring back to Fig. 179, if we take the sides as represented by a, b, and c, and the angles by A, B, and C, with the following approximate lengths of each, $a = 21{\cdot}838$ feet, $b = 60$ feet, and $c = 63{\cdot}851$ feet, we have the following results.

We have seen that $\frac{a}{b} = \tan \text{A}$, then $\tan \text{A} = \frac{a}{b} = \frac{21{\cdot}838}{60{\cdot}00}$ $= 0{\cdot}36397$, which by reference to a table of natural tangents indicates that the angle $\text{A} = 20°$. And since C is 90°, then $\text{B} = 90° - 20° = 70°$.

Take $b = 60$ and $c = 63{\cdot}851$. Then as $\frac{b}{c}$ is cos A,

$$\therefore \cos \text{A} = \frac{60}{63{\cdot}851} = 0{\cdot}93969.$$

Take $a = 21{\cdot}838$ and $\text{B} = 70°$, $c = \frac{a}{\cos \text{B}} = \frac{21{\cdot}838}{{\cdot}34202} = 63{\cdot}851$ ft.

Take $c = 63{\cdot}851$ and $\text{A} = 20°$. Then $a = c \sin \text{A} = 63{\cdot}851 \times 0{\cdot}34202 = 21{\cdot}838$ feet, and $b = c \cos \text{A} = 63{\cdot}851 \times 0{\cdot}93969 = 60$ ft.

Trigonometrical Ratios of Two Angles.—It has been clearly established that the relations between the sine, cosine, tangent, &c., of the sum or difference of two or more angles, and the sines, cosines, &c., of the angles themselves, are based on the following fundamental propositions:—

$$\text{Sin}\ (\text{A} + \text{B}) = \sin \text{A} \cos \text{B} + \cos \text{A} \sin \text{B}. \quad . \quad . \quad (28)$$
$$\text{Cos}\ (\text{A} + \text{B}) = \cos \text{A} \cos \text{B} - \sin \text{A} \sin \text{B}. \quad . \quad . \quad (29)$$
$$\text{Sin}\ (\text{A} - \text{B}) = \sin \text{A} \cos \text{B} - \cos \text{A} \sin \text{B}. \quad . \quad . \quad (30)$$
$$\text{Cos}\ (\text{A} - \text{B}) = \cos \text{A} \cos \text{B} + \sin \text{A} \sin \text{B}. \quad . \quad . \quad (31)$$

In this case (Fig. 192) A and B are the angles. Sin (A + B) is a fraction, but sin A + sin B is the sum of two fractions, and care should be taken to avoid any misunderstanding.

Then let us take H O G = angle A and G O F = the angle B. Then H O F = angle (A + B). In the line O F which bounds the angle (A + B) take any point P, and let drop the perpendicular P Q on O G, and P S on O H. Draw the perpendiculars Q R and Q T to the lines P S and O H.

Then

$$\text{QPR} = 90° - \text{RQP} = \text{RQO} = \text{HOG} = \text{A}$$

Now

$$\text{Sin }(A+B) = \sin HOF = \frac{PS}{OP} = \frac{SR+RP}{OP} = \frac{QT}{OP} + \frac{RP}{OP}$$

$$= \frac{QT}{OQ} \times \frac{OQ}{OP} + \frac{PR}{PQ} \times \frac{PQ}{OP}$$

$$= \sin HOG \cos GOF + \cos RPQ \sin GOF$$

$$= \sin A \cos B + \cos A \sin B.$$

Again

$$\text{Cos }(A+B) = \cos HOF = \frac{OS}{OP} = \frac{OT-ST}{OP} = \frac{OT}{OP} - \frac{RQ}{OP}$$

$$= \frac{OT}{OQ} \times \frac{OQ}{OP} - \frac{RQ}{QP} \times \frac{QP}{OP}$$

$$= \cos HOG \cos GOP - \sin RPQ \sin GOP$$

$$= \cos A \cos B - \sin A \sin B.$$

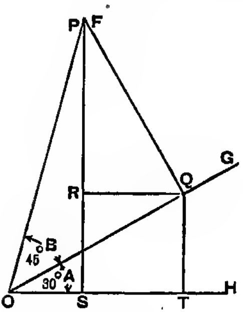


Fig. 192.

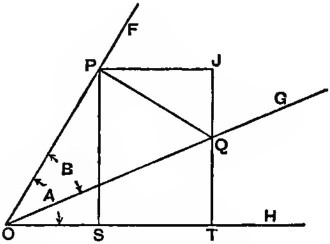


Fig. 193.

To prove that

Sin $(A-B) = \sin A \cos B - \cos A \sin B$, and $\cos(A-B) = \cos A \cos B + \sin A \sin B$.

Let HOF (Fig. 193) = the angle A and GOF = the angle B. Consequently HOG is the angle $(A-B)$.

In OG take any point Q, and from this let drop the perpendiculars QT, QP, on OH, OF. Then draw PJ at right angles to QT produced, and PS at right angles to OH.

Then the angle $PQJ = 90^\circ - JPQ = JPF = HOF$ = angle A. Thus

$$\text{Sin }(A-B) = \sin HOG = \frac{TQ}{OQ} = \frac{TJ-QJ}{OQ} = \frac{SP}{OQ} - \frac{QJ}{OQ}$$

$$= \frac{SP \times OP}{OP \times OQ} - \frac{QJ \times PQ}{PQ \times OQ} = \frac{SP}{OP} \times \frac{OP}{OQ} - \frac{QJ}{PQ} \times \frac{PQ}{OQ}$$

$$= \sin HOF \cos GOF - \cos JQP \sin GOF$$

$$= \sin A \cos B - \cos A \sin B.$$

Similarly

$$\text{Cos}\ (\text{A} - \text{B}) = \cos \text{HOG} = \frac{\text{OT}}{\text{OQ}} = \frac{\text{OS} + \text{ST}}{\text{OQ}} = \frac{\text{OS}}{\text{OQ}} + \frac{\text{PJ}}{\text{OQ}}$$

$$\frac{\text{OS} \times \text{OP}}{\text{OP} \times \text{OQ}} + \frac{\text{PJ} \times \text{PQ}}{\text{PQ} \times \text{OQ}} = \frac{\text{OS}}{\text{OP}} \times \frac{\text{OP}}{\text{OQ}} + \frac{\text{PJ}}{\text{PQ}} \times \frac{\text{PQ}}{\text{OQ}}$$

$$= \cos \text{HOF} \cos \text{GOF} + \sin \text{JQP} \sin \text{GOF}$$

$$= \cos \text{A} \cos \text{B} + \sin \text{A} \sin \text{B}.$$

To illustrate the foregoing formulæ we will find the value of sin 75°.

By the preceding

$\text{Sin}\ 75° = \sin (45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°.$

And we have seen (pp. 123, 124) that

$$\sin 45° = \frac{1}{\sqrt{2}};\ \cos 45° = \frac{1}{\sqrt{2}};\ \sin 30° = \frac{1}{2};\ \cos 30° = \frac{\sqrt{3}}{2}$$

Therefore

$\text{Sin}\ 75° = \sin 45° \cos 30° + \cos 45° \sin 30°$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} = \frac{\sqrt{2}\,(\sqrt{3} + 1)}{4}$$

$$= \frac{1{\cdot}41421\,(1{\cdot}73204 + 1)}{4} = \frac{3{\cdot}8636924305}{4} = 0{\cdot}96592.$$

Again

$\text{Cos}\ 75° = \cos 45° \times \cos 30° - \sin 45° \times \sin 30°$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} = 0{\cdot}25882.$$

From the foregoing remarks we have seen that :—

1st. The sine of the sum of two angles is equal to the sine of the first into the cosine of the second, together with the cosine of the first into the sine of the second.

2nd. The cosine of the sum of two angles is equal to the product of the cosines of the angles less the product of their sines.

3rd. The sine of the difference of two angles is equal to the sine of the first angle into the cosine of the second less the cosine of the first into the sine of the second.

4th. The cosine of the difference of the two angles is equal to the product of the cosines of the angles, together with the product of their sines.

Again

The tangent of the sum of two angles is equal to the sum of their tangents, divided by unity less the product of their tangents.

Take the angles A and B as before. Then

$$\text{Tan }(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

And in proof of this, if we use the foregoing formulæ, we have as follows:—

$$\text{Tan }(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$$

And dividing the numerator and denominator by cos A cos B, we have

$$\text{Tan }(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=1-\frac{\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}}{\dfrac{\sin A}{\cos A}\times\dfrac{\sin B}{\cos B}}$$

Therefore

$$\text{Tan }(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}\quad\ldots\quad(32)$$

And similarly

$$\text{Tan }(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\quad\ldots\quad(33)$$

We have seen by the fundamental formulæ (p. 128) that

$$\begin{aligned}\text{Sin }(A+B)&=\sin A\cos B+\cos A\sin B\\ \text{Sin }(A-B)&=\sin A\cos B-\cos A\sin B\\ \text{Cos }(A+B)&=\cos A\cos B-\sin A\sin B\\ \text{Cos }(A-B)&=\cos A\cos B+\sin A\sin B\end{aligned}$$

And from these, by addition and subtraction, we get

Sum and Difference of Sines and Cosines.—

$$\begin{aligned}\text{Sin }(A+B)+\sin(A-B)&=2\sin A\cos B\\ \text{Sin }(A+B)-\sin(A-B)&=2\cos A\sin B\\ \text{Cos }(A+B)+\cos(A-B)&=2\cos A\cos B\\ \text{Cos }(A-B)-\cos(A+B)&=2\sin A\sin B\end{aligned}$$

The sum of the sines of any two angles is to the difference of their sines in the same ratio as the tangent of half their sum is to the tangent of half their difference,

Or,

$$\text{Sin } A + \sin B : \sin A - \sin B :: \tan \tfrac{1}{2}(A + B) : \tan \tfrac{1}{2}(A - B).$$

For, from the preceding formulæ,

$$\frac{\text{Sin } A + \sin B}{\text{Sin } A - \sin B} = \frac{2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)}{2 \sin \frac{1}{2}(A - B) \cos \frac{1}{2}(A + B)}$$
$$= \text{Tan } \tfrac{1}{2}(A + B) \cot \tfrac{1}{2}(A - B).$$

Or in the form of proportion,

$$\text{Sin } A + \sin B : \sin A - \sin B :: \tan \tfrac{1}{2}(A + B) : \tan \tfrac{1}{2}(A - B).$$

The Sine and Cosine of Twice an Angle, in Terms of the Sine and Cosine of the Angle.—By putting $A = B$ in eq. (28) we get $\sin 2A = 2 \sin A \cos A$. In eq. (29) we get $\cos 2A = \cos^2 A - \sin^2 A$; and it was shown by eq. (1) that $1 = \cos^2 A + \sin^2 A$; whence by addition and subtraction we obtain

$$1 + \cos 2A = 2 \cos^2 A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$
$$\text{and } 1 - \cos 2A = 2 \sin^2 A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

By transposition the following expressions for the cosine of twice the angle are obtained:—

$$\text{Cos } 2A = 1 - 2 \sin^2 A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$
$$\text{Cos } 2A = 2 \cos^2 A - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

The Sine and Cosine of an Angle in Terms of Half the Angle.—Putting A for $2A$ on the left, and $\frac{1}{2}A$ for A on the right-hand side of the above equations

$$\text{Sin } A = 2 \sin \tfrac{1}{2}A \cos \tfrac{1}{2}A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$
$$1 + \cos A = 2 \cos^2 \tfrac{1}{2}A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$
$$1 - \cos A = 2 \sin^2 \tfrac{1}{2}A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g)$$
$$\text{Cos } A = 2 \cos^2 \tfrac{1}{2}A - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (h)$$
$$\text{Cos } A = 1 - 2 \sin^2 \tfrac{1}{2}A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

Sine, Cosine, and Tangent of the Sum of Three Angles.—

$$\text{Sin}(A + B + C) = \sin(A + B) \cos C + \cos(A + B) \sin C$$
$$= \sin A \cos B \cos C + \sin B \cos C \cos A$$
$$+ \sin C \cos A \cos B - \sin A \sin B \sin C. \quad . \quad . \quad . \quad (k)$$

$$\cos(A + B + C) = \cos(A + B) \cos C - \sin(A + B) \sin C$$
$$= \cos A \cos B \cos C - \cos A \sin B \sin C$$
$$- \cos B \sin A \sin C - \cos C \sin A \sin B. \quad . \quad . \quad (l)$$

$$\text{Tan}\ (\text{A}+\text{B}+\text{C}) = \frac{\sin\ (\text{A}+\text{B}+\text{C})}{\cos\ (\text{A}+\text{B}+\text{C})} =$$

$$\frac{\sin \text{A} \cos \text{B} \cos \text{C} + \sin \text{B} \cos \text{C} \cos \text{A} + \sin \text{C} \cos \text{A} \cos \text{B} - \sin \text{A} \sin \text{B} \sin \text{C}}{\cos \text{A} \cos \text{B} \cos \text{C} - \cos \text{A} \sin \text{B} \sin \text{C} - \cos \text{B} \sin \text{A} \sin \text{C} - \cos \text{C} \sin \text{A} \sin \text{B}}.$$

Dividing both numerator and denominator of the last expression by cos A cos B cos C, we obtain the tangent of the sum of three angles in terms of the tangents of the angles themselves—

$$\text{Tan}\,(\text{A}+\text{B}+\text{C}) = \frac{\tan \text{A} + \tan \text{B} + \tan \text{C} - \tan \text{A} \tan \text{B} \tan \text{C}}{1 - \tan \text{A} \tan \text{B} - \tan \text{B} \tan \text{C} - \tan \text{C} \tan \text{A}} \ . \quad (m)$$

The Sine, Cosine, and Tangent of Three Times an Angle.—In the above equations (*k*) (*l*) and (*m*), put A = B = C, then

$$\text{Sin}\ 3\,\text{A} = 3 \sin \text{A} - 4 \sin^3 \text{A} \quad . \ . \ . \ . \ . \ . \quad (n)$$

$$\text{Cos}\ 3\,\text{A} = 4 \cos^3 \text{A} - 3 \cos \text{A} \quad . \ . \ . \ . \ . \ . \quad (o)$$

$$\text{Tan}\ 3\,\text{A} = \frac{3 \tan \text{A} - \tan^3 \text{A}}{1 - 3 \tan^2 \text{A}} \quad . \ . \ . \ . \ . \ . \ . \quad (p)$$

As another proof the latter

$$\text{Tan}\ 3\,\text{A} = \tan\ (2\,\text{A} + \text{A}) = \frac{\tan 2\,\text{A} + \tan \text{A}}{1 - \tan 2\,\text{A} \tan \text{A}}$$

$$= \frac{\dfrac{2 \tan \text{A}}{1 - \tan^2 \text{A}} + \tan \text{A}}{1 - \dfrac{2 \tan \text{A}}{1 - \tan^2 \text{A}} \tan \text{A}} = \frac{2 \tan \text{A} + \tan \text{A} - \tan^3 \text{A}}{1 - \tan^2 \text{A} - 2 \tan^2 \text{A}}$$

$$= \frac{3 \tan \text{A} - \tan^3 \text{A}}{1 - 3 \tan^2 \text{A}}$$

Oblique-angled Triangles.—I now pass on to the consideration of oblique-angled triangles, which, in the limited space at my command, I can discuss only in brief terms. I will commence by submitting the following propositions:—

A. Any two sides of a plane triangle are in the same ratio as the sines of the opposite angles.

B. In a plane triangle, the sum of their sides is to their difference, as the tangent of half the sum of the angles at the base is to the tangent of half their difference.

C. In a plane triangle, the sum of the sides is to the base as the cosine of half the difference of the base angles is to the cosine of half their sum; and the difference of the sides is to the base as the sine of half the difference of the base angles is to the sine of half their sum.

D. The square on a side of a plane triangle, which is opposite an acute or obtuse angle, is equal to the sum of the squares on the sides which contain the angle, less twice the rectangle contained by them, into the cosine of the angle.

The foregoing propositions form the basis of the consideration of the formulæ for the solution of oblique angles, and we will briefly consider them *seriatim* :—

Proposition A. Take the triangle A B C (Figs. 194 and 195), and from C drop the perpendicular C D on to A B in Fig. 194 of A B produced in Fig. 195. Then

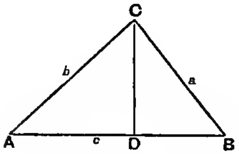


Fig. 194.

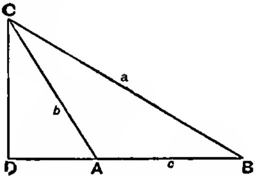


Fig. 195.

$$\frac{a}{b} = \frac{\sin \text{A}}{\sin \text{B}}; \text{ for } \sin \text{A} = \frac{\text{C D}}{b} \text{ and } \sin \text{B} = \frac{\text{C D}}{a}$$

Therefore

$$\frac{\sin \text{A}}{\sin \text{B}} = \frac{\frac{\text{C D}}{b}}{\frac{\text{C D}}{a}} = \frac{a}{b}$$

Similarly

$$\frac{a}{c} = \frac{\sin \text{A}}{\sin \text{C}}; \ \frac{b}{c} = \frac{\sin \text{B}}{\sin \text{C}}.$$

It should be noted that if the angle A or B be a right angle, there is no necessity to drop the perpendicular C D. From this proposition we may state the ratio between the sides and the sines of opposite angles. Thus

$$\frac{\sin \text{A}}{a} = \frac{\sin \text{B}}{b} = \frac{\sin \text{C}}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad (q)$$

Proposition B. From the preceding we have

$$\frac{a}{b} = \frac{\sin \text{A}}{\sin \text{B}}$$

Then by rule of proportion

$$\frac{a+b}{a-b} = \frac{\sin \text{A} + \sin \text{B}}{\sin \text{A} - \sin \text{B}}$$

Whence (see p. 130)

$$\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$$

which may be treated thus—

Since $\frac{1}{2}(A+B)=\frac{1}{2}(180^\circ-C);$

Therefore

$$\tan\tfrac{1}{2}(A+B)=\tan(90^\circ-\tfrac{1}{2}C)=\cot\tfrac{1}{2}C;$$

$$\therefore\ \frac{a+b}{a-b}=\frac{\cot\frac{1}{2}C}{\tan\frac{1}{2}(A-B)}=\cot\tfrac{1}{2}(A-B)\cot\tfrac{1}{2}C$$

Whence

$$\frac{a-b}{a+b}=\tan\tfrac{1}{2}(A-B)\tan\tfrac{1}{2}C$$

Proposition C.

$$A+B=180^\circ-C,\ \therefore\ \sin(A+B)=\sin C$$

$$\therefore\ \frac{a}{c}=\frac{\sin A}{\sin(A+B)},\ \text{and}\ \frac{b}{c}=\frac{\sin C}{\sin(A+B)}.$$

And by equations (*k*) *et seqq.* we get

$$\frac{a+b}{c}=\frac{\sin A+\sin B}{\sin(A+B)}=\frac{2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}{2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A+B)}$$

Consequently $$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}.$$

And similarly by subtracting the second from the first equation instead of adding,

$$\frac{a-b}{c}=\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}.$$

Proposition D. In the case of an acute angle, Fig. 194,

$$BC^2=AC^2+AB^2-2\,AB\times AD\ \text{(Euclid, ii. 13)}.$$

But $$\cos A=\frac{AD}{AC},\ \therefore\ AD=AC\cos A$$

and $$\therefore\ BC^2=AC^2+AB^2-2\,AB\times AC\cos A.$$

In the case of an obtuse angle, Fig. 195,

$$BC^2=AC^2+AB^2+2\,AB\times AD$$

But $$AD=AC\cos(180^\circ-A)=-AC\cos A$$

and $$\therefore\ BC^2=AC^2+AB^2-2\,AB\times AC\times\cos A$$

Therefore $$a^2=b^2+c^2-2\,b\,c\cos A$$

Similarly $$b^2=c^2+a^2-2\,c\,a\cos B$$

and $$c^2=a^2+b^2-2\,a\,b\cos C.$$

Sines and Cosines of Angles in Terms of Sides.—From the foregoing we get by transposition :—

$$\text{Cos A} = \frac{b^2 + c^2 - a^2}{2\,b\,c} \quad . \quad . \quad . \quad . \quad . \quad (r)$$

$$\text{Cos B} = \frac{c^2 + a^2 - b^2}{2\,c\,a}$$

$$\text{Cos C} = \frac{a^2 + b^2 - c^2}{2\,a\,b}$$

Now $\text{Sin}^2\,\text{A} = 1 - \cos^2\,\text{A}$

$$= (1 + \cos\text{A})\quad(1 - \cos\text{A})$$

$$= 1 - \frac{(b^2 + c^2 - a^2)^2}{4\,b^2\,c^2} = \frac{2\,b^2\,c^2 + 2\,c^2\,a^2 + 2\,a^2\,b^2 - a^4 - b^4 - c^4}{4\,b^2\,c^2},$$

therefore $\sin\text{A} = \dfrac{\sqrt{2\,b^2\,c^2 + 2\,c^2\,a^2 + 2\,a^2\,b^2 - a^4 - b^4 - c^4}}{2\,b\,c}.$

If, however, we substitute s for $\dfrac{a+b+c}{2}$ (or, as it is sometimes designated, the *semiperimeter* of the triangle) so that $(a + b + c) = 2\,s$, and

$$2\,(s - a) = b + c - a,$$
$$2\,(s - b) = a + c - b,$$
$$2\,(s - c) = a + b - c;$$

then by extracting the root we get

$$\text{Sin A} = \frac{2\sqrt{s\,(s-a)\,(s-b)\,(s-c)}}{b\,c},$$

$$\text{Sin B} = \frac{2\sqrt{s\,(s-a)\,(s-b)\,(s-c)}}{c\,a},$$

$$\text{Sin C} = \frac{2\sqrt{s\,(s-a)\,(s-b)\,(s-c)}}{a\,b}.$$

Sines and Cosines of Semi-angles.—We have (g) seen that

$$\text{Sin}^2\,\tfrac{1}{2}\,\text{A} = \tfrac{1}{2}\,(1 - \cos\text{A}) = \tfrac{1}{2}\left(1 - \frac{b^2 + c^2 - a^2}{2\,b\,c}\right)$$

$$= \frac{a^2 - (b - c)^2}{4\,b\,c} = \frac{(a - b + c)\,(a + b - c)}{4\,b\,c} = \frac{(s - b)\,(s - c)}{b\,c}$$

and extracting the square root we get

$$\text{Sin}\,\tfrac{1}{2}\,\text{A} = \sqrt{\frac{(s - b)\,(s - c)}{b\,c}}$$

and similarly

$$\text{Sin } \tfrac{1}{2}\,\text{B} = \sqrt{\frac{(s-c)\,(s-a)}{c\,a}},$$

and

$$\text{Sin } \tfrac{1}{2}\,\text{C} = \sqrt{\frac{(s-a)\,(s-b)}{a\,b}}.$$

Again, by (*f*) and (*r*)

$$\text{Cos } \tfrac{1}{2}\,\text{A} = \sqrt{\frac{s\,(s-a)}{b\,c}},$$

$$\text{Cos } \tfrac{1}{2}\,\text{B} = \sqrt{\frac{s\,(s-b)}{c\,a}},$$

$$\text{Cos } \tfrac{1}{2}\,\text{C} = \sqrt{\frac{s\,(s-c)}{a\,b}}.$$

Consequently, since $\tan \text{A} = \dfrac{\sin\ \text{A}}{\cos\ \text{A}}$

$$\therefore\ \text{Tan } \tfrac{1}{2}\,\text{A} = \frac{\sin \frac{1}{2}\,\text{A}}{\cos \frac{1}{2}\,\text{A}} = \sqrt{\frac{(s-b)\,(s-c)}{s\,(s-a)}},$$

and

$$\text{Tan } \tfrac{1}{2}\,\text{B} = \sqrt{\frac{(s-c)\,(s-a)}{s\,(s-b)}},$$

and

$$\text{Tan } \tfrac{1}{2}\,\text{C} = \sqrt{\frac{(s-a)\,(s-b)}{s\,(s-c)}}.$$

Logarithms.—It is necessary at this stage to say a few words regarding logarithms, or the ratio of numbers, without which it is impossible to consider the question of the solution of triangles. The principle is, that a fixed number called the base, raised to the proper power, may be made to represent any required number.

I must refer the student, who has yet to master the theory of logarithms, to the many suitable works upon the subject. In this present work space will only admit of an explanation of the use of tables of logarithms.

We propose to use the *common system* of logarithms, in which the base is 10. In calculations, they are usually designated by the abbreviated term " log."

Logarithms of numbers consist of two parts, viz. the *index or characteristic* and the *mantissa.* The *index* or *characteristic* is a numeral expressing the number of digits in the integral part of the number which the logarithm represents. It is one *less* than the number of those digits, and is placed immediately before the decimal part of the logarithm; thus, if there are seven integral figures, the characteristic is 6, if six figures 5, if five 4, and so on. If there are no integral figures, the characteristic is negative (the negative sign being placed over it), and is one *more* than the number of ciphers (if any) immediately following the decimal point. If there is no such cipher, the characteristic is $\bar{1}$; if there is a single cipher, the characteristic is $\bar{2}$; if two such ciphers, $\bar{3}$; and so on.

The *mantissa* is the decimal part of the logarithm, and is found, in the columns of mathematical tables, on a line with the number which the logarithm represents. It is the same whether that number is integral or not. Thus, ·6614151 is the mantissa of the log. of 45858, 4585·8, 458·58, and so on, the only difference being in the characteristic, as will be seen in the following example :—

Number.	Logarithm.
45858·	4·6614151
4585·8	3·6614151
458·58	2·6614151
45·858	1·6614151
4·5858	0·6614151
·45858	$\bar{1}$·6614151
·045858	$\bar{2}$·6614151
·0045858	$\bar{3}$·6614151
·00045858	$\bar{4}$·6614151

The mantissa alone appears in the tables, and it is always positive. The characteristic has to be supplied by the calculator.

Here let me explain that most tables of logarithms have numbers only to 9999, and by reference thereto they appear thus :—

No.	0	1	2	3	4	5	6	7	8	9	D
7695	8862086	2143	2199	2256	2312	2368	2425	2481	2538	2594	57

so that in reality we only get the logarithm of the first four of the five figures, viz. log. of 7695 = ·8862086 ; but we want the log. of 76952, to get which we must look in the column marked 2, and for the last four decimals, viz. 2086, substitute the four in column 2, viz. 2199, so that our log. of 76952 is ·8862199 ; equally if we wanted the logarithm of 76959 we should take 2594 in column 9 instead of the four last decimals opposite 7695, so that the logarithm of 76959 = ·8862594. Now in the last column, headed D, will be noticed one set of figures, viz. 57 ; this means that it is the difference between the logarithm of the number and that of the following unit.

Thus log. 7695 = ·8862086. Add 57.
57
Then log. 76951 = ·8862143. Add 57.
57
log. 76952 = ·8862200.* Add 57.
57
·8862257, and so on.

Multiplication by Logarithms.—*Rule.*—Find the logarithms of the numbers to be multiplied, and add them together. The sum will be the logarithm of the product. Thus—

Multiply 621 by 412.
log. 621 = 2·7930916
log. 412 = 2·6148972
log. of product = 5·4079888 = log. 255852
∴ product = 255852.

Division by Logarithms.—*Rule.*—Subtract the logarithm of the divisor from that of the dividend, and the remainder will be the logarithm of the quotient.

Example.

Divide 3882·2 by 4·7.
log. 3882·2 = 3·5890779
4·7 = 0·6720979
log. of quotient = 2·9169800 = log. 826
∴ quotient = 826.

Proportion by Logarithms.—*Rule.*—The logarithms of the two middle terms are to be added together, and from their sum the logarithm of the first must be subtracted, and the remainder will be the logarithm of the quantity required;

Or, instead of subtracting the logarithm of the first term from the sum of the second and third, add its *arithmetical complement* (*ar. comp.*), and from this sum deduct 10 from the characteristic.

Note.—The *arithmetical complement* of a logarithm is found by deducting it from 10. Thus, if the logarithm of 685 = 2·9469433, its *ar. comp.* = 10·0000000 − 2·9469433 = 7·0530567.

---

* Tables of logarithms are worked out to more decimals than are printed; and, the last printed figure being here and there increased because of the following figure (omitted in printing) exceeding 5, the difference between two successive logarithms occasionally varies from that given in the "difference" column. Thus, the difference between log. 76951 and log. 76952 is 56, between this and log. 76953 it is 57; between this and log. 76954 it is 56; but the printed difference can in general be used without material error resulting from this variation.

The following example will serve to illustrate the two methods of performing proportion :—

If the wages of a servant be £25 per annum, what amount should he receive for 87 days' service?

Then—

As 365 : 87 : : £25 : ?

	By Logarithms.		By Arithmetical Complement.
As log. 87 =	1·9395193		1·9395193
log. £25 =	1·3979400		1·3979400
log. 365 =	2·5622929	*ar. comp.*	7·4377071 *
	0·7751664		0·7751664

Answer, £5 19s. $2\frac{1}{7}$d.

Involution by Logarithms.—*Rule.*—Multiply the logarithm of the given number by the exponent of the power, and the product will be the logarithm of the required power.

Find the square of 75.

$$\begin{array}{rl} \log.\ 75 = & 1\cdot8750613 \\ & \underline{\qquad\qquad 2} \\ \therefore \log.\ \text{product} = & 3\cdot7501226 = \log.\ 5625 \\ \therefore & 75^2 = 5625. \end{array}$$

Similarly find the cube of 62.

$$\begin{array}{rl} \log.\ 62 = & 1\cdot7923917 \\ & \underline{\qquad\qquad 3} \\ \therefore \log.\ \text{product} = & 5\cdot3771751 = \log.\ 238328 \\ \therefore & 62^3 = 238328. \end{array}$$

Again, find the fifth power of 18.

$$\begin{array}{rl} \log.\ 18 = & 1\cdot2552725 \\ & \underline{\qquad\qquad 5} \\ \therefore \log.\ \text{product} = & 6\cdot2763625 = \log.\ 1889568 \\ \therefore & 18^5 = 1889568. \end{array}$$

Evolution by Logarithms.—*Rule.*—Divide the logarithm of the given number by the exponent of the root, and the quotient will be the logarithm of the required root.

Examples.

Find the square root of 256.

$$\begin{array}{rl} \log.\ \sqrt{256} = \frac{1}{2} \log.\ 256 = & \frac{1}{2} \times 2\cdot4082400 \\ = & 1\cdot2041200 \\ = & \log.\ 16. \end{array}$$

$$\text{And } \therefore \sqrt{256} = 16.$$

---

* 10·0000000 − 2·5622929 = 7·4377071.

Again, find cube root of 256

$$
\begin{aligned}
\log.\ \sqrt{256} &= \tfrac{1}{3} \times 2{\cdot}4082400 \\
&= 0{\cdot}8027466 \\
&= \log.\ 6{\cdot}3496 \\
\therefore\ \sqrt[3]{256} &= 6{\cdot}3496.
\end{aligned}
$$

And so evolution to any extent may be performed, simply by dividing the logarithm of the given number by the exponent of the root.

Natural and Logarithmic Sines, Cosines, &c.—We have seen that the ratio of the perpendicular to the hypotenuse, of that of the base to the hypotenuse, &c., give the natural sine, cosine, &c. As in the case of the angle of 45 deg., we found that

Sin 45° = 0·70711
Cos 45° = 0·70711
Tan 45° = 1·00000
Cotan 45° = 1·00000
Sec 45° = 1·41421
Cosec 45° = 1·41421

And similarly

Sin 60° = 0·86603
Cos 60° = 0·50000
Tan 60° = 1·73210
Cot 60° = 0·57735
Sec 60° = 2·00000
Cosec 60° = 1·15470 and so on.

We have further seen that these values express the lengths of the sines and cosines of arcs of a circle whose radius = 1.

Thus the natural sine of 37° = 0·60182, whilst the logarithmic sine of 37° = L 9·77946. In tables of logarithmic sines, cosines, &c., the logarithms are those representing the natural sines, cosines, &c., 10 being added to their characteristics in order to avoid the occurrence of negative ones in the tables: these logarithms are then termed *tabular logarithms*, and in calculations are denoted by the letter L instead of the term "log."

The natural sines, cosines, tangents, &c., may be found from the logarithmic sines, cosines, tangents, &c., by subtracting 10 from the indices of the latter, and then the number corresponding to this logarithm is the natural sine, cosine, tangent, &c., required.

Example.—The logarithmic sine of 37 deg. = 9·77946, from which it is required to find the natural sine.

L sin. 37° = 9·77946
Subtract 10·
log. nat. sin. = $\bar{1}$·77946
Hence natural sin. = ·60182.

It may be well here to state some of the peculiar properties of the lines in and about a circle as follows:—

1. The square of the diameter is equal to the sum of the squares of the chord of an arc, and of the chord of its supplement to a semicircle.
2. The square of the radius is equal to the sum of the squares of the sine and cosine.
3. The sum of the cosine and versed sine is equal to the radius.
4. Radius is to the sine as twice the cosine is to the sine of twice the arc, or as the secant is to the tangent.
5. As the cosine is to the sine, so is the radius to the tangent.
6. Radius is the mean proportional between the tangent and the co-tangent, and also between secant and cosine.

Solution of Triangles by Arithmetical Computation.—The terms of proportion must be stated according to rule, these terms consisting partly of the numbers which express the given lengths of sides, and partly of the sines &c., of the given angles.

Add together the logarithms of the second and third terms, and from their sum subtract the logarithm of the first term.

Or,—

To the sum of the logarithms of the second and third terms, add the arithmetical complement of that of the first term, and from the characteristic of the sum subtract 10.

The logarithm resulting from either of the above operations represents the natural number which is the fourth term of the proportion.

When the three angles of any triangle are given, but no side, the actual length of the sides cannot be determined, but only their ratio to one another.

I. Right-angled Triangles.

The solution of right-angled triangles has four cases, viz.—

1. When the hypotenuse and a side are given.
2. When the two sides are given.
3. When the hypotenuse and an acute angle are given.
4. When a side and an acute angle are given.

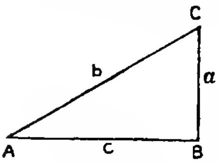


Fig. 196.

Let A B C (Fig. 196) be a right-angled triangle, B being the right angle and b the hypotenuse.

Case 1.—Given hypotenuse $b = 500$ links
side $a = 286{\cdot}788$ links.
Required angles A and C, and side c.

By logarithms—

$$\sin A = \frac{a}{b} = \log. a - \log. b$$

$$\begin{array}{rl} 10 + \log. a = & 12{\cdot}4575613 \\ \log. b = & \underline{2{\cdot}6989700} \\ & \underline{\underline{9{\cdot}7585913}} = L \sin A = 35^\circ. \end{array}$$

$$C = 90^\circ - 35^\circ = 55^\circ;$$
$$c = \sqrt{b^2 - a^2} = 409{\cdot}576 \text{ links.}$$

By Natural Sines &c.

Data as before. Required angles A and C, and side c.

$$\cos C = \frac{a}{b} = \frac{286{\cdot}788}{500} = 0{\cdot}7585913 = \text{nat. cos. } 55^\circ.$$

Case 2.—Given side $a = 286{\cdot}788$
,, $c = 409{\cdot}576$
Required angle A.

$$\tan A = \frac{a}{c}, \text{ and } L \tan A = 10 + \log. a - \log. c$$
$$= 12{\cdot}4575613 - 2{\cdot}6123345 = 9{\cdot}8452268$$
$$= L \tan 35^\circ.$$

Case 3.—Given hypotenuse $b = 500$
angle $A = 35^\circ$
Required C, a, and c.

$$C = 90^\circ - A = 55^\circ$$

$$\frac{a}{b} = \sin A, \therefore a = b \sin A$$

$$\begin{array}{rl} \log. b, 500 = & 2{\cdot}6989700 \\ L \sin A - 10 = & \underline{1{\cdot}7585913} \\ & \underline{\underline{2{\cdot}4575613}} = \log. 286{\cdot}788 = a. \end{array}$$

For base c:—

$$\frac{c}{b} = \sin 55^\circ, \therefore c = b \sin C = \log. b + L \sin C - 10$$

$$\begin{array}{rl} \log. b = & 2{\cdot}6989700 \\ L \sin C - 10 = & \underline{\bar{1}{\cdot}9133645} \\ & \underline{\underline{2{\cdot}6123345}} = \log. 409{\cdot}576 = c. \end{array}$$

CASE 4.—Given side $a = 286{\cdot}788$

angle $A = 35^\circ$

Required C, b, and c.

$C = 90^\circ - A = 55^\circ$

$\frac{a}{b} = \sin A, \therefore b = \frac{a}{\sin A}$, and $\log. b = \log. a - (L \sin A + 10)$

$$\begin{aligned} \log. a &= 2{\cdot}4575613 \\ L \sin A - 10 &= \bar{1}{\cdot}7585913 \\ & \underline{\underline{2{\cdot}6989700}} = \log. 500 = b. \end{aligned}$$

For base c:—

$$\frac{a}{c} = \tan A, \therefore c = \frac{a}{\tan A}$$

$$\begin{aligned} \log. c &= \log. a - (L \tan A + 10) \\ \log. a &= 2{\cdot}4575613 \\ L \tan A - 10 &= \bar{1}{\cdot}8452268 \\ & \underline{2{\cdot}6123345} = \log. 409{\cdot}576 = c. \end{aligned}$$

II. Oblique-angled Triangles.

The solution of oblique-angled triangles has four cases, viz.—

1. When two angles and a side opposite to one of them are given.
2. When two sides and the included angle are given.
3. When two sides and an angle opposite to one of them are given.
4. When three sides are given.

CASE 1.—*Rule.*—The sines of the angles are in the same ratio as their opposite sides.

In the triangle A B C (Fig. 197); given

side $c = 610$

angle $B = 115^\circ$

" $C = 42^\circ\ 30'$

To find the side b:—

$$\sin C : c :: \sin B\ (\text{suppl.} = 65^\circ) : b$$

$$\begin{aligned} L \sin 42^\circ\ 30'\ \textit{ar. comp.} &= 0{\cdot}1703167 \\ \log. 610 &= 2{\cdot}7853298 \\ L \sin 65^\circ &= 9{\cdot}9572757 \\ & \overline{2{\cdot}9129222} = \log. 818{\cdot}32 = b. \end{aligned}$$

To find the side a :—

$$L \sin 42^\circ\ 30'\ \textit{ar. comp.} = 0{\cdot}1703167$$
$$\log.\ 610 = 2{\cdot}7853298$$
$$L \sin \text{A}, 180^\circ - (115^\circ + 42^\circ 30') = 22^\circ 30' = \underline{9{\cdot}5828397}$$
$$\underline{\underline{2{\cdot}5384862}} = \log. 345{\cdot}53 = a.$$

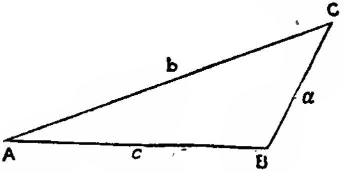


Fig. 197.

Case 2.—*Rule.*—As the sum of the two given sides is to the difference of those sides, so is the tangent of half the sum of their opposite angles to the tangent of half their difference.

This half difference added to the half sum will give the greater angle, and taken from the half sum will give the less angle.

In the triangle A B C (Fig. 198); given

side $a = 1272$
„ $c = 1636$
angle B $= 97^\circ\ 30'$

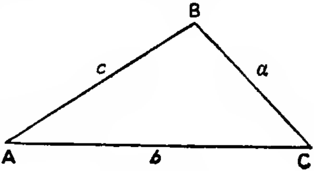


Fig. 198.

To find angles A and C :—

$$\log.\ c + a,\ (2908)\ \textit{ar. comp.} = 6{\cdot}5364056$$
$$\log.\ c - a,\ (364) = 2{\cdot}5611014$$

$$L \tan\left(\frac{\text{A} + \text{C}}{2}\right) = \left(\frac{180^\circ - \text{B}}{2}\right) = \underline{9{\cdot}9429879} = L \tan 41^\circ\ 15'$$

$$L \tan\left(\frac{\text{A} - \text{C}}{2}\right) = \left\{\begin{matrix}(\textit{sum of these} \\ \textit{logs} - 10)\end{matrix}\right\} = \underline{\underline{9{\cdot}0404949}} = L \tan \underline{6^\circ\ 15'\ 52''}$$

their sum $47^\circ\ 30'\ 52'' =$ C
their diff. $34^\circ\ 59'\ 8'' =$ A

To find the side b :—

$$L \sin \text{A}\ \textit{ar. comp.} = 0{\cdot}2415645$$
$$\log.\ a = 3{\cdot}1044872$$
$$L \sin \text{suppl. B},\ 82^\circ\ 30' = \underline{9{\cdot}9962686}$$
$$\underline{\underline{3{\cdot}3423203}} = \log.\ 2199{\cdot}48 = b.$$

CASE 3.—*Rule.*—Same as in Case 2.

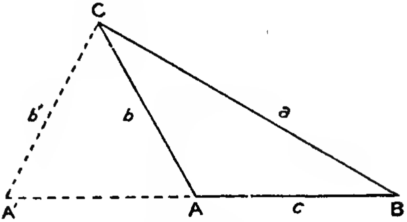


Fig. 199.

In the triangle A B C (Fig. 199); given

$$\text{side } a = 923{\cdot}6$$
$$\text{side } b = 530$$
$$\text{angle B} = 29^\circ\ 26'$$

Required angles A and C.

$a + b = 923{\cdot}6 + 530 = 1453{\cdot}6$

log. $1453{\cdot}6$ *ar. comp.* = $6{\cdot}8375551$ = sum

$a - b = 923{\cdot}6 - 530 = 393{\cdot}6$

log. $393{\cdot}6$ = $2{\cdot}5950551$ = diff.

$$\frac{\text{A} + \text{C}}{2} = \frac{180^\circ - 29^\circ\ 26'}{2} = 75^\circ\ 17',\ L \tan = \frac{10{\cdot}5806126}{10{\cdot}0132228}$$

$$= L \tan 45^\circ\ 52'\ 20'' = \frac{\text{A} - \text{C}}{2}$$

Then $75^\circ\ 17' + 45^\circ\ 52'\ 20'' = 121^\circ\ 9'\ 20'' = \text{A}$

$75^\circ\ 17' - 45^\circ\ 52'\ 20'' = 29^\circ\ 24'\ 40'' = \text{C}.$

Case 3 is called *the ambiguous case* when the given angle is less than a right angle, because, as shown by the dotted lines in Fig. 199, the data apply to two distinct triangles A B C and A' B C, where the given angle B and the given sides *a* and *b* retain their dimensions, *b'* being equal to *b*, the unknown side *c* extending from B to A'. The ambiguity can scarcely occur in ordinary practice, because in a properly conducted survey other measurements obtained in the course of the work will determine whether the given angle is obtuse or acute. In cases of this sort, it is best to solve the triangle by the rule given below under Case 4, or by the method given for triangle C A B, Fig. 276.

CASE 4.—*Rule.*—From the greatest angle let fall a perpendicular to the opposite side (which we will call the base), dividing it into two parts and the whole triangle into two right-angled triangles. Then

As the whole base : the sum of the other two sides :: the difference of those sides : the difference of the parts of the base.

Half the difference of the parts, added to half the base, will give the greater part; and subtracted from half the base will give the less part.

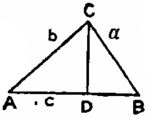


Fig. 200.

In the triangle A B C (Fig. 200); given

$$a = 1272 \ (\log. = 3\cdot1044871)$$
$$b = 1636 \ (\log. = 3\cdot2137833)$$
$$c = 2200$$

Required the parts A D and D B and the angles.

As $2200 : 2908 :: 364 : \underline{481}$ diff. of parts A D and D B.

Half diff. of parts $= 240\cdot5$

adding to and subtracting from half the base

$$1100 + 240\cdot5 = 1340\cdot5 = \text{A D} \ (\log. = 3\cdot1272668)$$
$$1100 - 240\cdot5 = \ 859\cdot5 = \text{D B} \ (\log. = 2\cdot9342459).$$

For angle A :—

$$\cos \text{A} = \frac{\text{rad} \times \text{A D}}{b} = 10 + 3\cdot1272668 - 3\cdot2137833$$
$$= 9\cdot9134835 = L \cos 34^\circ\ 58'\ 39''.$$

For angle B :—

$$\cos \text{B} = \frac{\text{rad} \times \text{D B}}{a} = 10 + 2\cdot9342459 - 3\cdot1044871$$
$$= 9\cdot8297588 = L \cos 47^\circ\ 29'\ 27''$$

and angle $\text{C} = 180^\circ - (\text{A} + \text{B}) = 97^\circ\ 31'\ 54''$.

If s be put to denote half the sum of the three sides, the case can also be solved by the formula

$$\sin \frac{\text{A}}{2} = \sqrt{\frac{(s - b)\ (s - c)}{b\ c}}$$

which by due change of letters holds for the other half-angles.

Heights and Distances.—The trigonometrical measurement of height and distance of an object is a not unimportant part of surveying, and involves various problems arising out of the special conditions of different cases: of these we shall now consider the principal ones.

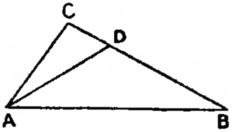


Fig. 201.

In the following examples, the angles of triangles are denoted by capital letters, and the sides by italic small ones; it being understood, in order to avoid multiplicity of lettering, that where two or more angles meet in the same point, the angle and the side or sides referred-to are those belonging to the triangle specified. Thus, in the triangle A B C (Fig. 201), the angle A is C A B, the side *a* being B C; whereas in the triangle A B D the angle A is D A B, the side *a* being B D; and so on. The position of the theodolite or other instrument of observation is represented by a little tripod.

It is further to be noted, that in practice all linear and angular measurements must be made with the most scrupulous carefulness and precision, the correctness of the result depending upon the accuracy of measurement of a line or an angle of sometimes very small dimensions.

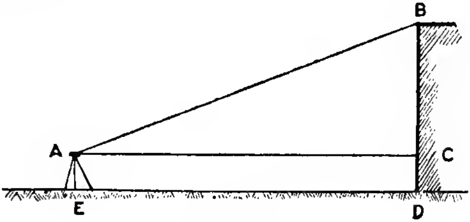


Fig. 202.

Problem I.—To find the height of an object having a vertical face B D, accessible to the observer; the ground line E D being horizontal (Fig. 202).

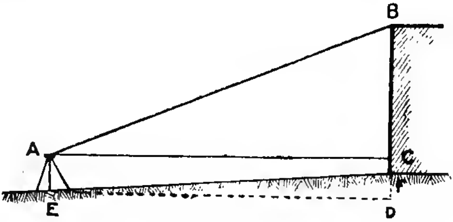


Fig. 203.

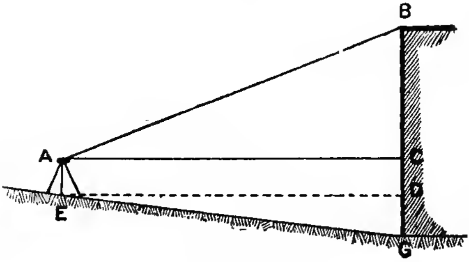


Fig. 204.

Measure E D = AC, and the angle of elevation B A C. Then, in the right-angled triangle A C B, the side b and the acute angle A are given, and a = tan A × b (Case I., 4, p. 142).

To this add A E = C D. Then a + C D = B D the height required.

Note.—If the ground slopes, as E F or E G (Figs. 203 and 204), the difference of level C F or C G can be ascertained by levelling, and the length of slope E F or E G measured. Then, in the right-angled triangle E D F (or E D G), the sides e and d are given, whence E D = $\sqrt{d^2 - e^2}$, and the required height of building = B C + C F (or B C + C G).

If the ground slopes upwards as E H (Fig. 205) so that the foot of the building is above the horizontal A C, ascertain the difference of level between E and H = D H by levelling, and measure length of slope E H. Hence is obtained E D = A C; and A C × tan B A C − (A C × tan H A C) = B H the height required.

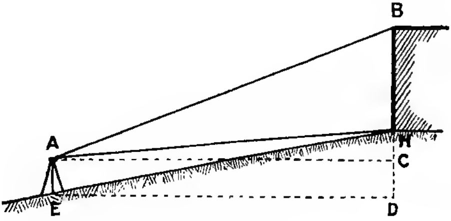


Fig. 205.

Problem II.—To find the height of an object having a vertical face C B inaccessible to the observer.

At the stations selected for the observations (in the same vertical plane as the point C of the object C B), from the nearer one of which stations the foot of the object is visible, drive pegs E and F (Figs. 206 and 207), their heads level with surface of ground; and measure the length of base E F.* Set up theodolite over peg F, measure F D, the height of its horizontal axis D, and from D observe the angle of elevation H D C. Remove theodolite to station E; and from A observe the angle D A H′ between the reading F D on a levelling-staff set on peg F and the horizontal A H′.

---

* The height of a theodolite above the ground varies according to the spread given to the legs, and it is difficult to place it accurately in position over two pegs in succession with its axis at the same height above each. A usual and an unusual spread occasion a difference in height of about 7½ inches, corresponding to a difference of 0·00196 per cent. between the length of base as measured on the ground and that of a line joining the axis of the instrument at the two stations, the length of the latter being that whereon the triangulation is founded. To avoid prolixity in working the problems, the correction for this difference has been omitted in the examples, the two lines being taken as parallel and equal.

H D and A H′ being horizontal and therefore parallel lines, H D A and D A H′ are equal; and C D H + H D A (Fig. 206) or C D H − A D H (Fig. 207) = C D A.

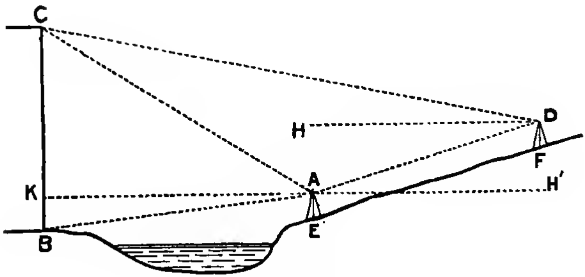


Fig. 206.

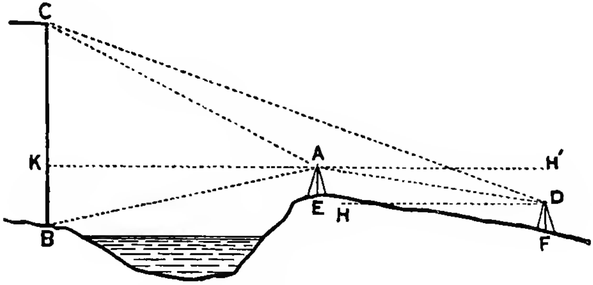


Fig. 207.

From A, observe angles C A D, K A C, and K A B.

Then, in the triangle A C D, $180^\circ - (A + D) = C$;
and $\sin C : c :: \sin D : d$ (Case II. 1, p. 144).

In the right-angled triangle A K C, $C = 90^\circ - A$;
and $\sin K : k :: \sin A : a$ (= C K).

In the right-angled triangle A K B,
$b \tan A = a$ (= K B),
whence C K + K B = C B = the height sought.

In the above two examples, the foot of the object is visible from the nearer station. When it is visible from the further one only (Fig. 208) the problem may be solved as follows:—

From D, observe the angles C D B and and C D H; and measure

D F the height of instrument above peg F. Remove instrument to station E; and from A observe the angles C A D and D A H′.

$$CDH + HDA = CDA.$$

Then, in the triangle A C D, $180^\circ - (A + D) = C$;

and $\sin C : c :: \sin D : d.$

In the right-angled triangle C H D, $90^\circ - D = C$.

In the triangle C B D, $180^\circ - (C + D) = B$,

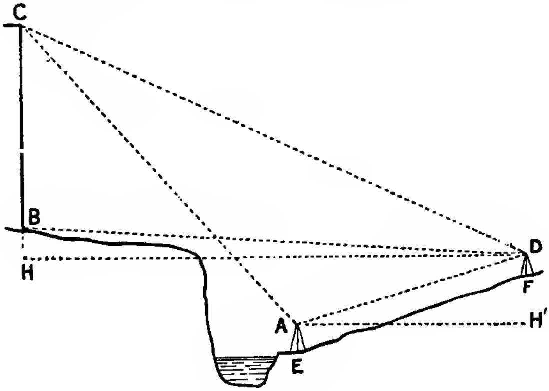


Fig. 208.

and $\sin B : b :: \sin D : d$;

and $d = CB$, the height sought.

Problem III.—To find the surface-length of an inaccessible slope C D, as that of a steeple on a tower (Fig. 209).

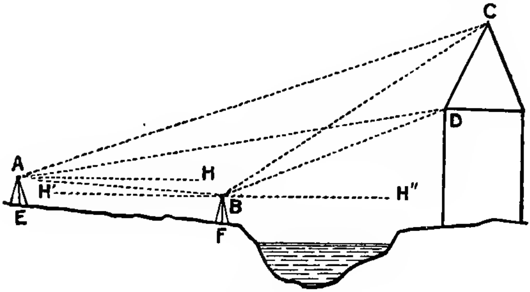


Fig. 209.

Set out and measure base-line E F, and place theodolite at E and F successively. From A observe angles C A D, C A H, and D A H; and from B the angles C B D, C B H″, D B H″, C B A, and A B H′.

$$\begin{aligned} DBH'' - DAH &= ADB, \\ CBH'' - CAH &= ACB, \\ CAH + ABH' &= CAB. \end{aligned}$$

In the triangle A B C,

$$\begin{aligned} \sin C : c &:: \sin A : a\ (= BC), \\ \sin C : c &:: \sin B : b\ (= AC). \end{aligned}$$

In the triangle A D B,

$$\begin{aligned} \sin D : d &:: \sin A : a\ (= BD), \\ \sin D : d &:: \sin B : b\ (= AD). \end{aligned}$$

In the triangle C B D we have given

side d (= B C),
side c (= B D),
and angle B;

then (Case II. 2, p. 145)

$$d + c : d - c :: \tan\left(\frac{D + C}{2}\right) : \tan\left(\frac{D - C}{2}\right),$$

whence
$$\tan\left(\frac{D + C}{2}\right) + \tan\left(\frac{D - C}{2}\right) = \tan D,$$

$$\tan\left(\frac{D + C}{2}\right) - \tan\left(\frac{D - C}{2}\right) = \tan C,$$

$$\sin C : c :: \sin B : b,$$

and b = C D = the length of slope sought.

In all the foregoing examples the base-line is assumed to be set out in the same vertical plane as the point or points of the object. When, owing to configuration of the ground, or to other circumstances, this cannot be done, as in Fig. 210, where the base-line has to be set out on a narrow road bounded by precipitous cliffs on one hand and a river or lake on the other:—

From A observe the vertical angles C A D, D A H, and the horizontal angle H A B. From B observe the horizontal angle H B A.

The angle A H B = 180° − (H A B + H B A).

In the triangle A H B,

$$\sin H : h :: \sin B : b\ (= HA).$$

And with H A as base, the heights C D and D H are ascertained, as in Problem I.

The measurement of inaccessible distances is performed on the same principle as that of inaccessible heights. Thus, if the heights in Figs. 202 to 208 are known, or ascertained, their dimension serves as a base-line, and the distances are obtained by a process the converse of that followed in the examples. Fig. 210 illustrates a case wherein the height of the observed object does not enter into the calculation, the distance H A (and thence D A) being obtained independently of it. In fact, by far the greater number of such cases are solved by a base-line on the ground, and thus by angles in azimuth only.

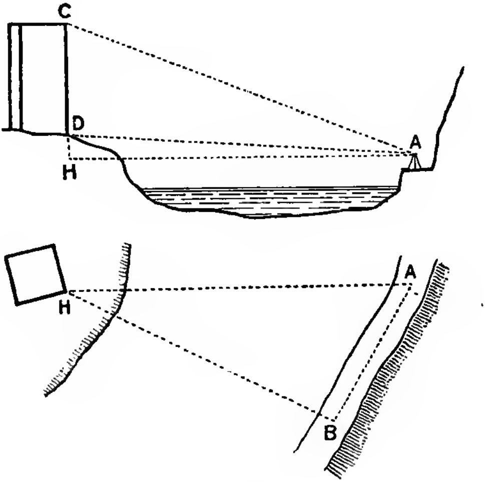


Fig. 210.

Suppose it be necessary to ascertain the length between two trees C D, but it is impossible to approach them by reason of the river. Having measured the base-line A B very accurately, the angles C A B, C B A, D B A, and D A B must be observed; from which, by preceding problems, the sides C A, D B, C B, and D A must be calculated (see Oblique Triangle, p. 144), together with the angles A C D, B D C, C D A, and D A C. With these, as has been shown, the length C D may be calculated.

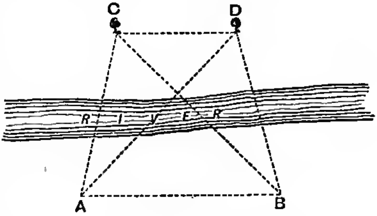


Fig. 211.

TRIGONOMETRICAL SURVEYING.

Trigonometrical Survey.—The determination of various points in a survey, to be afterwards filled-in by detailed chain or other surveys, is performed by triangulation founded upon observations starting from a measured base-line. For all triangulation work, the measurement of this needs, as already mentioned, special carefulness. In great undertakings such as the Ordnance and other official surveys, in which the largest base yet measured is short in comparison with the sides of triangles thereon built up—some of these upwards of eighty and even a hundred miles in length—the matter is one of national importance. To ensure its utmost attainable accuracy, extraordinary pains have been bestowed upon it; particulars whereof, too lengthy to be here adequately described, are to be found detailed in several works dealing with the subject.*

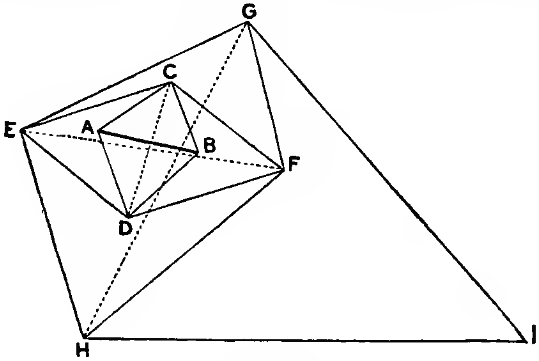


Fig. 212.

From the measured base, triangles are successively deduced, the length of their sides being increased as rapidly as possible; and upon these, secondary and tertiary systems of triangulation are in turn continued. In the setting-out of all these, attention has to be paid to their being "well-conditioned," *i.e.* that their angles are neither too obtuse nor too acute. The limits recommended as the maximum and minimum of those for great surveys, 120° and 30°, cannot, however, always be adhered-to in minor work nor in observations for calculation of heights and distances.

"A B [Fig. 212] is supposed to be the measured base, of two or three miles, or as long as can conveniently be obtained; and C and D the nearest trigonometrical points. All the angles being observed, the distances of C and D from the extremities of the base are very carefully calculated. Then in each of the triangles, D A C and D B C, we have two sides and the contained angles, to find D C; one calculation acting as a check upon the other. This line, D C, is again made the base from which the distances of the stations E and F are computed from D and C; and the length of E F is afterwards obtained in the two triangles E D F and E C F. In like manner the relative positions of the points H, G, I, &c. are obtained; and some such system should be pursued until the stations arrive at the required distance apart. . . . The length of the sides of the smallest triangles must depend upon the intended method of filling in the interior. If the contents of parishes, estates, &c. are to be computed, the distances between the points must be diminished to one or two miles, for an enclosed country; and to two or three perhaps, for one more open. If no contents are required, and the object of the triangulation is solely to ensure the accuracy of a topographical survey, the distances may be augmented according to the degree of minuteness required." * The average length of side of the primary triangles of the Ordnance survey of the United Kingdom was from 40 to 60 miles; of the secondary ones about 10 to 12 miles; and of the tertiary ones 1 mile to 3 miles.

---

* Frome, "Outline of the Method of conducting a Trigonometrical Survey;" Yolland, "Account of the Measurement of the Lough Foyle Base;" Bourns, "Principles and Practice of Surveying."

The triangles thus established by observation being plane triangles, while their actual nature is that of spherical ones, a correction has to be applied for spherical excess—a correction unnecessary in small surveys by reason of the minuteness of the difference, sufficient, however, to occasion in extensive ones a measurable variation from absolute correctness, since in a spherical triangle the sum of the three angles exceeds 180°, the amount of this excess in any given case being proportional to the area of the triangle. Let E represent the spherical excess in seconds, A the area of the triangle calculated as a plane one, and R the mean radius of the earth (these two being expressed in terms of the same unit of measurement), and π circumference in terms of diameter; then

$$E = \frac{A \times 648000}{R^2 \pi}.$$

Another rule is: From the log of the area of the triangle in square feet subtract the number 9·3267737, and the remainder will be the log of the spherical excess in seconds.

A simple method of determining the spherical excess, when very great accuracy is not required, is by dividing the area of the triangle in square miles by 76, the result being the spherical excess in seconds.

For the practical application of the correction, the simplest of

---

* Bourns, "Principles and Practice of Surveying," pp. 271 273.

three possible methods is that of Legendre, viz.: "In any spherical triangle, the sides of which are very small compared with the radius of the sphere, if each of the angles be diminished by one-third of the true spherical excess, the sines of these angles will be proportional to the lengths of the opposite sides, and the triangle may therefore be calculated as if it were plane." The area of the triangle having been calculated as a plane one from the *observed* data, and the spherical excess E obtained by the formula, the sum of the observed angles should $= 180^\circ + E$. The difference (if any) is due to error of observation; and, if an error of excess, one-third of it is to be deducted from, and if of defect to be added to, each of the angles. From each of them is then to be deducted one-third of the spherical excess; and from the thus corrected angles and a given side the other sides are calculated as those of a plane triangle.

The latest figures representing the radius of the earth arrived at in 1880 by Colonel A. R. Clarke, C.B., F.R.S., are

Major Semiaxis	20,926,202 feet,
Minor Semiaxis	20,854,895 feet.

The mean radius may therefore, without material error, be taken as

20,890,548·5 feet = 3956·54 miles.

CHAPTER V.

CHAIN-SURVEYING.

Surveying with Chain only.—I have in the previous chapters elected to treat all the preliminary questions together, leaving the present exclusively for the consideration of chain-surveying of estates, &c., and the method of keeping the field-book, with such other matters as may appear necessary.

Field-book.—First I will deal with the field-book, because this is a very essential element in surveying. I may here say that the manner in which the field-book is kept is in the highest degree important, bearing as it does upon the accuracy with which the survey is made and plotted. It is quite a mistaken theory (commonly held by old-fashioned surveyors) that the field-book should be so kept as to be only understood by them. Those days have gone by, and the modern surveyor must be so qualified that his work is not only as clean and simple as possible, but is capable of the most searching scrutiny.

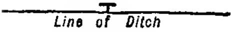
Ordnance Field-book.—The Ordnance surveyors are obliged to keep their field books in ink, and so particular have they to be, that when the survey is completed the books are sent in to Southampton, and possibly are never seen again by the surveyor, for the work is plotted by special draughtsmen, who may never have seen the ground they have to plot; so that unless the book has been kept clean and accurate it would be impossible to plot the survey.

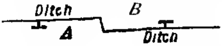
Necessity for Reconnoitre.—I have strongly recommended a reconnaissance previous to commencing a survey, for the purpose of determining the base and other lines, for establishing stations, and to make a sketch of the chief boundaries and features of the property. The latter is very important, not only to enable you to lay down the various lines, with their relative directions and positions, but in plotting will be found to be of the greatest assistance.

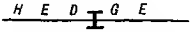
Survey Lines to be numbered consecutively.—The lines should be numbered consecutively from 1 upwards; and it is a great help to the surveyor if he represents his principal stations by letters; as A, B, &c., for one cannot have too much detail in one's field-book, bearing forcibly in mind the fact that others than yourself may have to plot the work.

Conventional Signs.—It may be well at this point to refer to conventional signs which are usually adopted by surveyors to indicate special features:—

1. Ditch and hedge are shown by a straight line, which line represents the edge of the ditch; the hedge being delineated by a T, showing on which side it belongs.





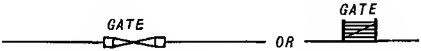


2. Where a change of position of ditch and hedge occurs, it should be carefully noted as in the sketch, which shows that at a certain point the ditch passes to the other side of the hedge, so that on the left the hedge belongs to A and on the right to B.

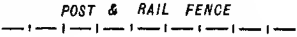
3. When a hedge alone separates two properties and on neither side is there a ditch, it is called a "foot-set" fence, and is shown in the third illustration above.

4. In most cases it is desirable to show gates, and they may be delineated in either of the ways indicated.

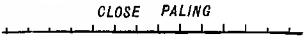
Gates thus:—



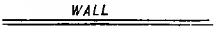
5. Post-and-rail fencing is shown thus:—



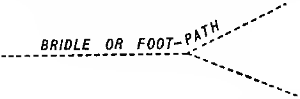
6. Close-paling thus:—



7. Walls by a double line.

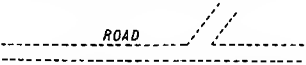


8. Footpaths are shown by a single dotted line.

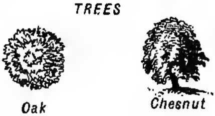


9. Cart-track or bridle-path by a double dotted line; but in

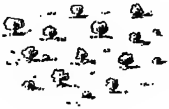
measuring upon the ground it is usual only to take the centre of the track, and allow twelve to fifteen links for the width.



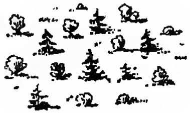
10. Trees are shown thus, and are described :—



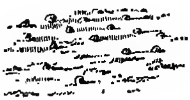
11. Orchards are sketched thus :—



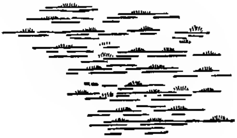
12. Woods.



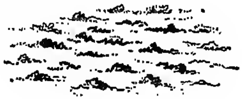
13. Brushwood.



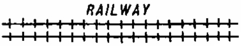
14. Marshy ground.



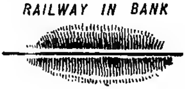
15. Heath or gorse.



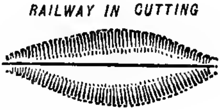
16. Railways, or preferably by a strong blue line.



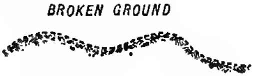
17. Railway embankment.



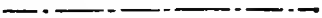
18. Railway cutting.



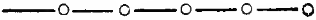
19. Broken ground or cliff.



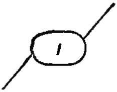
20. Parish boundaries.



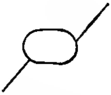
21. County boundaries.

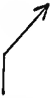


22. Surveying stations.



23. Direction of line.





Field-book.—The usual kind of field-book is 8 inches long by 4½ inches wide, opening lengthwise, and having a central column about ¾ inch wide for the longitudinal measurement, whilst the right and left columns are for marking the offsets, sketching in the fences, buildings, &c., and any memoranda that may be necessary, as in the following example:—

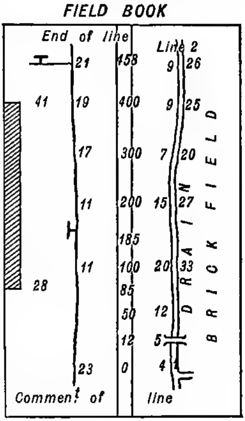


Fig. 213.

In Fig. 213 I have given but a very simple illustration of the use of such a field-book, and so long as all is plain sailing there may be little or no objection to this system; but in complicated work, where we have fences crossing our lines in all directions, and to take note of a large amount of detail, neither the size nor arrangement of the book can be recommended. For instance, supposing we have a fence crossing our chain-line obliquely, it would have to be entered in the book as in Fig. 214; or if a fence crosses our chain-line at right angles, but at the point of intersection another fence joins in an oblique direction, it would have to appear as in Fig. 215, the word "at" written against the sketch distinguishing that at 316 the oblique fence C joins A B at the point where it is intersected by the chain. Again, if our chain-line runs at a point on the edge of the ditch, so that in plotting at such a point the fence will impinge on the survey-line, it will have to be shown in the field-book as in Fig. 216, the word "at" at 300 signifying that this is the point of impingement. Then as to noting the stations, I maintain that the double column is anything but convenient; and to illustrate my argument I have given (Fig. 217) a portion of a field-book the system of which is advocated by one of the best authorities on modern surveying, in which it will be seen that stations occur at 1025 for line No. 3 to the left; at 1425 for No. 9; 1740 and 1875 for lines Nos. 5 and 10: whilst at 2185

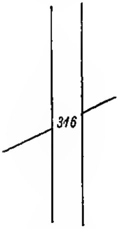


Fig. 214.

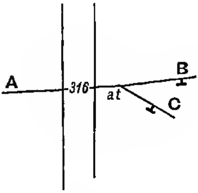


Fig. 215.

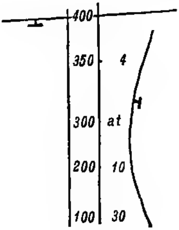


Fig. 216.

we have a station for the intersection of lines 13 and 14, and 3325 a station for No. 21; all being on the left side of the chain-line; the point of the station being delineated by a small circle outside the column against the chainage, with a dotted line to represent the direction of the line diverging from this station, whilst a circle enclosing a number indicates the line to which it refers. Can anything more troublesome be conceived—this extraneous sketching on the book to represent so little? so that to indicate that at 2185 there is a station whence two lines diverge involves three circles, two dotted lines, and two sets of figures, as in Fig. 218. I have taken the liberty of drawing a horizontal line above and below the station in the central column, which is the custom of many surveyors, and it is sometimes done as in Fig. 217.

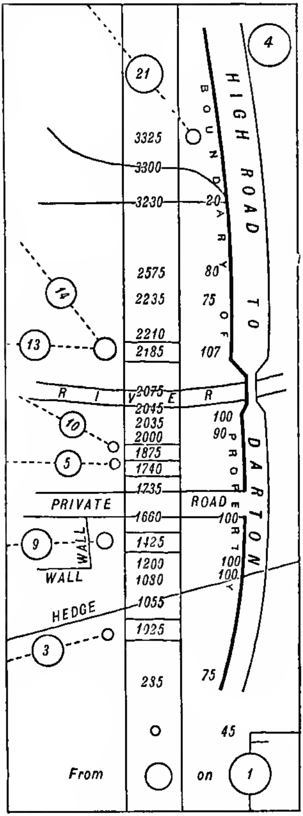


Fig. 217.

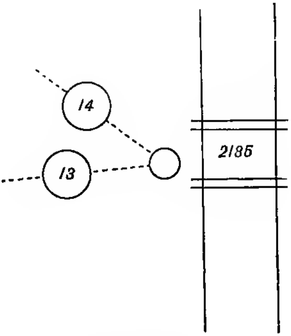
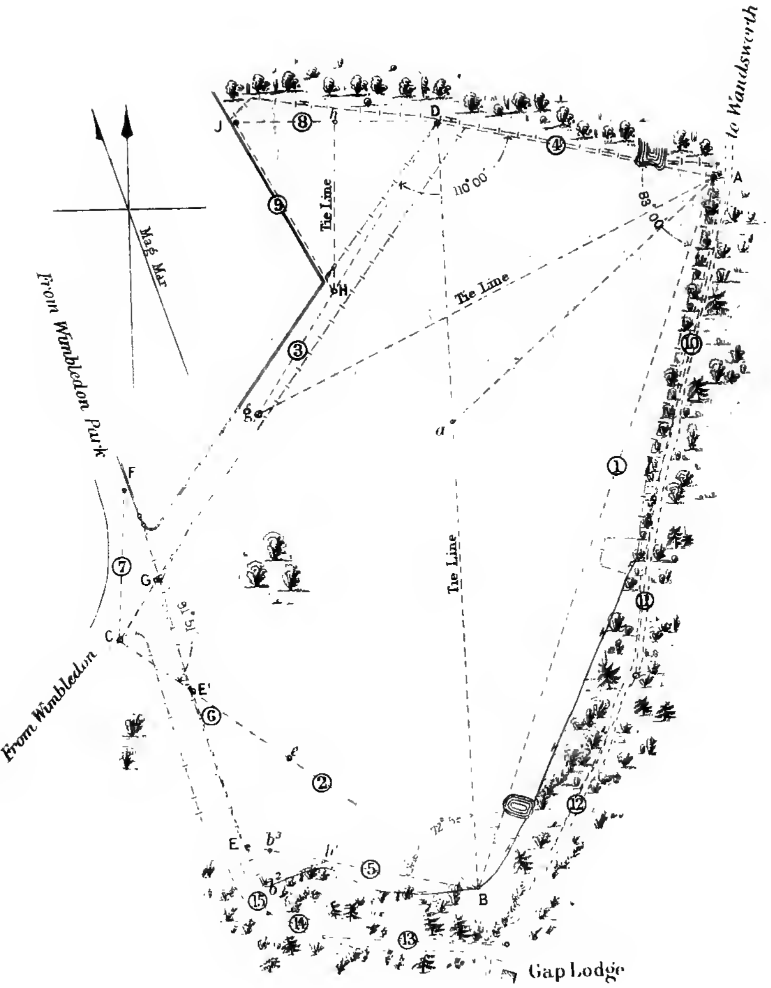


Fig. 218.

Best Size of Field-book.—In the first place, I maintain that the field-book is too small. I prefer a quarto size (opening lengthwise), which gives plenty of room for sketching in detail any features that may require to be taken, and for remarks, either as to the name of the field, &c, the description as to whether it is arable or pasture (distinguished by *ara.* or *pas.*), the county and parish or township, the occupiers, and the proprietors of the adjoining lands, &c.

PART OF WIMBLEDON PARK



Scale, 4 Chains to an Inch.

(To face page 163)

Crosby Lockwood & Son 7 Stationers Hall Court, London

Single Line preferable to Double Column.—Instead of the central column, *I recommend a single line upon which the longitudinal measurements may be marked.* This line represents in the book what the chain does in the field, and any crossing or intersection of a fence can be accurately shown in its proper position and direction, and a station may be represented with greater facility by drawing a circle or oval round the distance.

To illustrate my meaning, I reproduce in Plate II. (p. 165) a field-book adapted to the system I advocate, which is at once simple and intelligible, and one to which one soon gets accustomed. I have found it the most useful in my own practice; and in preparing a large number of pupils I have had ample evidence of the great facilities it affords.

Chain Survey of Part of Wimbledon Park.—I give here also an example of a complete survey in Wimbledon Park recently executed by one of my pupils (see Plate I.). This is a survey of somewhat undulating ground, the rise from B to G being about 90 feet. Commencing at A at the north-eastern end of the property for line 1, it was found impossible to restrict the offsets to fifty links, as the point B was an important station; consequently we had offsets of ninety-nine links, which, as a rule, is too much; but as this survey was for a special purpose, connected with the higher ground, the absolute accuracy of this particular fence, to the left of the line 1, was not a matter of great moment, especially as in the subsequent operations of traversing the road this fence was carefully adjusted. On reaching B (at the end of line 1), we ran the line No. 2 to C; thence a third line to D, and from A to D by line 4. This trapesium was tied by the base-line B D and a check-line from *g* to A; an additional check-line E′ G completely secures the accuracy of this figure. The south-western corner of the property had to be taken by a triangle B E E′, tied by E *e*; whilst a further small triangle was necessary, b^1 b^2 E, tied by b^2, b^3. Line No. 6 from E to F, passing through B C at E′ and C D at G, was a survey-line to take up the post-and-rail of the fencing of the road to Wimbledon Park. A small triangle is formed by line 7 from C to F, as much to keep up the curve of the fence on the western side as to accurately fix the position of the line E F. The north-western indent was taken up by means of a triangle H J D on the line C D, with a check-line H *h*.*

Few Lines as possible.—Thus it will be seen that the whole of this figure has been accurately surveyed by means of as few lines as possible, and the accompanying field-book (Plate II., p. 165), which is given in detail, will enable the student to plot this work for himself. Referring to line 1, it will be seen that the first point of importance at 550 is the gate, the position of which should be fixed by a small triangle upon the chain-line formed by 60 and 67 links at 600; the width of the gate in links between the posts to be noted in the field-book next. At 700 is a point on the chain-line which it is necessary to measure from to the corner where the small stack fence cuts the main fence. Similarly, each of the other corners should be fixed upon the chain-line by means of triangles as shown; and finally the small pond near the end of line 1 should be so treated. It should be noted that any defined point, such as an indentation in a fence, the position of a gate-post, the intersection of one fence with another, should be accurately fixed upon the survey-line by means of a triangle, and certainly on no account should such an important point be trusted to a simple offset.

---

* In the field-book (Plate II.) the lines 8 and 9 are given on page 5 representing end of line 3.

Tape not to be used for Offsets.—In Chapter II. I have expressed a decided opinion against the use of a tape for taking offsets, and I shall here emphasise that opinion by remarking that the accuracy of a survey, however simple or elaborate, will best be assured by arranging the survey-lines so that the offsets shall be as short as possible.

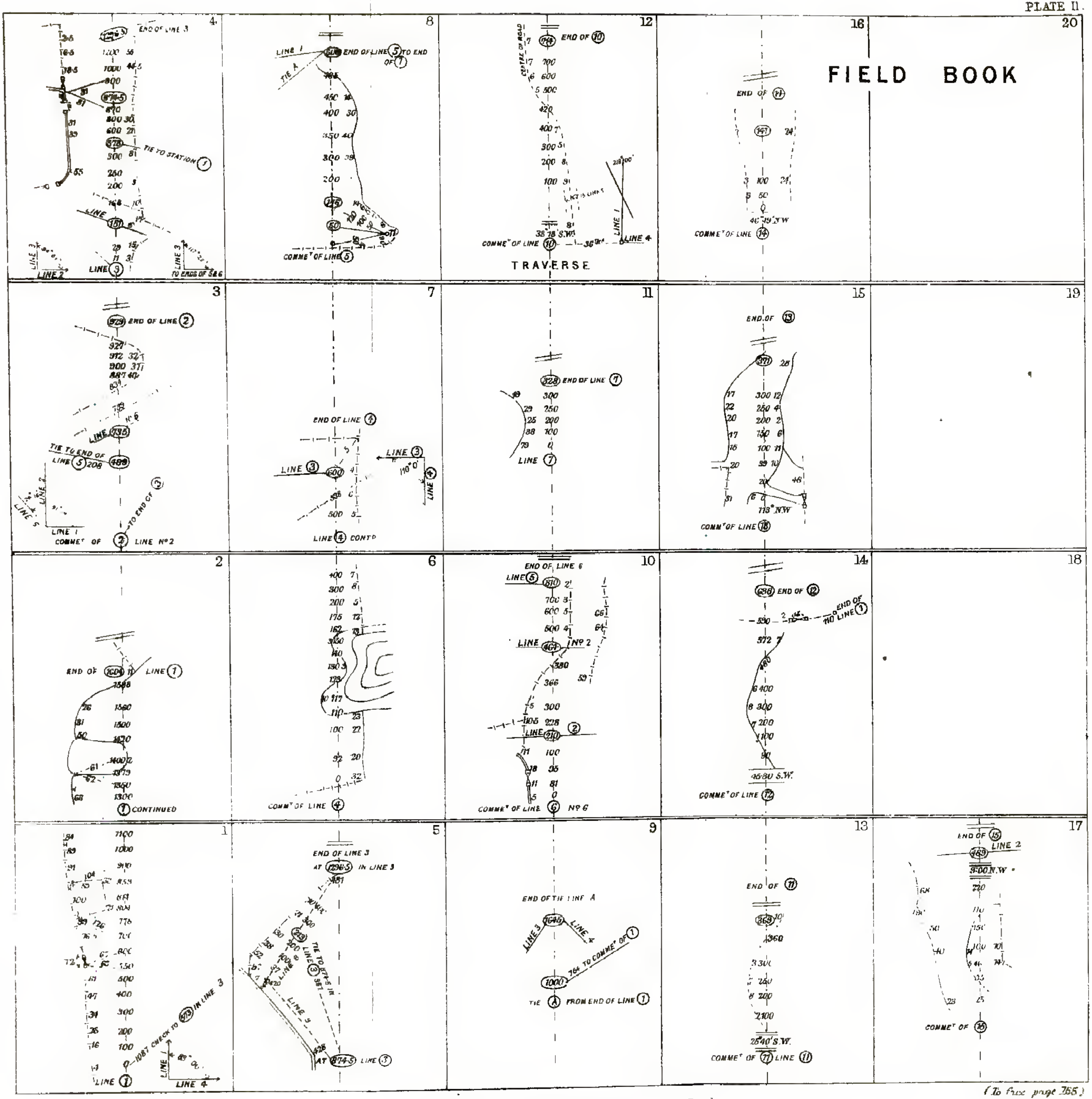
Chain-men should be instructed as to their Duties.—In commencing a survey it is necessary that the surveyor should satisfy himself that his chain-men are thoroughly conversant with their duties, and that his chain has been properly tested.

Enter every Ten Chains in Field-book.—At the completion of every ten chains, the surveyor should enter that number in his field-book, seeing that the leader receives from the follower ten arrows, and, placing his foot against the end of the tenth chain, take care that the eleventh arrow is duly put in position.

Boning-out Lines with Laths recommended.—It is a considerable saving of time if each line is well boned-out by means of laths, before referred to, especially where the ground is of an undulating character, as they are of great value in guiding both the leader and follower to keep well in line. At any point where it is deemed necessary to make a station, either a peg or a lath with a paper duly figured, or some distinguishing mark, should be left on the chain-line for future reference.

Best Form of Stations.—It is quite a mistake to imagine that by kicking a hole or cutting a mark in the turf the work will be facilitated, as often the time lost in trying to find this point subsequently is a matter of serious moment. If the survey is of an extensive character, occupying some considerable time, all stations and minor stations should be marked by pegs, each of which

PLATE II.



FIELD BOOK

(To face page 265.)

Crosby Lockwood & Son 7, Stationers Hall Court London

should have a distinguishing letter or number, as shown by Fig. 7 in Chapter II.

Begin at the End of Book and work upwards.—Referring to the field-book (Plate II.) in connection with Plate I., it will be seen that it is necessary to begin at the foot of the first page of the book, working upwards, using one side only of the paper; and that, as in the case of line 1, on reaching the top of the first page (at 1100), the line may have to be carried over on to another leaf, where it terminates at 1604; and it is derirable to draw two dashes across the book to represent that you have finished that line, taking care to write at the beginning, "Commencement of line 1," and at the finish, "End of line 1."

Let each Line have a Separate Page.—On no account attempt to commence another line on the same page, as paper is cheap enough to obviate such a necessity. It will be seen that all the offsets are on the left-hand side. Line 2 on the third page should be designated "Commencement of line 2," "End of line 1, right." At 489 is a station for a check-line to the end of line 5, and again at 735 there is another in connection with line 5. 739, 834, and 927, in line 2, intersect the post-and-rail fence which forms the boundary of the road, and between 834 and 927 there are points where it will be found necessary to take offsets to the right of the line to pick up the curvature of the aforesaid fence, whilst the final station of line 2 is at its termination 929. Here again it is necessary to draw two dashes across the book to show the completion of this line, and I would here say that I find it most convenient to indicate all stations by an oval enclosing the figures, thus (929), and, by means of one or more lines as the case may require, indicating the direction and nature of other lines connected with that station. Line 3, which commences at the end of line 2, crosses the road to Wimbledon Park and intersects line 6 at 151; a small line from the commencement of line 3 to the end of line 6 forms a triangle as much to check the position of these lines as to take up the curved fence on the left-hand side. Line 3 crosses the post-and-rail fence running alongside line 6, and thence, at the various points indicated, there are offsets on the right to the post-and-rail fence, and on the left to the boundary wall; at 573 there is a station for a tie-line to the commencement of line 1. At 870 and 900 are points whence a small triangle is formed to take up the corner of the boundary wall, whilst at 874·5 is a station for line 9 for the triangle necessary to take up the indentation at the north-west portion of the survey, the end of line 3 being the other point of the triangle on this line at 1296·5, for line 8. From this point also the base-line to the end of line 1 is commenced. Following this, upon page marked (5), is a detailed sketch

(Plate II.) of lines 8 and 9 before referred to, which needs no explanation. Line 4, beginning at the commencement of line 1, runs to the end of line 3, and crosses the edge of a pond on the right-hand side, the boundaries of which have been fixed by the points where it crossed, and also by offsets; and, further on the right-hand side, the post-and-rail fence was taken up by offsets, and on reaching the end of this line the junction of the two fences was determined by a diagonal offset from the station. From this point the tie-line to the end of line 1 was carefully measured over very undulating ground. The reason for taking this step will now be seen, as from the end of line 1 we were able to survey the two triangles on the left-hand side of line 2 on lines 5, 6, and 7.

In order that the student may the better follow and understand the system of procedure illustrated in the foregoing example, he is recommended to plot this survey from the field-book, to a scale of 2 chains to an inch, which will afford him excellent practice both in plotting and in the *modus operandi* with the chain only.

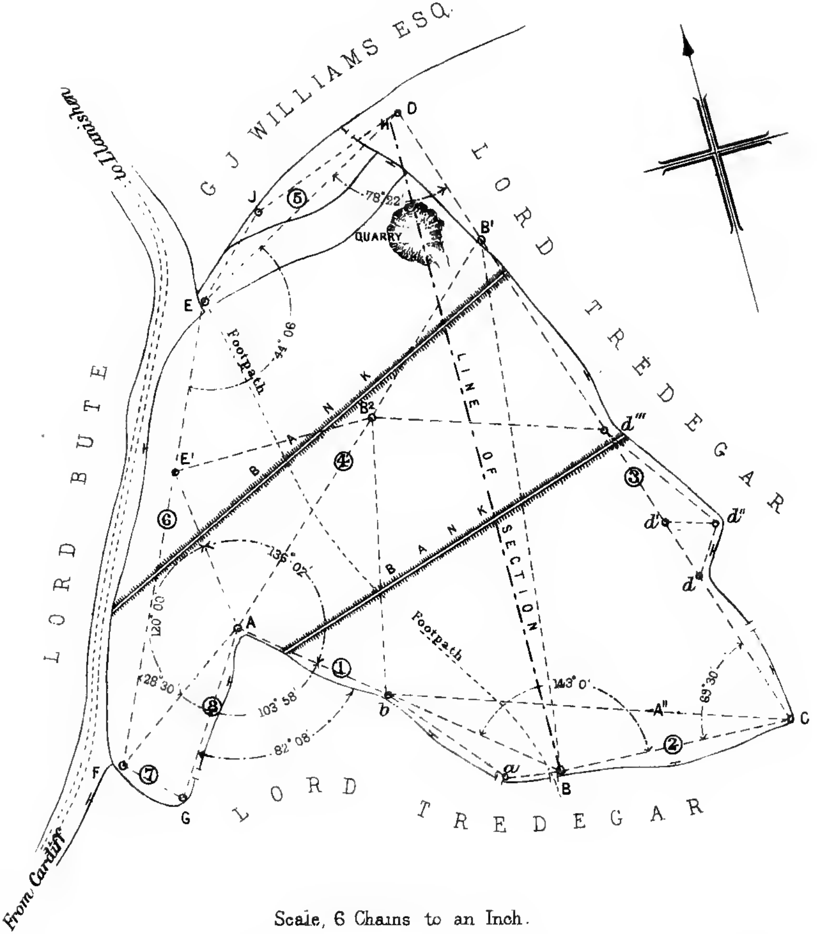
Mark Intersection of Lines by Small Circles.—In plotting a survey, at all points of intersection of lines with stations, it is desirable to draw a very small circle round a point of intersection, and, after the principal lines have been carefully plotted, the exact length being determined by a puncture with a very fine needle before any detail is plotted, it is absolutely necessary that these lines be finally drawn in with lake or carmine, and on no account should a survey be plotted from pencil lines.

Best Form of Base-lines.—In the early part of this book I have expressed an opinion that a survey is best accomplished by treating its two main base-lines as intersecting the estate surveyed in the form of the letter X, and I cannot impress too strongly upon the student the desirability of doing this wherever practicable. As these lines should form the basis of a complete network of triangulation, it need hardly be said that where possible it is always desirable that the figures formed upon them should be triangles.

Plate III. (p. 167) is an illustration of a part chain and a part theodolite survey, the result of a course of lectures I delivered at Cardiff; and, having been first surveyed with the chain only, it is applicable to the present consideration.

Line 1 commences at an acute angle of a fence A and runs to B. A station is left at b, for the purpose of tying-in other lines. Line 2 from B to C is tied to line 1 by the line marked A''. Line 3 from C to D is the longest line of the survey, and has upon it stations at d, d', and d''', and B'. From the stations d and d''', a triangle d, d'', d''' is set out for the purpose of taking up an indented fence on the eastern side of line 3, which triangle is tied

SURVEY OF FIELD PEN-Y-LAN, CARDIFF



Scale, 6 Chains to an Inch.

(To face page 167)

Crosby Lockwood & Son 7, Stationers Hall Court London

by the line d' d''. Line 4 from B' is really a tie-line to complete the construction of the chain survey proper, and the lines 3 and 1 are tied in by lines B^2 d''' on line 4 and B^2 *b* on line 1, whilst the diagonal line from the end of line 1 at B to B' in line 3 completely secures the figure.

I should here say that for practical purposes it is possible to survey this figure with the chain alone by a less number of tie-lines; but seeing that I was addressing myself to a number of pupils, I dwelt with greater emphasis upon this question of tying in figures, as I wanted to prove to them that if care and judgment be observed, it is possible under almost any circumstances to make a survey by means of lines which may or may not be in the form of triangles, triangles being, however, preferable. I wanted to prove that the lines forming the outside or boundaries of a survey may have their relative positions one to another accurately determined by such means, and (as I subsequently show under the head of "Theodolite Surveying") if a survey be so conducted the instrumental observations will confirm the accuracy of the chain survey.

From the end of line 3, line 5 from D to E, and line 6 from E to F, line 7 from F to G, and line 8 from G to A, complete the exterior boundaries of the survey. Lines 6, 7, and 8 are fixed to the other portions of the survey by the tie-lines F A, E' A, and E' B^2. It will be seen that line 6 passes out of the field through a fence into the waste land adjoining the Pen-y-lan road and again into the field through the fence running alongside this road. It may suggest itself to the student that such a step might have been obviated by moving the station E further inside the field, but the object I had in view was a double one: first to show how such a difficulty of crossing a fence at a very awkward point might be overcome; and secondly, that by the trouble occasioned thereby I sought to impress upon them the fact that the reason which actuated me in taking all that trouble was to carry out my principle of reducing the length of the offsets as much as possible.

I might here explain that the dotted line B H was advisedly laid out for the purpose not only of taking a section over it, but to enable me to demonstrate the method of measuring very undulating and broken ground. In this case we had to measure across a disused quarry of nearly two chains in width, and this being partly filled with water rendered our task somewhat difficult, but it had the result of further testing the accuracy of the survey, because its intersection with the tie-lines *b* C, B^2 d''', and B^1 A was identical when it came to be plotted, and we had the satisfaction also of finding that on arriving on line 5 at H it measured exactly in its proper position. It will be seen that running nearly parallel east and west are two banks or mounds and a footpath shown by a dotted line from E to B. This should be shown in the field-book by a sketch in the margin.

Foot-paths and Cart-tracks.—Foot-paths should always be shown by a single dotted line, cart-tracks by a double dotted line; but in taking the latter it is customary to ascertain the average width, the offsets of which are always taken and booked to the centre thereof unless for very exceptional reasons to the contrary.

Gates.—In picking up a gate in a fence it is necessary to fix the position of one of the posts accurately by means of a triangle and then to ascertain the width of the gate; it is not absolutely necessary to take both posts.

How to mark Hedge and Ditch.—It will be seen in the course of this survey that the fences are shown by a strong line, which indicates that it is a hedge; the little T's indicate the position of the hedge. In the case of Plates I. and II. it will be seen that the northern and a greater part of the eastern fence are shown by dotted lines, with crossed dashes; this indicates that it is a post-and-rail fence, and where the line is firm it is evident that it is an ordinary hedge. The north-western fence F H J is a double line, from which it is to be understood that it is a wall.

Avoid as much as possible crossing Fences.—On a large survey it frequently happens that many of the lines cut through a large number of fences, but it is very desirable to minimise this as much as possible, and it not unfrequently happens that, if one stands on an eminence at the commencement of (say) line 1, it is possible to command a long stretch of country to the termination of that line, passing, it may be, through ten or twelve fields. It is wise, therefore, for the surveyor, having determined upon his stations at the commencement and termination of this line, to dispatch his assistant with laths or other means of marking, with instructions that in front of every fence through which the line passes he is there to leave some distinguishing mark according to directions given by means of signalling right or left as the case may be. This should be done at every fence, for it is not at all an uncommon thing, in the process of chaining such a line, especially in a valley, that it is not only found impossible to command a view of the end of the line, but the hedges themselves may obscure the view also. But another reason in favour of marking the exact point of intersection is, that the chain-men can see the exact place through which the chain should pass, for which purpose the offset staff has a hook arrangement (as illustrated at Figs. 1 and 2, Chap. I.) to facilitate getting it through.

Be careful not to cut Fences unnecessarily.—There are many parts of England, especially in Leicestershire, where the hedges are not only very thick but exceedingly high; and in a survey for a railway which I made some years ago, of about twenty miles in length, with the snow on the ground, my patience and that of my assistants was very severely taxed by the constant necessity of passing through such fences; and here I would repeat the warning I have given elsewhere, that the surveyor must exercise very great judgment as to how he passes through such fences. I have seen most wanton damage done to a fine, handsome, fully-grown hedge by thoughtless and often wilful cutting of huge gaps. No good surveyor would descend to such a questionable practice, and it is to obviate such expedients that I recommend the line to be accurately ranged out before proceeding to chain. Here again my theory of becoming intimately acquainted beforehand with all the characteristics of the property, holds good, as unless the surveyor has walked completely round the boundaries and made mental note of the position and form of the various fences and other circumstances, he must not be surprised if after the expenditure of some hours' work he is brought face to face with the fact that the line, which he thought would be clear of a fence running parallel therewith, at an unexpected point projects apparently right into the fence, involving a fresh line being set out and all the previous work thrown away.

Don't cut down a Tree to save moving a Line.—Again, by a reconnoitre such as I have recommended, the necessity of cutting down trees (which intercept the line) is avoided. I speak somewhat feelingly on this subject, as in one case the reckless carelessness of one of my assistants—in cutting down a valuable oak-tree in my absence—not only involved me in heavy pecuniary loss and other unpleasantness, but very nearly was the means of throwing an important project out of Parliament.

In conclusion, it only remains for me to say that when a surveyor goes on a property—no matter whether at the instance of the owner or occupier, or whether he is really a trespasser—there are certain courtesies which devolve upon him, which, if neglected, may involve him in unpleasantness if not in more serious results. If it be necessary to pass through a gate, it is equally desirable that you should close it after you; the same remark applies to doors. If curiosity prompts individuals to interrogate you as to what you are doing, a little tact may evade the necessity of your divulging your business, and protect you from the mortification of afterwards finding out that a discourteous answer was given to a person who not only had a right to know what you were doing, but who had the power to make things very unpleasant.

Clear up the Ground after you.—After having completed the survey, before leaving the ground insist upon the chain-men removing all pegs and laths, which are often considered not worth carrying away, and pieces of paper that may have been used in the operations. In fact, leave the ground as nearly as possible in the state in which you found it.

Cautions.—It is not only *not* desirable to throw stones at dogs on the property, but the time occupied in so doing may be devoted to better purposes without the risk of giving offence to those to whom they belong! In putting pegs in the ground, especially in meadow land, care should be observed that they project very slightly above the surface, as otherwise serious injury is often done to cattle and horses grazing thereon.

The chain should be tested every morning before commencing operations.

If a station has been made by driving a peg into the ground, it is necessary to remove the peg if a rod is to remain there for the purpose of chaining to, as it should be exactly in the same position as the peg.

CHAPTER VI.

THEODOLITE-SURVEYING.

It seems hardly necessary to say, that the long lines in many important and extensive surveys can be best ranged, and are now executed, with the theodolite or other instrument for obtaining the angles which a line or lines make with another. In Chapters II. and V. I have endeavoured to show how surveying may be accomplished with the chain only; and for small surveys in open country, perhaps the base-lines are most accurately connected by chain measurements; but in the present chapter I propose to demonstrate how any large or complicated survey can be checked and considerably expedited by means of the theodolite.

Check-lines reduced.—In the first place we have seen that in the simple case of a four-sided figure, whose sides may have been carefully chained, it is impossible to plot the same except by diagonal or other check-lines—the only means of testing the accuracy of the work—whereas with a theodolite check-lines can be reduced in number, and in the field the accuracy of the relative positions of the four stations is made absolute by the addition of the four angles together, the sum of which should give 360 deg.

Accurately mark Station.—In commencing a theodolite survey, it is necessary to establish the chief stations in the first case, and at these points to drive stout pegs well into the ground, and into the centre of each should be driven a nail to mark the exact point of intersection of the lines, which is absolutely necessary.

When to take Angles.—It is a matter entirely of choice whether the angles be taken at the commencement of the survey or not; but it will be found most convenient to take them altogether (and possibly it is preferable to do so the last thing), as it is not desirable to keep the instrument knocking about in the field, where accidents, often of a serious nature, easily happen.

The Necessary Number of Angles.—The number of angles necessary to be taken depends so much upon configuration of the ground, extensiveness of the survey, and complexity or otherwise of the system of survey-lines and the tie-lines needful for checking them, that only a general rule can be laid down, viz.:—In all cases, the taking of angles serves as a useful check, but ought not to be employed to the exclusion of tie-lines where these can be run without undue increase of time and expense.

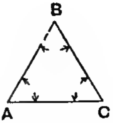


Fig. 219.

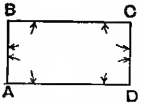


Fig. 220.

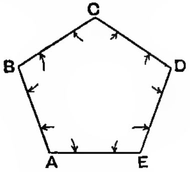


Fig. 221.

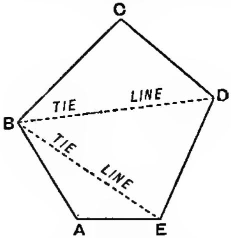


Fig. 222.

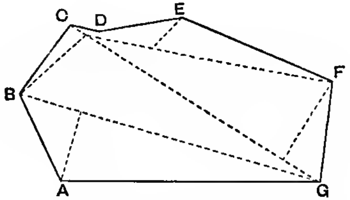


Fig. 223.

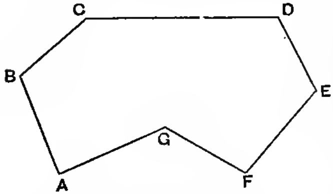


Fig. 224.

In the case of Fig. 219, if the side A C and the angles at A and C are given, it is possible to calculate the sides A B and B C; or if the angle B and the sides A B and B C are given, so may A C be found. Therefore in the field it is not absolutely necessary to take more than the angle B in the one case, or the angles A and C in the other, to check the accuracy of the sides A B, B C, A C; but this is a very primitive illustration, and really to do the thing properly I should recommend that *all* the angles be taken. Again, in Fig. 220, if the angles A and B are taken, then it will be possible to test the accuracy of the line B C; but it would be better to take the angles at A and C and run a tie-line from B to D. In the case of a five-sided figure (Figs. 221 and 222), tie-lines B D and B E and the angles at A and C would answer as well as taking all the five angles. In a figure such as Fig. 223, by taking the angles B A G, B C D, C D E, D E F, and F G A, some of the tie-lines shown might be omitted. And in Fig. 224, even if the seven angles at A, B, C, D, E, F, and G were taken, this would not dispense with the need of tie-lines from C to G and from G to D, or angular observations by which their length could be calculated.

It is in making a survey of a large estate that the greatest judgment is required as to what angles should be taken or not. And as a simple illustration I reproduce a part of a survey at Cardiff (see Plate III.), executed by the pupils attending my lectures. Here it will be seen that the general outline of the estate is one of seven sides, A B, B C, C D, D E, E F, F G, and G A, whilst the indentations are dealt with by small triangles *b a* B, *d d″ d‴*, and D J E. Although this is only a sketch from memory, yet it is fairly proportional, and serves to illustrate how the long offsets on lines A B, C D, and D E were avoided. I do not say that the angles of these small triangles should not be taken—indeed, if time permitted, it would be very desirable to do so—but I offer this sketch as a type of those angles which should be taken and those which may be avoided.

Angles necessary.—Thus angles 1, 2, 3, 4, 5, 6, and 7 are indispensable to the accuracy of the survey, whilst the triangles may be treated in the ordinary way. So in the survey of an estate, large or small, a similar treatment will be found desirable.

Requirements by the Examiners of the Surveyors' Institution.—In the instruction to candidates, under the head of "Land Surveying and Levelling," issued by the Surveyors' Institution, each candidate for Associateship in Subdivisions I. and II. is required "to make a survey with the chain of about 20 acres of land, more or less, (situated in any locality most convenient) comprising not less than four separate fields or inclosures, and having a minimum variation of 5 feet in the surface levels, and to take the angles of the principal inclosing and check lines with the theodolite, entering them in the proper place in the field-books." The whole of the work has to be executed from actual survey by the candidate, unaided by any other surveyor or skilled assistant. His general knowledge in surveying is thus tested in making the survey complete with the chain alone, and his acquaintance with the use of the theodolite by taking the angles as mentioned; while at the same time the value of that instrument as a check upon the accuracy of the chain-survey is well emphasized by the enjoinment of the latter operation.

What to avoid.—In Fig. 225 I reproduce an example given in an old work upon surveying which, I think, will illustrate what to avoid in theodolite-surveying. It will be seen that by a more judicious use of the instrument the irregular boundaries of this property might have been more accurately determined than by the system illustrated.

We have an estate consisting of three large fields and one small one, irregularly formed, and encompassed by fourteen main survey-lines. I have reproduced (Figs. 226 and 227) the field-book of lines 1, 2, 3, and 4. Now, commencing line 1, we have the angle

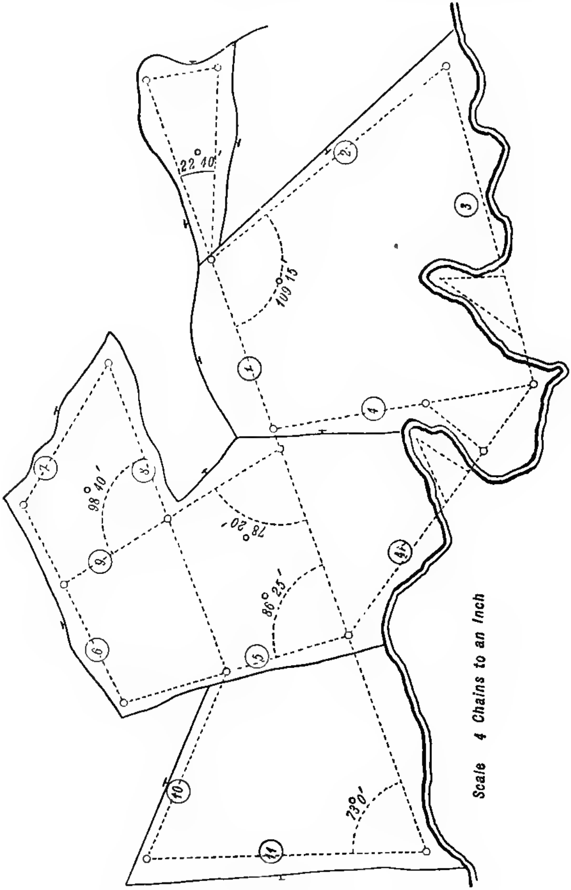


Fig. 225.

which line 11 makes with it, viz. 73 deg.; and at 490 we have line 5 making an angle of 86 deg. 25 min. with line No. 1; and at 910 line No. 9 makes an angle of 78 deg. 20 min. with line No. 1; but at 980, the station for line No. 4 on the right, it is not deemed necessary to take this angle, nor indeed is line No. 4A regarded as sufficiently important to have its position fixed with the theodolite. It is true that from 490 and 980 in line No. 1 the lines 4A and 4 have at 175 in the former, and at 222 in the latter, a check-line of 160;

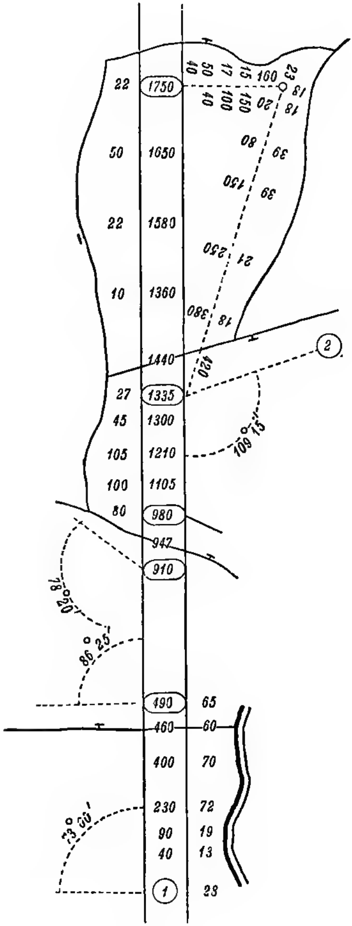


Fig. 226.

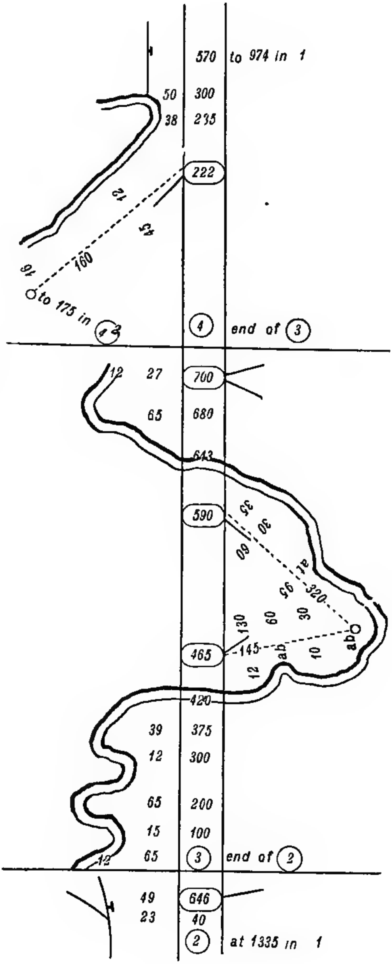


Fig. 227.

but the importance of having the meandering stream accurately fixed would surely justify, whilst the instrument was fixed at 490 to observe line No. 5, the taking of the angle of the line 4A. Now instead of forming two stations close together on line No. 1 at 910 and 980 for lines 9 and 4 respectively, by slewing line 9 round (which would be more convenient for the small fence) we should have only one instead of two stations for lines 9 and 4, and the angles formed by lines 9 and 4 respectively with line No. 1 could be taken at the same time. At 1335 in line No. 1 we have line No. 2 making an angle of 109 deg. 15 min., but instead of the small triangular field being fixed by the line 22 deg. 40 min. from 1335 in line No. 1 it would have been quite as well to check the actual position by finding the intermediate angle, without which I am of opinion the position of this triangular field is not sufficiently reliable. So much for what angles have been taken. I now turn to those that have been omitted, which lines in my judgment are essential to the satisfactory and indeed accurate completion of the survey. The angles between lines Nos. 2 and 3, 3 and 4, 4 and 4A, 5 and 10, 5 and 6, 7 and 8, 10 and 11, and 1 and 4.

Surveying a River.—In surveying a river, I do not know that I can suggest a better method of recording its serpentine course, than that suggested in Fig. 228. Here, we have line No. 2 forming an angle of 95 deg. 38 min. with No. 1, line No. 3 forming an angle of 61 deg. 50 min. with No. 2, line No. 4 forming an angle of 43 deg. 40 min. with No. 3, and line No. 5 forming an angle of 51 deg. 5 min. with No. 4. The various small triangles on lines Nos. 2, 3, and 4 required for the purpose of taking up the bends of the river will serve as additional checks to the work.

Don't spare the Use of the Theodolite.—Thus I trust I have established a rule that the theodolite, when once called into requisition on a survey, should not be used sparingly, but all the chief lines, constructing as it were the main network, should be systematically connected by means of ascertaining their various included angles.

Corroboration of Observation.—What can be more satisfactory, to take a simple illustration, than to find the sum of three observed angles of a triangle make 180 deg.; and much greater corroboration of your work in the field will attend a large number of angles giving a similar result, as I have shown in a preceding illustration.

Now let me guard against any possible misinterpretation of my meaning in the foregoing paragraphs. There are cases, as in Fig. 229, where it is quite unnecessary to take more than the six angles, A, B, C, D, E, and F, which govern the lines that absolutely affect the external boundaries of the estate, such as 1, 2, 3, 4, 7, and 8. The truncated cone formed by lines 1, 2, 3, and 4 should give by the sum of the angles A, B, E, and F 360 deg., whilst the angles C and D serve to determine the exact position of a portion of line 3 and line 7.

Line 5, by reason of each of its extremities being fixed by the chainage on lines 2 and 4, should by its length be an additional check of the accuracy of the survey, whilst it serves to pick up the fence which runs alongside it. The same applies to line 6, whilst if the angles C and D and the lines 3 and 7 have been accurately taken and plotted, then line 8 should exactly fit in at their extremities.

By reference to Plate No. I. it will be seen that a portion of my ground at Wimbledon Park is here delineated to illustrate the method of testing a chain-survey. The estate, bounded on the east and south by a wood, on the west by roads, and the north by a plantation, has been surveyed by chain only, on the lines 1, 2, 3, 4, 5, 6, 7, 8, and 9 with the various check-lines as shown. Now, having thus made an accurate chain-survey, it was desirable to show my pupils how I should have proceeded with a theodolite, and at the same time to check the other work. The following

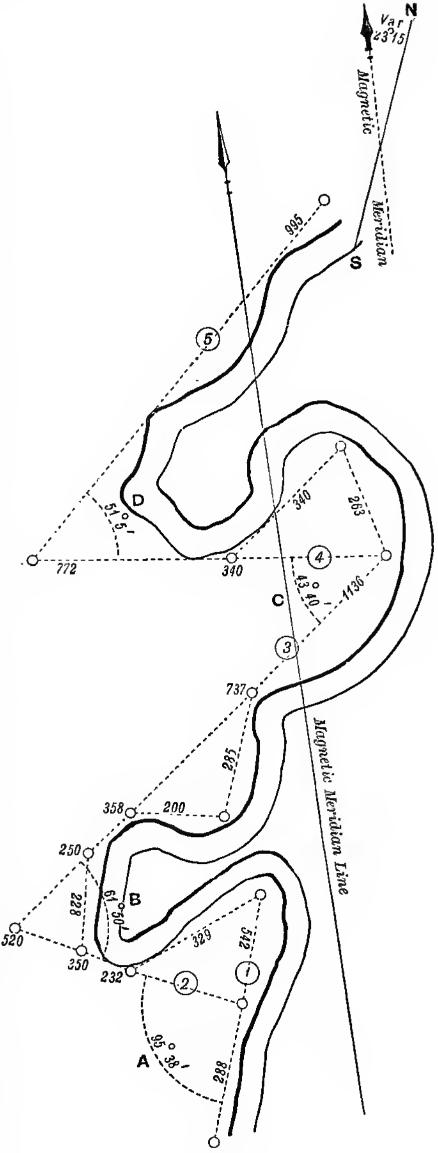


Fig. 228.

angles were necessary: DAB, ABC, CBE, BCD, JHD, and CDA, by means of which it was shown that the tie-lines DB, *a* A, E *e*, CG, and H *h* were obviated. As under the head of "Traversing" I shall have to deal with that part of this survey which has reference to the roads in the wood, I shall not at the present say anything about them. I have reproduced the field-book in connection with this survey, which will better illustrate the *modus operandi.*

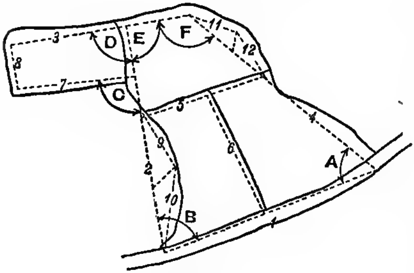


Fig. 229.

A few brief hints as to the practical part of theodolite work will form a useful conclusion of this chapter.

Hints on the Use of the Theodolite.—1. It is of little use attempting to use the theodolite on a foggy, rainy, or windy day. I need not dilate on my reasons in the first-mentioned case; but in the second, the wet gets into the glasses, and the constant necessity to take them out and wipe them is not only a source of delay but a very great tax on patience; and with regard to wind, not only does it affect the steadiness of the telescope, but the chief difficulty is to keep the plumb-bob from swaying about, and unless it is perfectly plumb over the nail or cross-cut the accuracy of the observations will be impaired.

2. Before planting the instrument, see that the point of the plumb-bob is exactly over the point of intersection of the line.

3. Always plant the legs of your instrument firmly in the ground as nearly level as your judgment directs. Don't force all three legs in at once by pressing from the apex, but take each leg separately, and with both hands press it into the ground.

4. Having planted the instrument, before you proceed to level it take care to clamp the upper plate to the lower one at zero.

5. Now level the instrument by means of the parallel screws, having previously attended to the adjustments for collimation, parallax, &c. (referred to in Chapter III.).

6. Now direct the telescope in direction of the extremity of the first line which forms the angle as B (Fig. 230), and when as near upon the point as is possible, clamp the lower plate, and bring it exactly to allow the cross-wires to intersect the point B by means of the lower tangent or slow motion. NOTE.—Do not on any account touch any other than the lower clamp and tangent screws in this operation.

7. Now (having entirely done with the lower clamp and tangent-screws) unclamp the upper plate and gently turn the telescope in direction of C, then clamp it at as near the point as possible, and with the upper tangent or slow-motion screw bring the cross-wires until they exactly intersect the point C.

8. Now proceed to read the number of degrees and subdivisions of degrees on the lower plate, and the number of minutes and subdivisions in the vernier.

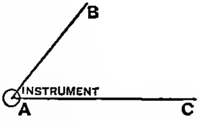


Fig. 230.

9. Always take the lowest point of a rod, and preferably the point of it, or an arrow held upon the nail or cross-cut in the peg. In the case of a church steeple it is advisable to take the apex.*

10. The observer should not talk or be listening to conversation during instrumental observations, as the distraction of his attention often leads to serious mistakes.

11. Most theodolites are graduated in the direction of the motion of the hands of a watch. When an angle has to be taken in the opposite direction, it has to be deducted from the instrumental reading at which it starts: from 360° if that reading is zero. Thus, if at starting the instrument is set at zero, an angle of $10^\circ\ 25'$ to the left of the direction in which the telescope points will read $360^\circ - 10^\circ\ 25' = 349^\circ\ 35'$. If the instrument is set at (say) 195°, an angle of $12^\circ\ 40'$ will read $195^\circ - 12^\circ\ 40' = 182^\circ\ 20'$. If at $11^\circ\ 25'$, an angle of $32^\circ\ 56'$ to the left will read $360^\circ + 11^\circ 25' - 32^\circ\ 56' = 338^\circ\ 29'$. Working to the left is often a difficulty to a beginner; but it is really a simple affair, requiring only care and attention.

---

* Chesterfield church excepted.

CHAPTER VII.

TRAVERSING.

WHILST surveying proper is entirely dependent upon a system of triangles or other figures, whose sides must be accurately measured, and whose relative points of intersection must be tied in with the greatest care, traversing may be termed a method of following the meandering of any irregular figure, whose sides shall be determined by angular observation.

Traversing with Chain.—Traversing may be accomplished with a chain only, but this mode of proceeding is open to great objection, as inaccuracies may find their way into the work itself, and there is no real security for its accuracy.

I illustrate by Fig. 231 the general principles of a chain traverse, and I think it will be manifest to those who have read the preceding chapters that little or no dependence should be placed upon the relative positions of lines to each other, which rely solely upon the measurement of a short length at the extremities of lines. Take the lines A B, B C, C D, D E, and E F (Fig. 231), whose directions are entirely dependent upon the care with which the triangles *a b* B, *c* C D, D *d e*, and E *f g* are taken, not only as affecting the measurement upon the ground, but more particularly the after operation of plotting; for, unlike a chain survey of a series of triangles and check lines, there is nothing in a chain traverse to guarantee the accuracy of the work. Upon fairly level ground, in the enforced absence of instruments, it may be admissible to ascertain the relative positions of diverging lines by some such method, to do even which I should strongly advise the use of an optical square to establish the triangles, which, wherever practicable, should be *right angled;* but in undulating ground I do not hesitate to say that chain traversing is inadmissible.

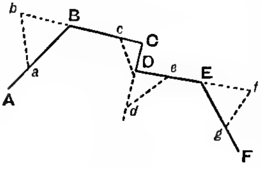


Fig. 231.

Traversing by Included Angles.—Traversing may also be performed by taking the included angles A B C, B C D, C D E, and D E F (Fig. 231) either with a box-sextant or, preferably, a theodolite. These angles having been accurately observed, and the lengths, A B, B C, C D, D E, and E F carefully

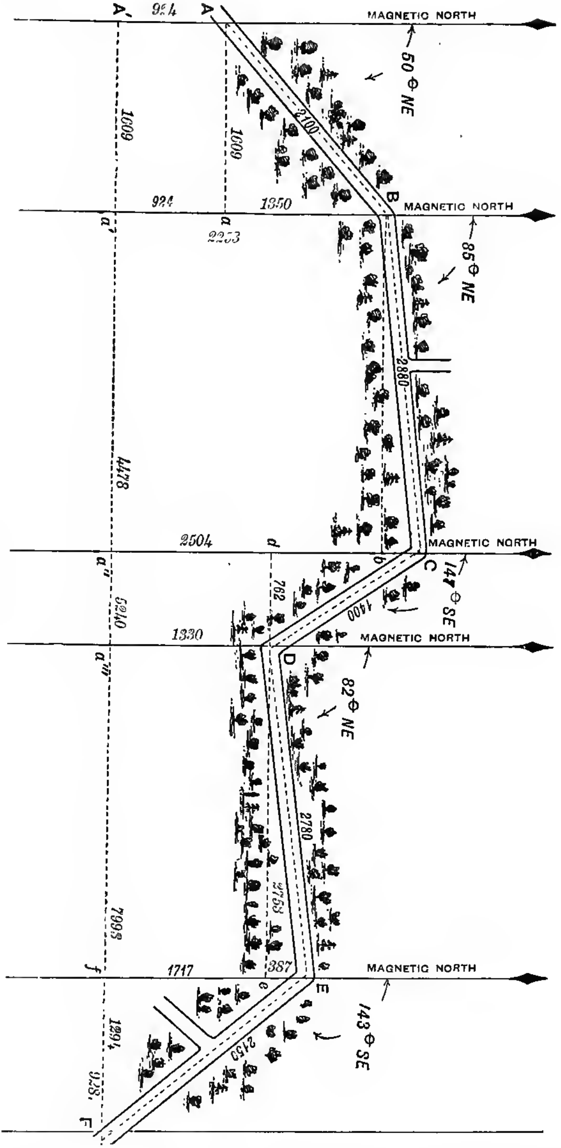


Fig. 232.

measured, the survey may be plotted with a straight-edge and protractor, but the greatest minuteness is necessary, for it is only what is called an "unclosed" traverse.

The most generally adopted system of traversing is by observations from magnetic north, as is illustrated in Fig. 232, which is an unclosed traverse; in other words, the survey has no means of being adjusted to its starting point, either from real cause or option. If we were to take such a figure as an octagon (Fig. 233), and work all round its eight sides at the points A, B, C, D, E, F, G, and H, then, if we had observed the necessary care in taking the angles, when we closed from H upon A we should find our work prove itself. But in the case of Fig. 232, which is the traverse survey of a meandering road on either side of which are dense plantations, in terminating our work at F we have nothing to guarantee its accuracy, as it is impossible to command the starting point A, which, if we could do so, would enable us to test our work.

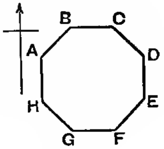


Fig. 233.

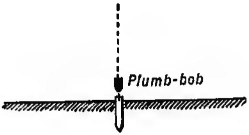


Fig. 234.

Now, in commencing a traverse, or any operations in which the compass is used, it is imperative to guard against any metallic attraction, as even with the most studious care traversing is a very delicate process. It is necessary to carefully select your stations, and by means of pegs or other means to mark the various points, as A, B, C, D, E, and F; the measuring of the lines between these points, together with the necessary offsets right and left, may be performed in the first instance or subsequent to the instrumental observations, but the one operation should be distinct from the other. Possibly it would be more convenient to have the survey made first, so that the angles and other information may be neatly entered in the book in their proper order and place. It should be here noted that after the instrument has been adjusted, the upper and lower plates being clamped at zero (and duly levelled, care having been taken to firmly plant it exactly over the point of intersection of the line*), and when the zero of the upper and lower plate has been made to coincide with magnetic north, that the lower plate should be firmly clamped, and on no account must it be touched either by accident or intent, otherwise the work will be in error. Now having taken all these necessary precautions, the instrument being placed at A (Fig 232), direct the telescope to a rod held on the peg at B, being careful that the wires intersect the spike of the rod. In the illustration before us the angle which B makes with magnetic north at A is 50 deg. on the A vernier and 310 deg. on that at B;* now remove the instrument to B, with the upper plate still clamped at 50 deg., and, after having adjusted it, direct the telescope back to A, and by means of the tangent-screw see that the wires exactly cut the bottom of the rod.

---

* This is best accomplished by driving a brass-headed nail in the centre of the peg, and let the point of the plumb-bob be coincident with it. See Fig 234.

Plenty of Assistance required.—Here let me say that plenty of assistance is required in traversing, as I am opposed to leaving a rod either stuck in the hole of the peg or behind the peg itself, either of which in the case of road or town surveying is impossible. Consequently I prefer that the spike of a rod should be held by an assistant on the nail in the peg. Having intersected the point A, unclamp the upper plate and bring it to zero; the result should be that the needle will record magnetic north, if not, something wrong has occurred, which must be attended to at once, even to the commencement *de novo*. Having satisfied ourselves that the needle is in its normal position, unclamp the upper plate and turn the telescope to C, which will give 135 deg. or 85 deg. from magnetic north. Keeping 135 deg. in the instrument, remove it to C, observe back upon B, bring the top plate to zero, and the needle should again assume magnetic north. Next direct the telescope to D, when the reading will be 282 deg. or 147 deg. from magnetic north, and so proceed at the points D, E, and F; the various angles should be entered as follows:—

A = 360°	2100	links.
B = 50°		
C = 85°	2880	,,
D = 147°	1400	,,
E = 82°	2780	,,
F = 143°	2150	,,

Northings and Southings.—Now in plotting the foregoing it is necessary, to ensure accuracy, to draw a series of vertical and horizontal lines intersecting the various points, and readily converting them into a series of right-angled triangles, whose base and perpendicular are the sines and cosines of the complements of the various angles; they are also designated "northings" and "southings" for the perpendiculars, and "eastings" and "westings" for the horizontal lines. In the first case draw the vertical line representing magnetic north at the point A. Now we have seen that the sine and cosine of the complement of an angle will give us the lengths of the base and perpendicular as A *a*, *a* B (Fig. 232), therefore $90^\circ - 50^\circ = 40^\circ$, and the natural sine of 40° is 0·64279, which, if multiplied by the length $\text{A B} = 2100$, will give 1350 links as the length *a* B; and the cosine of $40^\circ = 0{\cdot}76604 \times 2100 = 1609 = \text{A}\ a$. Again, B C makes an angle of 85° with magnetic north, consequently $90^\circ - 85^\circ = 5^\circ$, then nat. sin. $5^\circ = 0{\cdot}08716 \times 2880 = 251 = b$ C, or nat. cos. $5^\circ = 0{\cdot}99619 \times 2880 = 2869 = \text{B}\ b$. Now if the angle be greater than a right angle it must be deducted from 180 deg., and if greater than two right angles then from 270 deg., and if greater than 270 deg. then from 360 deg. Thus in the case of D the angle being 147 deg., we must deduct it from 180 deg.; thus $180^\circ - 147^\circ = 33^\circ$, and nat. sin. $33^\circ = 0{\cdot}54464 \times 1400 = 762 = d$ D, nat. cos. $33^\circ = 0{\cdot}83867 \times 1400 = 1174 = d$ C; and in like manner all the various sides may be calculated which are tabulated as under:—

---

* Most theodolites have their verniers marked A and B, the former being used to take the angle proper and the latter as a check.

		HYP.	BASE.	PER.
A B	$90^\circ - 50^\circ = 40^\circ$	2100	1609	1350
C	$90^\circ - 85^\circ = 5^\circ$	2880	2869	251
D	$180^\circ - 145^\circ = 33^\circ$	1400	762	1174
E	$90^\circ - 82^\circ = 8^\circ$	2780	2753	387
F	$180^\circ - 143^\circ = 37^\circ$	2150	1294	1717

But these calculations are not alone sufficient to ensure accuracy, as it is necessary to treat an unclosed traverse somewhat in a similar manner to plotting a section. Referring again to Fig. 232, it will be seen that *f* E is 1717, and *e* E is 387, therefore *e f* is $1717 - 387 = 1330$; *d* D, D *e* are in one straight line, consequently *a″ d* is 1330, and *d* C is 1174, whilst *d b* is $1174 - 251 = 923$, and *b* B is parallel to A *a*, therefore $a'' d + d b = 1330 + 923 = 2253 = d'$ B; consequently if we mark on the line A′ F the horizontal distances A′ *a′*, A′ *a″*, A′ *a‴*, A′ *f*, and *f* F, which are 1609, 4478, 5240, 7993, and 9287, and then plot A′ A = 924, *a′* B = 2253, *a″* C = 2504, *a‴* D = 1330, *f* E = 1717, we shall have satisfactorily accomplished our traverse, and assured ourselves as to its accuracy. If it be possible, with the instrument at F, to command a station at A′, then taking the last angle, viz. 143° from $180^\circ = 37^\circ$, consequently E F A′ = 53°; if, therefore, from F an angle E F A′ of 53° be set out it should give a point 924 links below A, which is of course an important check equally as the length A′ F could it be accurately chained, which would give 9287 links.

As to closing a Traverse.—Of course, if it is possible, it is always desirable to close a traverse, even to the extent of working back to your starting point by a circuitous route, as illustrated in Fig. 235, whereby, after having run from A to B, C, D, E, and F, which was work requiring to be done, it would be satisfactory to continue back to A by the zigzag route F G, G H, H J, J K, and K A;

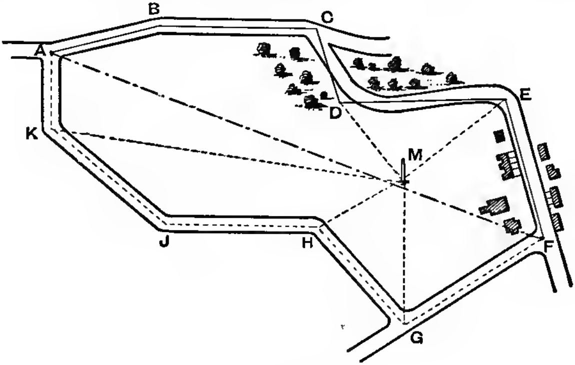


Fig. 235.

and although it would be more satisfactory to have the lengths of these lines as well as their bearings, yet it is not absolutely necessary, as the sum of the angles will give, if the observations be carefully taken, the result of working back on to A as we commenced. By such a method the necessity of calculating the sines and cosines is obviated.

Care in Checking.—In taking angles from magnetic north it is necessary to be very careful that the readings are correct; and as an additional check upon the work, especially in a close survey, it is desirable to take frequent objects, such as the chimney at M in Fig. 235, to which observations may be made at the points D, E, F, G, H, and K.

Relative Position of Bearings.—In booking the bearings, it is desirable to have them in their proper order. For instance, all angles less than 90 deg. will be N.E.; between 90 deg. and 180 deg. S.E.; between 180 deg. and 270 deg. S.W.; and between 270 deg. and 360 deg. N.W. When it is possible to take the included angle between points such as E F G (Fig. 235), it is, of course, very desirable to do so.

Magnetic Variation.—It is necessary to make allowance for what is termed the magnetic declination or variation, which alters every year. This at Greenwich, at the beginning of the present year (1904), is variously given as 16° 5′ and 16° 17′ W., decreasing 6·2′ annually. It is needless to say that the declination varies with the geographical position of the point of observation.

CHAPTER VIII.

TOWN-SURVEYING.

To make a survey of a town or even a village is by no means an easy task, added to which it is a very tedious proceeding, for it seldom happens that lines of any great length can be arranged. It is desirable, however, that when possible a base-line should be taken through the town from end to end, in order to tie all the other lines on to it. Triangulation is almost impossible owing to the irregularity of the streets. It is equally out of the question to do town-surveying without an instrument for taking the angles of the various lines.

The surveyor should provide himself with a skeleton plan of the principal thoroughfares, upon which he should lay out such lines as appear to him feasible and then proceed to examine them upon the ground. Having determined upon some of the chief lines, he should establish stations, where possible using hydrants or sewer-ventilators to mark the spot. In the absence of such, he will have to drive down iron spikes or "dogs" into the pavement, for which purpose he should be provided with a small steel bar and a fairly heavy hammer. The spikes should be of $\frac{3}{8}$-in. iron and from $2\frac{1}{2}$ in. to 4 in. long, pointed at one end. They should be driven well home and their position very carefully observed by means of a detail sketch, with several measurements from well-defined points, as in Fig. 236, taking distances from the four angles of Cross Street

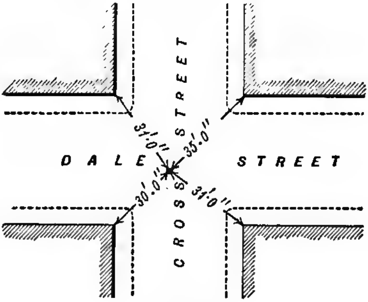


Fig. 236.

and Dale Street; or, as in Fig. 237, with two distances from the angles of Church Lane and High Street, and from the end of the "Crown Inn" and from a point measured along the face of the hotel from George Yard; or, in Fig. 238, from the two angles of the Market Place and those of Market Street.

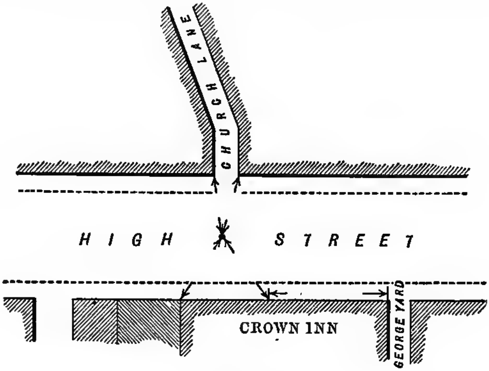


Fig. 237.

It is recommended by some writers to take "lamp-posts, corners of buildings, &c.," as "objects at a distance," forgetting that inasmuch as instrumental observation will be necessary at all points of divergence, such points will be of very slight service, independent of their somewhat questionable applicability. Town-surveying requires great care and patience, with a very considerable amount of method. It resolves itself into three distinct operations after the lines and stations have been determined: 1, the observation of the angles; 2, the chainage of the lines between these points; and 3, the detail measurement of the yards, gardens, buildings, &c.

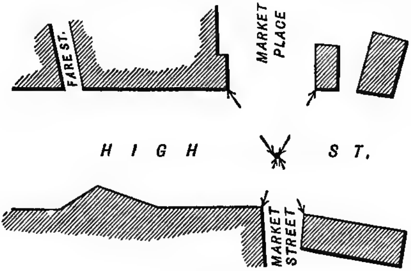


Fig. 238.

Taking Angles.—There are two ways of taking the angles.

First by taking (with theodolite or prismatic compass) the angle which a street or road makes with the magnetic meridian; but this cannot be recommended in towns (although in villages it may be more practicable), in consequence of the numerous sources of attraction to the needle, such as tram-rails, lamp-posts, hydrants, man-holes, iron railways, &c. By the second and most reliable method the included angles of one or more lines are taken with the theodolite as illustrated in Fig. 239, where a line along Station Road terminates at the junction of three streets. Here the theodolite should be planted, and after being carefully adjusted, the angle between Station Road and High Street (90° 30′), between High Street and West Gate (71°), and between West Gate and Mill Street (46°), should be observed; the sum of which should be 207° 30′. Now take the angle between Station Road and Mill Street, which should be 152° 30′, or the difference between 360° and 207° 30′.

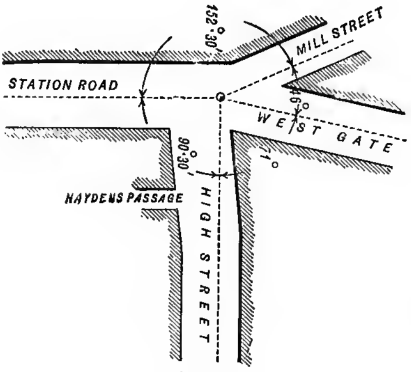


Fig. 239.

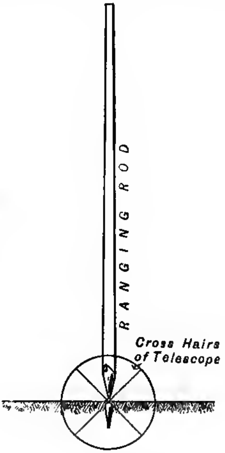


Fig. 240.

Objection to Lamp-posts, &c.—My reason for *not* taking lamp-posts, corners of houses, &c., as distant points upon which to fix the telescope, is that in the first place they can only be of a temporary character, and a lamp-post especially is not sufficiently defined for the purpose even if it be perfectly perpendicular. If spikes are driven in the streets or roads at points of intersection, it is surely the most accurate method for a chain-man to hold the point of the rod upon the spike, which point only is to be taken, for I cannot impress upon the student too strongly the necessity of observing the bottom of the rod as in Fig. 240 in *all* surveying operations, whether it be simple chain-surveying or with a theodolite. By this means we have an absolute point upon which our instrument will in turn be placed, so that with necessary care all our observations should be accurate, and judgment (often very misleading) as to which is the actual centre of a far distant lamp-post is obviated.

In consequence of the circuitous nature of many streets in European towns—which, unlike American cities, were evidently never laid out with any idea that it would be necessary to survey them—it is often impossible to get a straight line from end to end. Take the case of Fig. 241. Here we have, at A, to take the two angles right and left equal to 180°. At B we should take the angle between A and Bemer Street, and that between Bemer Street and C, whilst to test our work we must observe the angle C B A, all three being equal to 360°; at C, the included angle B C D and its supplement; at D, all four angles, which should equal 360 deg.

Now a very natural question might be asked: "Yes, I see how you do such a street, and if I have taken the angles and distances between the points correctly, all well and good: but how do I know that it will all fit on to the other parts of the survey?" I will endeavour to clear this question up.

In Fig. 242 we have a sketch map of part of the town of Leatherhead, of which it was desired to make a detailed survey. It was found impossible to run a larger base-line through the principal streets than the line A B, about 1,200 ft.; but C D, 2,050 ft., could be tied on to the other portion of the survey outside the

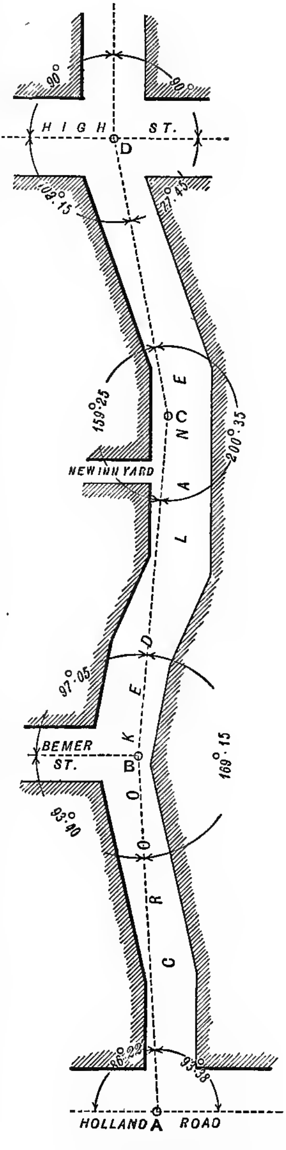
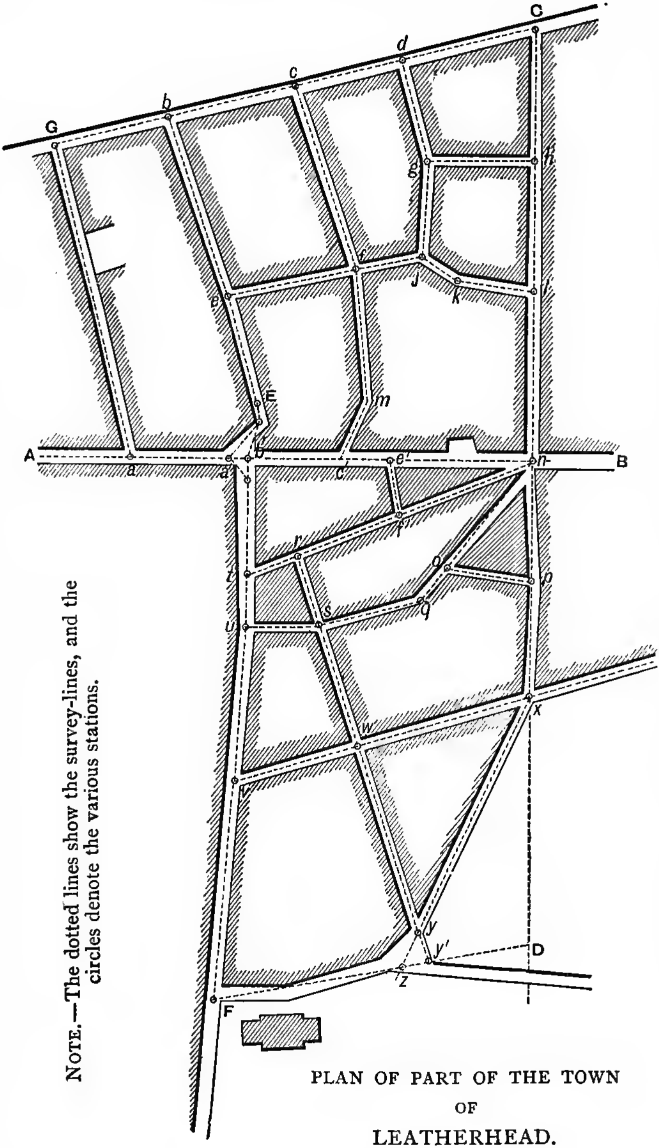


Fig. 241.



NOTE.—The dotted lines show the survey-lines, and the circles denote the various stations.

PLAN OF PART OF THE TOWN
OF
LEATHERHEAD.

Fig. 242.

town, and as it is always best to take the longest line for a base we adopted C D. It so happened that A B is so situated that it was possible to set out the line at right angles to C D, which of course was of immense advantage. But with the exception of the short line *g h*, this is the only case in which it was possible.

Taking the upper portion first, it will be seen that G C at the ends of C D and of *a* G with A B circumscribed this portion of the town; on the line A B, stations at *a*, *a′*, *b′*, *c′*, *e′*, and *n* were left, whilst on G C, stations *b*, *c*, and *d*; and on the upper part of C D, *h* and *l*.

Strictly speaking, the angles *a* G C and G C D should be taken as well as G *a* B and C *n* A; although it is argued that if these latter two angles are accurately taken, and the distances G *a*, *a n*, and *n* C are carefully measured, then by calculation in the one case and measurement in the other the length G C will be proved. I say it is so argued, but my own opinion is that whilst about it the most satisfactory way will be to take the angles with the theodolite, especially as we must take the angles G *b e*, G *c f*, G *d g*. It is not absolutely necessary to take the angle *b e f*, but those *c f j*, *f j g*, *g j k*, and *k l* C are imperative; as are also A *b′* E and A *c′ m*. The angle A *a′* E is not necessary, but the line *a′* E should be carefully measured as a tie; *g h* needs only to be measured from its two terminal points and will act as a check on *d g* and C *h*.

Similarly, if the angles A *b′ t* and A *n* D be carefully observed in the lower portion, it is not absolutely necessary to take more than *b′ t n* and *u s q*, as all the other lines tend to check the trapesium *b′* F D *n*; for *t n* and *v x* in one direction and *r y′* and *x z* in the other are as complete checks as can be wanted.

Thus will be seen the relative systems to be adopted in street surveying, but let it never be forgotten that there should be no question about the angle any street may form with another. The line C D was able to be produced until it fitted into the system of triangulation for the survey of the district around the town.

The traffic in the streets is a considerable drawback to the operations of the surveyor, whilst from twelve till two and after four o'clock are periods towards which he looks with dread, as at these times he is sure to be accompanied or surrounded by a powerful contingent of the rising generation, whose inquisitiveness and love of mischief are of the greatest impediment to his progress, and test his patience and temper to the utmost.

As to the Chain.—For ordinary small scale plans the measurements may be taken with a 66-ft. chain, but when great detail and accuracy are requisite the 100-ft. chain is the best. The offsets should be taken in feet and inches with a tape; those at right angles to the chain-line require the greatest care and are best set out with an ordinary square (as it is seldom, from the narrowness of the streets, that an optical square can be used) having one arm 6 ft. and the other 4 ft. long (see Fig. 243). This should be laid on the ground and adjusted until the long arm is in line with the point to which the offset is to be taken. But it is not sufficient to trust to such offsets to fix the corners or angles of buildings. A tie-line is necessary, as in sketch.

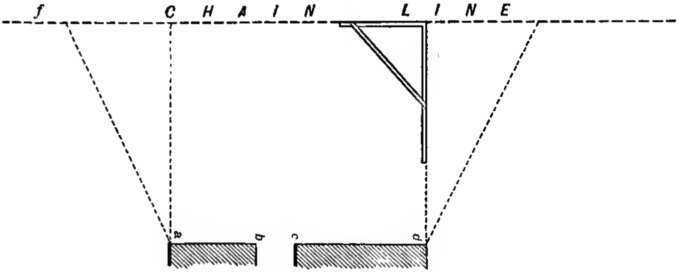


Fig. 243.

It is very seldom that the frontages of streets are straight or that they are of equal width. It more frequently happens that indentations of all kinds occur as in Fig. 244, where it will be seen that in order to accurately take up the various angles and indentations a very elaborate network of triangulation is necessary, as shown by the dotted lines.

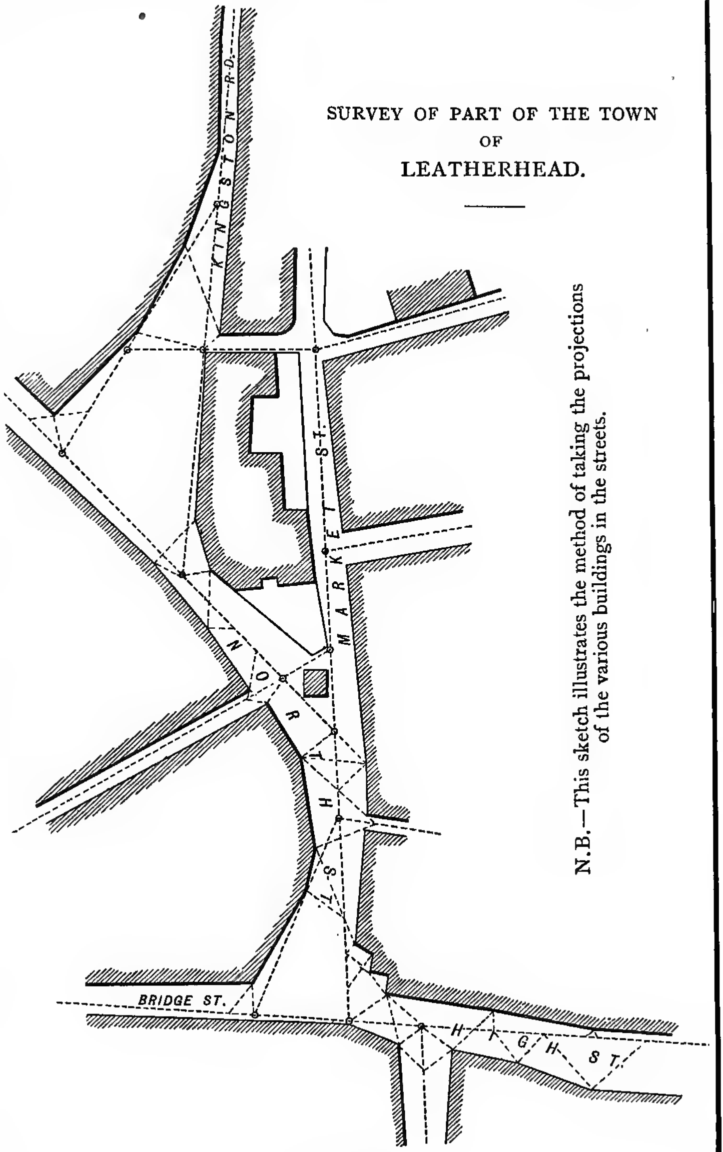
It is not sufficient at the angles formed by one street running out of another to take an offset at right angles, and form a right-angled triangle as a check. It is necessary to make an independent triangle such as A *b c*, A *a b*, A *c d*, A *e d*, A *e f*, A *g f*, A *g h*, or A *h a* in Fig. 245.

The diamond formed by those triangles which are hatched need not necessarily be taken, but it is quite as well to have the thing complete, especially at important points.

When the outlines of the streets have been surveyed and plotted, the surveyor should make a careful tracing of sections of the work, and then carefully walk over the route to examine every detail, so as to be satisfied that nothing has been omitted.

Then a station plan, drawn to a large scale, should be prepared and mounted, in sizes of about 18 in. square, on a board, so that the details of the houses and outbuildings may be accurately drawn to scale as the measurements proceed. A steel tape or a 10-ft. rod is the best thing for this purpose.

When to take Angles.—In busy thoroughfares it is always desirable to take the angles soon after daybreak, so that the operations may not be impeded by the traffic.



SURVEY OF PART OF THE TOWN
OF
LEATHERHEAD.

N.B.—This sketch illustrates the method of taking the projections of the various buildings in the streets.

Fig. 244.

In measuring buildings the greatest care is necessary to see that the total length of a series of frontages is equal to the sum of the separate frontages. For this purpose the addition should always be made on the side of the field-book or upon the detail drawing, and in ink if possible.

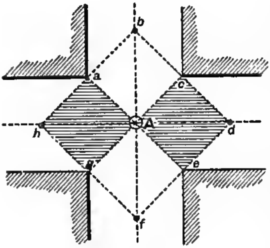


Fig. 245.

Do not erase Figures.—In all branches of surveying it is important to bear in mind that figures when once written down should on no account be erased, but if it is necessary to alter them then draw the pencil through the existing figures, and over or by the side make the alteration. I have seen some very serious mistakes occur by rubbing-out figures which after all have proved to have been right.

If you cannot drive a peg or spike into the road, as in the case of asphalte roads, then the intersection of lines should be arranged so as to cut at some point on the curb or pavement, in order that a nail or spike may be driven in at a joint.

Use Arrows for counting.—In measuring a line along a street an arrow should be stuck in if possible, or if not, it should be left to denote the number of chains, and the leader (who should always have plenty of chalk about him) should mark with a "crow's-foot" the end of the chain together with the number, with chalk, either upon the pavement or on the walls of the buildings.

As to Buildings.—Outhouses should be specified in the field-book. Churches, chapels, schools, and all public buildings should be carefully noted. Also public-houses, beer-houses, "on" and "off" licences, &c.

Lamp-posts, Gullies, &c.—The position of lamp-posts, gullies, ventilators, sluice-valves, hydrants, manholes, &c., must be taken *en route* and carefully plotted on the plan.

As to Streams.—Should a street or road cross over a river or stream the full particulars thereof must be noted; and by an arrow the direction of the flow should be indicated. Or in the case of a railway crossing over or being crossed by a street, the name and particulars of the railway, together with the direction of its commencement and termination, should be ascertained and marked upon the plan. The nature of the street or road should be observed—whether gravel, macadam, granite-pitched, wood, asphalte, &c. And the pavement, whether York paving, artificial stone, asphalte, concrete, brick-on-edge, gravel, &c. The boundaries of the various parishes must be ascertained and carefully plotted, even in such a case as occurred to me at Hereford, where I found that the intersections of three parishes occurred in one of the bedrooms of a school-house. The parliamentary or municipal boundaries, or those of wards, must also be shown. Each road or street must be plainly marked with its name, and the thoroughfares at the outside of the survey should have written in italics the places to or whence they lead.

As to Plotting.—The survey of a town or parish should *always* be plotted so as to be north and south; in other words, the top of the sheet is north and the left and right sides are west and east respectively.

Photographic Surveying.—The employment of photographic views as data upon which to construct a topographical survey, was originally suggested by Colonel Laussedat a French officer, some time Professor in the École Polytechnique and Director of the École Centrale des Arts et Metiers, whose exposition of the theory and procedure still forms the foundation of its application in practice. Various points and features of the ground surveyed are shown in views photographed from various stations, and are located by intersection of the sight-lines from two or more such stations; the position of the several stations being fixed by a trigonometrical survey. The general principle is the same as that of plane-table surveying; but whereas the plotting of the work in the case of the latter is, practically speaking, done on the ground, that of a photographic survey is performed at leisure in the office. The process of this plotting is one of considerable complexity, and the business altogether requires no small acquaintance with the theory and practice of photography. To teach a learner the work of such a survey by a brief description is impossible: a course of instruction possessing any real value would form a treatise in itself, and even this would be of little use unless supplemented by illustrative teaching and practical example. The student is referred to an excellent handbook by Mr. E. Deville, Surveyor-General of Dominion Lands in Canada,* which goes fully into the subject, and appears to carry the learner as far as is possible in print, beyond which nothing but actual experience in practice can qualify him in knowledge.

Photography has been extensively employed in Government surveys in Canada; and would probably be found advantageous for a general topographical survey of mountainous and rough country, as a basis for subsequent detailed surveys by the ordinary methods. In the preface to his book, Mr. Deville has stated and discussed the advantages and the difficulties of the photographic method, and regards it as possessing great superiority over that of the plane-table. It must, however, be added that the Geographer of the U.S.A. Geological Survey has recorded * a distinctly unfavourable opinion of it as compared with plane-table surveying.

---

* "Photographic Surveying," 8vo, Ottawa 1895.

---

* H. M. Wilson, "Topographic Surveying," 8vo, New York 1900.

CHAPTER IX.

LEVELLING.

LEVELLING is the art of finding the difference between two points which are vertically at different distances from a plane parallel with the horizon. Take the ocean or a sheet of water, the calm surface of which is in a plane parallel with the horizon, then the bank or beach that is above the water-line at certain points is relatively higher in level than the water itself. Thus in Fig. 246, where A represents the impingement of the water upon the slopes of the stream, B is relatively higher, and C and D lower, than the horizontal line L L′.

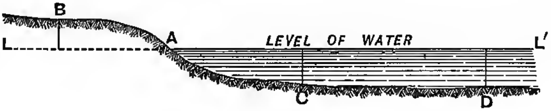


Fig. 246.

This is a very primitive description of what levelling means, but it is nevertheless a true one.

As to the Earth's Curvature.—But there is a very important consideration in reference to this question, and that is, that the earth being spherical in form, strictly speaking two points are only truly level when they are equidistant from the centre of the earth.

Also, one place is higher than another, or out of level with it, when it is further from the centre of the earth ; and a line equally distant from that centre, in all its points, is called the *line of true level.* Hence, because the earth is round, that line must be a curve, and make a part of the earth's circumference, or at least be parallel to it and concentrical with it, as the line P F D B C E Q (Fig. 247), which has all its points equally distant from A, the centre of the earth considered as a perfect globe.

But the line of sight F′ D′ B C′ E′, given by the operation of levels, is a tangent or right line perpendicular to the semi-diameter of the earth at the point of contact B, rising always higher above the true line of level the farther the distances, and is called *the apparent true level.* Thus $C'C$ is the height of apparent above the true level, at the distance BC from B; also $E'E$ is the excess of height at E. The difference between the true and the apparent

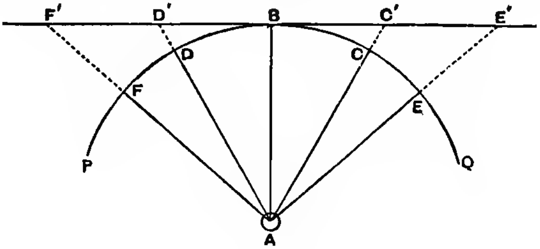


Fig. 247.

level, it is evident, is always equal to the excess of the secant of the arc of distance above the radius of the earth.

Now the difference CC' between the true and apparent level, at any distance BC or BC', may be found thus: by a well-known property of the circle $2AC + CC' : BC' :: BC' : CC'$; or, because the diameter of the earth is so great with respect to the line CC' at all distances to which the operation of levelling commonly extends, that $2AC$ may be safely taken for $2AC + CC'$ in that proportion, without any sensible error, it will be, as $2AC : BC' ::$ BC' to CC'; CC' therefore $= \frac{BC'^2}{2AC'}$ or $\frac{BC^2}{2AC}$ nearly; that is, the difference between the true and apparent level is equal to the square of the distance between the places, divided by the diameter of the earth; and consequently is always proportional to the square of the distance.

Taking the mean diameter of the earth ($2AC$), as 41,781,097 feet = 501,373,164 inches, and the distance $BC = 1$ mile = 63,360 inches:—

$$\frac{BC^2}{2AC} = \frac{63{,}360^2}{501{,}373{,}164} = \frac{4{,}014{,}489{,}600}{501{,}373{,}164} = 8 \text{ inches in one mile,}$$

which is the difference between the apparent and the true level.

Refraction.—There is also another matter that has to be considered, and that is "atmospheric refraction." The line of sight, being the line along which the light proceeds from the object looked at to the telescope, is not perfectly straight, being made slightly concave downwards by the refracting action of the air. Hence the point seen on the staff apparently in the line of collimation produced is not exactly in that line, but is below it by an amount called the error from refraction, and thus the error arising from curvature is partly neutralised; and the correction to be subtracted for curvature and refraction usually is somewhat less than the correction for curvature alone.

The error produced by refraction varies very much with the state of the atmosphere, having been found to range from one-half to one-tenth of the correction for curvature, and in some cases to vary even more. Its value cannot be expressed with certainty by any known formula; but when it becomes necessary to allow for it, it may be assumed to be on an average 0·154 of the correction for curvature; so that the joint correction for curvature and refraction to be subtracted from the reading of the staff is as shown in the following table:—

TABLE OF DEDUCTIONS FROM STAFF-READINGS FOR CORRECTION OF CURVATURE AND REFRACTION TOGETHER.

Distance, or B C.	Deduct.	Distance, or B C.	Deduct.
Feet.	Decimals of a Foot.	Miles.	Feet. Dec.
300	0·002	¼	0·035
600	0.007	½	0·141
900	0·016	¾	0·318
1200	0·029	1	0·564
1500	0·046	1½	1·270
1800	0·066	2	2·258
2100	0·089	2½	3·528
2400	0·117	3	5·081
2700	0·148	3½	6·914
3000	0·182	4	9 032
3300	0·220	4½	11·431
3600	0·262	5	14·112
3900	0·308	5½	17·077
		6	20·322

Thus, if the staff be 600 feet from the instrument, and the cross-wires cut 10·50 feet, we must deduct for correction of curvature and refraction 0·007 of a foot from this reading, which should now be 10·493 feet.

Professor Rankine expresses an opinion that "the errors produced by curvature and refraction are neutralised when back and fore sights are taken to staves at equal or nearly equal distances from the level. At distances not exceeding ten chains they are so small that they may be neglected. The uncertainty of the curvature and refraction makes it advisable to avoid, in exact levelling, all sights at distances exceeding about a quarter of a mile."

Adjustments.—Before proceeding to level it is necessary to attend to the temporary adjustments, which require to be made each time the instrument is set up, as follows:—

1. To plant the legs of the instrument firmly in the ground, taking care that the parallel plates are made as horizontal as possible.

2. To level "the instrument," that is, to place the vertical axis truly vertical.

3. To adjust the telescope for the prevention of "parallax," that is, to bring the foci of the glasses to the cross-wires, look through the telescope, and shift the eye-piece in and out until the cross-wires are seen with perfect distinctness. Then direct the telescope to some well-defined distant object, and by means of the milled-head screw, shift the inner tube in and out until the image of the object is seen sharp and clear, coinciding apparently with the cross-wire. This latter part of the adjustment must be made anew for each new object at a different distance from the preceding one. The nearer the object the further the inner tube must be drawn out.

A good test of the adjustment for parallax is to move the head from side to side while looking through the telescope. If the adjustment is perfect, the image of the object will seem steadily to coincide with the cross-wires; if imperfect, the image will seem to waver as the head is moved. If the image seems to shift to the opposite direction to the head, the inner tube must be drawn out further; if in the same direction, it must be drawn inwards.

Levelling is of two kinds, simple and compound. Simple levelling has only one line of collimation, whilst compound levelling entails constant changes of collimation, and hence the necessity for extreme accuracy in the work and care in the adjustment of the instrument. In the case of Fig. 248 the instrument is placed equally between A and B, and the telescope being directed towards A, the line of collimation cuts the staff at 5·70 (this being the first reading is called the "back-sight"); * the telescope is then reversed, and the reading appears 11·68, consequently, by the invariable rule that if the intermediate or fore-sights are greater than the back-sights they are "falls," and if less "rises;" in the present case it is a fall of 5·98 feet from A to B. Here I would refer to a query which is frequently put by students:

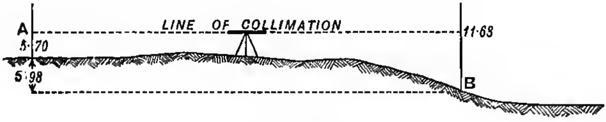


Fig. 248.

---

* In simple levelling, the first sight after the level has been planted and adjusted is always the "back-sight," and the very last sight before the instrument is removed is the "fore-sight:" all others are "intermediates."

"How does the height of the instrument affect the result?" The height of the instrument has nothing whatever to do with the operation of levelling, and the only thing that will account for this fallacy is either that it is known to be requisite in levelling with a theodolite to have the distance of the axis of the telescope from the ground, or that in the early days of levelling it was usual to note the height of the instrument. But I think I shall have no difficulty in showing that, as in the case of Fig. 248, the line of collimation being an imaginary line parallel with the horizon, the heights which are taken at A and B are in reality the depths of the surface of the ground at those points below the line of collimation, consequently it does not matter whether the instrument is 4 or 40 feet above the surface of the ground.

Compound Levelling consists of following the undulation of the ground along a line of section, by means of varying lines of collimation, according to the rise or fall of the ground.

Fig. 249 is a simple illustration of my meaning. The instrument is placed equidistant between A and B, and the reading of the staff at A is 4·10, whilst that at B is 10·15, showing a fall of 6·05. Next remove the level to *b* and establish a new line of collimation. Now where in the previous case was a fore-sight 10·15, the instrument now reads on the same staff 5·30 as a back-sight, consequently the line of collimation is 4·85 lower than that from A to B. Now turn the telescope towards C for a fore-sight 8·19, and then move the instrument to *c*. Here again our line of collimation is lower, its exact depth being determined by reading off the staff at C a back-sight of 3·50, which gives a fall 4·69. Reverse the telescope for a fore-sight at D of 0·17, now move the level to *d* on higher ground, and we find that the line of collimation cuts the staff at D for a back-sight at 7·18, or a rise of 7·01. At E the fore-sight is 0·30, whilst a back-sight from the level at *e* to E is only 0·40, showing the last line of collimation to be only 0·10 higher than the one from D to E, the staff at F showing a fore-sight of 6·15 shows a fall of 5·75 from E to F. We will now tabulate these results, and for the moment I shall only deal with two columns for the readings of back- and fore-sights, and the ordinary "rise" and "fall" columns.

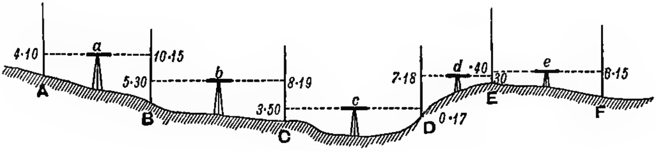


Fig. 249.

Back-sight.	Fore-sight.	Rise.	Fall.
4·10	10·15		6·05
5·30	8·19		2·89
3·50	0·17	3·33	
7·18	0·30	6·88	
0·40	6·15		5·75
20·48	24·96	10·21	14·69
	20·48		10·21
	4·48		4·48

So that we see that by taking the less from the greater we get rises or falls, as follows: 10·15 being greater than 4·10 is a fall of 6·05, 8·19 being greater than 5·30 gives a fall of 2·89, whilst 0·17 being less than 3·50 we have a rise of 3·33; and similarly 0·30 being less than 7·18 we have a rise of 6·88, and 6·15 being again greater than the back-sight 0·40 we have a fall of 5·75. Now, to prove our calculations, if we take the sum of the rises from that of the falls we get the same result as deducting the sum of the back-sights from that of the fore-sights, or 4·48, which shows that there is a total fall from A to F of 4·48 feet, regardless of the fact that the ground rises at D and E.

Now before I proceed to elaborate the subject of compound levelling, I think it advisable to deal with two primary questions, which may well be introduced at this point. I refer to datum and bench-marks.

Datum.—First, as to datum. It is an imaginary line parallel with the horizon, and with the several lines of collimation. Its object is to simplify all calculations in levelling operations by referring all the observations to one fixed standard, which is fixed at some convenient depth below a well-known and clearly defined mark (called usually a bench-mark), and from this standard line all heights are relatively adjusted.

Ordnance Datum.—The ordnance datum of this country was determined by the ordnance authorities to be "the approximate mean water at Liverpool," and all the levels marked upon the ordnance maps are the "altitudes in feet above this datum," so that in the north of Scotland or in Wales, equally with Southampton or Yarmouth, all over the United Kingdom, whenever a figure preceded by a dot, thus ·33·6 or ·336·0, is upon a map, it shows in the one case 33 ft. and $\frac{60}{100}$th of a foot, and in the other 336 ft.

above this datum respectively. It is not, however, usual or necessary to adopt the ordnance datum in ordinary levelling operations, but to assume some convenient depth below the lowest point of the section, the reason for which is obviously that all altitudes shall be above this datum, so that they will always be positive and never negative heights.

Now, as an illustration, referring to Fig. 249, seeing that by the level-book F is 4·48 ft. below A, we may safely assume our datum to be 20 ft. below A, and to elucidate the operation it is necessary here to explain that the calculated heights above datum are called *reduced levels*, and appear in another column next to the "fall" column. I repeat the level-book to show this.

Back-sight.	Fore-sight.	Rise.	Fall.	Reduced Levels.	Remarks.
4·10				20·00	Below A
	10·15		6·05	13·95	At B
5·30					
	8·19		2·89	11·06	„ C
3·50					
	0·17	3·33		14·39	„ D
7·18					
	0·30	6·88		21·27	„ E
0·40					
	6·15		5·75	15·52	„ F
20·48	24·96	10·21	14·69	20·00	
	20·48		10·21	15·52	
	4·48		4·48	4·48	

Thus if upon a piece of paper a straight line be drawn, and at the points thereon A, B, C, D, E, and F, as in Fig. 250, vertical lines be drawn up, then if the reduced levels be plotted to the value given in the last column, you will have these points relative to a uniform datum of 20 ft. below A. It will be seen that so far as the instrument is concerned, the operation in the field is confined exclusively to the back- and fore-sight columns, whilst the rise and fall and reduced level have reference to the office. I say this advisedly, because this book is for the information of the

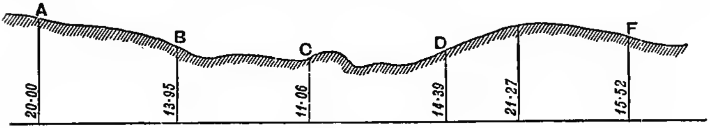


Fig. 250.

uninitiated, and I want to make it clear that, having accurately taken the readings of the staff at the back- and fore-sight stations, the identity of the instrument now ceases from the work. This is the real answer to the question so often put as to whether any notice is to be taken of the height of the instrument from the ground.

Bench-marks.—I leave this portion of the question for the present, to consider the next important process, viz. that of bench-marks. It has been laid down as an invariable rule, that to secure a perfect system of levelling, some clearly defined and immovable point shall be established to serve as the basis of all operations. In other words, whether it is the top of a mile-stone, a corner of the top step of some well-known building, a boundary stone, the hinge-post of a gate, the trunk of a tree, or a mark cut on a wall, that such should represent the commencement of a series of levels, which shall be so accurately described and located as to enable the greatest stranger to easily determine its whereabouts.

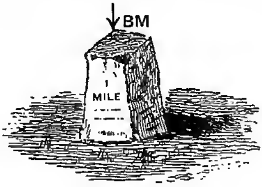


Fig. 251.

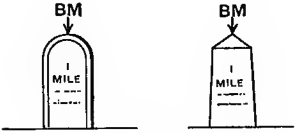


Fig. 252.

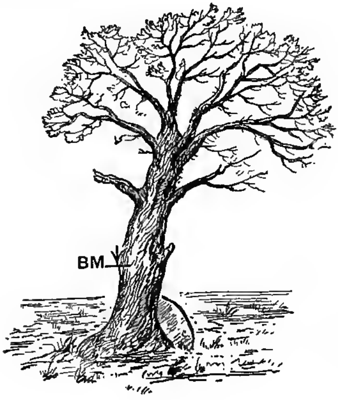


Fig. 253.

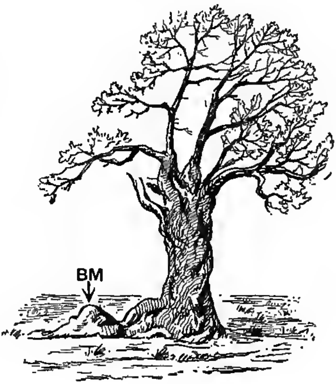


Fig. 254.

In selecting a bench-mark, if on a mile-stone or a gate-post, the highest point is always to be taken; or, in the case of a stone post, whose top may be uneven, by intention or wear, then select the extreme point, as shown in Fig. 251; and in the case of iron or round stone posts the apex, as in Fig. 252. Let me say one word regarding the habit of driving nails into the trunks of trees (Fig. 253). It is by no means a satisfactory one, and should be avoided except under most exceptional circumstances. It may be necessary to utilise a tree in close proximity to the work, in which case it is always advisable to cut a cross or crow's foot on the root, as in Fig. 254. Again, it is usual to advise students to make bench-marks of gate-posts, the favourite expression being the "top hook of the hanging post," as in Fig. 255. I can only say that this is a mistake, as the constant opening and shutting of the gate must loosen the hook and destroy the identity of the mark. The hanging post of gates, in the absence of any more suitable fixtures, may do very well, but instead of being the hook, as in Fig. 255, it should be on the top of the post itself, as in Fig. 256. The door-steps of churches, chapels, public-houses, farmhouses, &c., are frequently adopted for bench-marks, in which case it is always usual to take the top step (Fig. 257), and to be extremely careful to describe whether it is north, south, east, or west. Ordnance bench-marks are invariably cut in the walls of buildings, public or private, or in stone or wooden mile- or gate-posts, and are in the form of a crow's-foot, similar to Fig. 258.

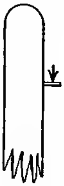


Fig. 255.

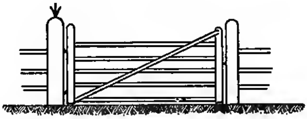


Fig. 256.

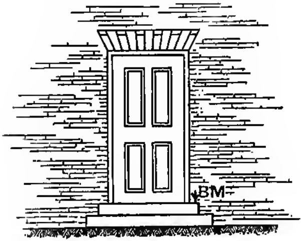


Fig. 257.

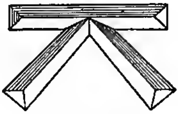


Fig. 258.

Position of Bench-marks.—Bench-marks need not necessarily be exactly on the line of section, nor is it essential they should be at the commencement of the work. In starting to take levels the staff is held upon some convenient permanent mark, such as I have mentioned, as near to the work as possible. I have known cases where the only fixed point suitable for a bench-mark has been a considerable distance away, in which case it has been necessary to level expressly from this point to that of the commencement of the section, even if it be a mile off or more. Upon the Sligo, Leitrim, and Northern Counties Railway we had only two bench-marks in 42½ miles' length, and each was some considerable distance from the commencement and termination of the railway, and was on the top of iron mile-posts.

My advice is always to have frequent bench-marks, say one at every furlong, as they are invaluable at the time the section is taken or in after times for reference. If the operation of levelling takes longer than the one day, when leaving off always do so upon a bench-mark, from which you may safely resume your levelling at a subsequent date. In entering the position of a bench-mark in the level-book it needs to be described very minutely, somewhat thus: "B M on top of doorstep, N E corner of Coach and Horses P H" or "B M on top of sixth mile-post from Dover;" or "B M on top of hanging post of gate leading from main road to Cedar Farm."

Different Kinds of Levelling.—Levelling may be done in several ways: 1st, by taking observations of altitude at measured points upon a given line, which is called a section; 2nd, by taking observations of altitude at points along a road; 3rd, relative levels at points of an estate, whose positions are fixed, upon plan, and whose relative values to the datum are marked thereon.

First, as to a line of section. It is usual to set out a line either straight or curved, which shall comprehend a line of country of which it is necessary to determine the various features of undulation, commenced at a fixed point, as A. After having held the staff upon a bench-mark at A, it is removed to the point which is the commencement of the section.

Level-book.—Before going into details, however, it is necessary that I should say a few words as to the level-book and the method of taking observations. The following is in my judgment the form of level-book best adapted to modern practice. It consists of seven columns on the left page and one column and a large space on the right page. The first three columns, viz. "back-sight," "intermediate," and "fore-sight," are exclusively for the observations with the instrument; and these, together with the seventh column on the left page and the whole of the right for "distance," "total distance," and "remarks," have reference only to field operations, whilst the fourth, fifth, and sixth columns, for "rise," "fall," and "reduced levels," need not necessarily be worked out in the field, but it is always as well to do so if time and circumstances permit.

LEVELS TAKEN IN WIMBLEDON PARK, JUNE 30TH, 1886.

Back Sight.	Inter-mediate.	Fore Sight.	Rise.	Fall.	Reduced Levels.	Distance.	Total Distance.	Remarks.
6·30					50·00*			B.M on root of tree at A on plan.
	1·60		4·70		54·70			On peg at end of line 5.
	1·45		0·15		54·85			On peg No. 2.
	0·55		0·90		55·75			Centre of road.
		0·59		0·04	55·71			Peg No. 3.
9·80					—			
	2·20		7·60		63·31	000		Commencement of section.
	4·30			2·10	61·21	100		At peg.
	5·90			1·60	59·61	150		,,
	8·30			2·40	57·21	180		,,
	10·00			1·70	55·51	200		,,
	7·30		2·70		58·21	300		,,
	4·90		2·40		60·61	400		,,
	3·50		1·40		62·01	500		,,
	0·10		3·40		65·41	600		,,
	10·50			10·40	55·01	700		,,
		12·53		2·03	52·98	—		,,
5·02					—	—		,,
	4·70		0·32		53·30	800		,,
	8·60			3·90	49·40	900		,,
	11·80			3·20	46.20	1000		,,
	13·50			1·70	44·50	1100		,,
		5·02	8·48		52·98	—		,,
7·30					—	—		,,
	7·40			0·10	52·88	1200		End of section.
		10·27		2·87	50·01*			B.M. on tree.
28·42		28·41	32·05	32·04				

Now referring to the level-book just described, the instrument is planted in some convenient position to command the bench-mark on the root of a tree, marked A on plan. Direct the staff to be held thereon, and direct the telescope towards it. Carefully observe the reading where the cross-wire cuts the staff—in this case it is 6·30. This is a back-sight. And here let me again impress upon the student that the *first sight* he takes after fixing the instrument is always a *back-sight*, and *the last* he takes before he removes the instrument is always a *fore-sight*, and all other sights are intermediate. Again, a back-sight signifies the commencement of a series of levels and fore-sight its termination. Now 6·30 is the first reading, therefore book it in the first column, and having entered it take another look to satisfy yourself that the reading is correct.* Now there are three points at which it is desirable to have readings before moving the instrument—1·60, 1·45, and 0·55. These being connected with the same line of collimation will appear in the second or "intermediate" column, and for convenience of sight it is arranged that the chain-man should hold the staff at a point the reading of which is 0·59, which, being the last, will appear in the third or "fore-sight" column, and we have now done with this line of collimation, and must proceed to establish another. But the staff must remain at the last point, only be careful, in turning the figures towards the new position of the level, that it is exactly upon the same spot.† To better illustrate my meaning by reference to Fig. 259, the instrument is at A (for the first line of collimation), and B is the point deemed desirable for a change of collimation, the staff being held on some fixed point at B and the sight taken at *a*, the reading of which is 0·59. The second line of collimation is established by planting the level at C, and reading the staff still held at B, but cutting it at *b*, which reads 9·80. Now the 0·59 goes in the third column and 9·80 in the first, but whilst the readings are different the point B is just the same, the staff never having moved (except to turn its face towards C). The difference lies in the alteration of the lines of collimation, and it is most important to impress this fact, that the accuracy of the levels is entirely dependent upon the care with

---

* See p. 211.

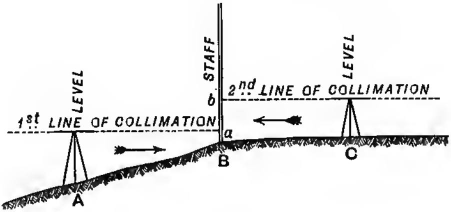


Fig. 259.

---

* To carefully observe a reading and make a mental note thereof enables the leveller to accurately record it in the book; and looking again, after having booked it, will prove a corroboration of the observation.

† I always prefer, in cases of change, before establishing my fore-sight to select a stone, peg, or root of tree, in fact anything firm upon which the staff may be held. If in pasture land, instruct your man to carry a stone, and to well kick it into the ground before placing the staff upon it.

which the changes of collimation are made, so that if there is the slightest alteration in the point at P, where the sights *a* and *b* are observed—in other words, if the staff in the process of turning has shifted only ever so slightly—the accuracy of the work is jeopardised, nay, destroyed. Let me further emphasise this. According to the reading of the staff at B the value of *a* is 0·59 when the staff is held on a stone (as *a*, Fig. 260). Now if the chain-man is not careful when he raises the staff to turn it towards the instrument, although he may place it back on the same stone, yet if from want of care instead of doing so at *a* he puts it upon a lower part of the stone, as *b*, then the difference of the lines of collimation will be ½ in. out, and the identity of *a* and *b* at B, in Fig. 259, destroyed, for by this error of ½ in. they are not taken on the self-same spot.

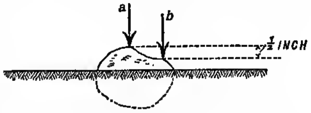


Fig. 260.

Foot-plates.—To obviate such an unfortunate contingency it is very desirable that the chain-man should carry slung on his arm an iron foot-plate, such as Fig. 261, or, for soft ground, a foot-peg, as Fig. 262.

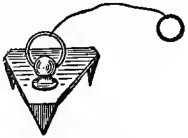


Fig. 261.

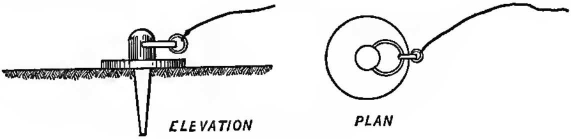


Fig. 262.

I think I have sufficiently explained the importance of these precautions, and now proceed with the second line of collimation, with 9·80 as the back-sight. By reference to the level-book it will be seen that the real commencement of the section is not until the first intermediate in the second line of collimation, viz. 2·20, and it is here that the seventh column is brought into use, and three cyphers are booked to notify the zero of the horizontal measurement. At 1 chain occurs the second intermediate 4·30, and following at 150, 180, 200, 300, 400, 500, 600, and 700 links are eight intermediate sights, 5·90, 8·30, 10·00, 7·30, 4·90, 3·50, 0·10, and 10·50: and for the convenience of shifting the instrument we now make a fore-sight (12·53) on a peg put in for that purpose only, having no chainage because not intended to be plotted. This ends the second line of collimation. The third line of collimation begins with a back-sight of 5·02, has four intermediates, 4·70, 8·60, 11·80, 13·50, at 800, 900, 1000, and 1100 links, and is terminated by a fore-sight (5·02) also on a peg not to be plotted. The third line of collimation begins with a back-sight (7·30) on that peg, has an intermediate (7·40) at 1200 links of chainage, and terminates with a fore-sight (10·27) on the bench-mark from which we started: this also forms no part of the plotted section, and therefore has no chainage. I have given this illustration, taken from actual practice over a portion of a section of a railway, which by being for the first 1200 links round a very sharp curve, gave the section the form in which it appears in Fig. 263, and also enabled us to tie upon our original bench-mark.

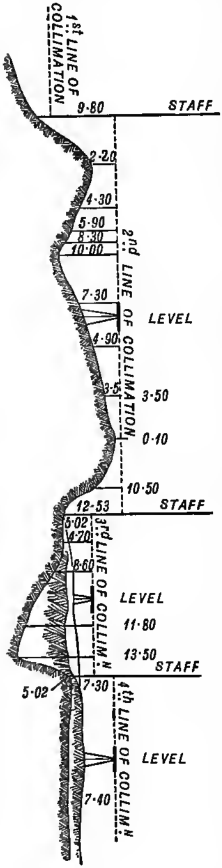


Fig. 263.

Keeping the Level-book.—I now wish to speak of the method of keeping the level-book, and shall take p. 207 for illustration. On p. 200 I have explained that if the "intermediate or fore-sights are greater than the back-sights they are falls, and if less rises," and thus in the present case we shall have no difficulty in making up our book as follows. Working diagonally downwards from left to right, 1·60 being less than 6·30, is a rise of 4·70; 1·45 being less than 1·60 is also a rise; 0·55 being less than 1·45 is a rise; but 0·59 being greater than 0·55 is a fall of 0·04. We have now done with the first series; and although the fore-sight 0·59 and the back-sight 9·80 are taken on the same point, I prefer to start a fresh line, as a better illustration that each series is independent of the other. Thus 9·80 back-sight being greater than 2·20 (intermediate) is a rise, but 2·20 being less than 4·30 shows a fall of 2·10, and 4·30 less than 5·90 a fall of 1·60, and so on until 10·00 being greater than 7·30 we have a rise of 2·70, 2·40, 1·40, 3·40, a fall of 10·40, and finally the fore-sight 12·53 being greater than the last intermediate (10·50) shows a fall of 2·03. Now a new line of collimation, with a back-sight of 5·02, we have a rise of 0·32, the three intermediates showing falls of 3·90, 3·20, 1·70 respectively, whilst the fore-sight gives a rise of 8·48, and the fourth and last line of collimation has a fall of 0·10 from the back-sight, and also on to the B.M. a fall of 2·87.

Making up Level-book.—It is here necessary to explain how to make up the level-book. We have seen that, commencing with a back-sight of 6·30 on the bench-mark, we terminate upon the same point with a fore-sight of 10·27, and that we have four back-sights of 6·30, 9·80, 5·02, and 7·30, giving a total of 28·42 ft., and also four fore-sights of 0·59, 12·53, 5·02, and 10·27, in all 28·41 ft. Thus the back-sight being greater by 0·01 than the fore-sight shows a discrepancy of $\frac{1}{100}$th of a foot, or $\frac{1}{8}$th of an inch. In so short a distance this should not occur, as 1 in. in four miles is considered the allowance for errors. I have purposely shown it thus to illustrate my meaning. Now if we have correctly reduced the intermediate and fore-sight from the back-sight, the rises and falls if added together should give the same difference as that existing between the back- and fore-sights, or 32·05 rise − 32·04 fall = 0·01. Now on p. 202 I have spoken about datum, and in the present case I have assumed a datum of 50 ft. below the bench-mark. This 50 ft. appears in the sixth column, opposite the 6·30 in the first, and it will be necessary to carry forward the system of reduced levels by adding or deducting the consecutive rises or falls as follows: 50·00 + 4·70 = 54·70, 54·70 + 0·15 = 54·85, 54·85 + 90 = 55·75, 55·75 − 0·04 = 55·71. This last being a fall must be deducted. There is no reduced level opposite 9·80 in the back-sight column, as being identical with 0·59 in the fore-sight column; its value is just the same* above datum of 55·71, and to save confusion I simply draw a dash across the space. Then to 55·71 must be added 7·60 = 63·31; from 63·31 − 2·10 = 61·21; from 61·21 − 1·60 = 59·61; 59·61 − 2·40 = 57·21; 57·21 − 1·70 = 55·51; 55·51 + 2·70 = 58·21, and so on until the last fall of 2·87, opposite the last fore-sight 10·27, gives a result of 50·01, from which should be taken the height above datum, viz. 50·00 = 0·01, or $\frac{1}{100}$th of a foot. Having thus obtained all our reduced levels, we now proceed to plot our section, of which I shall have something to say later on. But I want to explain how to avoid any complications or inaccuracies with the level-book in cases where there are a large number of intermediate sights, so much so as to go over on to the next page. The following is a very simple illustration.

---

* Some surveyors prefer to place their back- and fore-sights upon the same line, as in example A; but I prefer to devote a separate line to each observation, as in example B, which shows more clearly the various lines of collimation. The back-sight 9·80 is placed one lower than fore-sight 0·59, being taken on the same spot, for the new collimation-line.

A

B. S.	Inter.	F. S.
9·80		0·59

B

B. S.	Inter.	F. S.
		0·59
9·80		

TWO METHODS OF KEEPING THE LEVEL-BOOK.

(USUAL METHOD.)

PAGE NO. 1.			PAGE NO. 2.		
Back-sight.	Inter-mediate.	Fore-sight.	Back-sight.	Inter-mediate.	Fore-sight.
			16·10		0·59
6·30				3·50	
	1·60			0·10	
	1·45			10·50	
	0·55		5·02		12·53
9·80		0·59		4·70	
	2·20			8·60	
	4·30			11·80	
	5·90			13·50	
	8·30		7·30		5·02
	10·00			7·40	
	7·30				10·27
	4·90				
16·10		0·59	28·42		28·41

(AUTHOR'S METHOD.)*

PAGE NO. 1.			PAGE NO. 2.		
Back-sight.	Inter-mediate.	Fore-sight.	Back-sight.	Inter-mediate.	Fore-sight.
			16·10		5·49
6·30			4·90		
	1·60			3·50	
	1·45			0·10	
	0·55	0·59		10·50	
9·80					12·53
	2·20		5·02		
	4·30			4·70	
	5·90			8·60	
	8·30			11·80	
	10·00			13·50	
	7·30				5·02
		4·90	7·30		
				7·40	
					10·27
16·10		5·49	33·32		33·31

---

* See Note, p. 211.

In the first case at the bottom of Example No. 1 is an intermediate 4·90, and at top of page No. 2 is also an intermediate immediately following, of 3·50, so that the fore-sight 12·53 does not occur until the fifth line. Now it is absolutely necessary to make each page of the level-book balance, so that the difference between the back- and fore-sights, rise and fall, and reduced levels, should correspond. But page No. 1 (Example 1) will not enable you to do this, for you have only one fore-sight, 0·59, as compared with two back-sights, or 16·10, or a rise of 15·51; and if you deduct the falls from the rises down to 4·90, you will find 18·45 – 7·84 = 10·61, and this is not only confusing (although not absolutely wrong), but may lead to serious errors; whereas if, as in the Example 2, when you get to the bottom of the page, you make 4·90 a temporary fore-sight—taking care that what you have borrowed on page 1 you repay on page 2, so that 4·90 appears there as a back-sight—by this means each page can be made to balance, and the facility in making up the book is immense. It may be asked how is it that the total back-sights in case No. 1 are 28·42 and the fore-sights 28·41, whilst in case No. 2 they are 33·32 and 33·31 respectively? The reason undoubtedly is because we have added one more back- and fore-sight: thus 28·42 + 4·90 = 33·32, and 28·41 + 4·90 = 33·31; but the difference between the back- and fore-sight is exactly the same in both cases, or $\frac{1}{100}$th of a foot. It may from the foregoing example seem hardly a matter of much importance, but I can assure my readers that a whole level-book will involve a very much greater amount of trouble in making up by the old way, and errors in casting up will creep in unawares which could not possibly occur if each page is made to properly balance. Let me here say again that on no account must figures be rubbed out in the back-, intermediate, fore-sight, or distance columns, as any alteration can be made by drawing the pencil through the figures that are wrong, and re-written.

Collimation Method.—A method of keeping the level-book without "rise" and "fall" columns is termed sometimes the "height of instrument method" and sometimes the "collimation method." The principle is, that all sights taken at the first "set" of the instrument are referred to the height of its collimation above the first starting-point; and those taken at each successive "set," to the height of such new collimation above the spot on which the fore-sight of the previous "set" was taken, the new collimation-height being determined by adding to the reduced level of that fore-sight the reading of the back-sight taken after the shift of the instrument. From each collimation-height, the intermediates and the fore-sight taken at that "set," are deducted, the remainders being the reduced levels of the several points. As the intermediates in some "sets" are numerous, each successive collimation-height is entered (on the same line as the new back-sight) in a column so headed, without which it would have to be either noted on a slip of paper or carried in the memory. The subjoined version of the level-book (see page 207), when kept according to the collimation method, exemplifies the mode of procedure. It will be observed that the collimation-height, unlike the reduced levels, is not entered at each sight, but given only at the commencement of the "set" to which it relates, thus effecting a saving of trouble and of possible confusion with the reduced levels of the several points.

COLLIMATION METHOD LEVEL-BOOK.

Back-sight.	Inter-mediate.	Fore-sight.	Collimation Height.	Reduced Level.	Distance.	Remarks.
				50·00		B.M. on root of tree at A on plan.
6·30			56·30	—		
	1·60			54·70		On peg at end of line 5.
	1·45			54·85		On peg No. 2.
	0·55			55·75		Centre of road.
		0·59		55·71		Peg No. 3.
9·80			65·51	—		
	2·20			63·31	000	Commencement of section.
	4·30			61·21	100	At peg.
	5·90			59·61	150	,,
	8·30			57·21	180	,,
	10·00			55·51	200	,,
	7·30			58·21	300	,,
	4·90			60·61	400	,,
	3·50			62·01	500	,,
	0·10			65·41	600	,,
	10·50			55·01	700	,,
		12·53		52·98	—	,, (not for plotting).
5·02			58·00	—	—	,,
	4·70			53·30	800	,,
	8·60			49·40	900	,,
	11·80			46·20	1000	,,
	13·50			44·50	1100	,,
		5·02		52·98	—	,, (not for plotting).
7·30			60·28	—	—	,,
	7·40			52·88	1200	End of section.
		10·27		50·01		B.M. on tree.
28·42		28·41				

[NOTE.—The discrepancy of 0·01, to which attention is drawn on p. 211, is here purposely retained.]

The datum being in this example fixed at 50 ft. below bench-mark A, that dimension is entered as the reduced level of the bench-mark; and back-sight 6·30 is added thereto and entered as collimation-height for the sights of the first "set." From this, the intermediates are one by one deducted, and finally the fore-sight 0·59 at peg No. 3 leaves the reduced level of that peg 55·71. The instrument is then shifted, and the back-sight 9·80 (on peg No. 3) is added to 55·71 the final reduced level of the previous "set," giving a new collimation-height of 65·51. As a back-sight has no reduced level, a line is there drawn across the reduced level and the distance columns. The intermediates and the fore-sight throughout the second "set" are then one by one deducted from the new collimation height 65·51; and the like procedure is followed at each shift, and so on, to the end of the section. It will be noticed that each sight—back, intermediate, or fore—is entered on a separate line, and that a new collimation-height always stands on the same line as a new back-sight: adherence to this practice will be found conducive to clearness in the entries.

The collimation-method is by many surveyors considered a great improvement on the old "rise" and "fall" system; and, by keeping the "distance" and "remarks" columns on the right-hand page, the level-book can be reduced to a width of 3⅝ inches (a considerable gain in handiness) without cramping the space available for the several entries, while at the same time all risk of confusing the chainage figures with those of the staff-readings is avoided.

It is further claimed as a merit of the method, that the surveyor can reduce his levels as he proceeds, and thus save time in office-work. The actual gain on this score, however, is at best only slight, and seems to be more than counterbalanced by the inevitable risk of error attending such work in the field, whatever be the form in which the level-book is kept. On this point, the opinion of an old and deservedly esteemed authority merits careful remembrance: "Some surveyors reduce their levels in the field, but it is not a commendable practice; there is plenty to occupy a man's attention without that." *

Levelling-staff.—I proceed now to speak of the levelling-staff (fully described in Chapter III.) and its manipulation. It has been explained that the most approved staves are those upon the telescopic principle in three pieces—5 ft., 4 ft. 6 in., and 4 ft. 6 in. for the 14-ft.; and 6 ft., 5 ft. 6 in., and 4 ft. 6 in. for the 16-ft. staff. At the top of the two lower members there is a spring-clip (Fig. 264) which presses its way into the oval hole, so as to keep it from slipping up or down. Some makers have a spring-clip which, upon the member being drawn out, closes accurately over the top of the lower portion, as in Fig. 265, whilst some surveyors prefer to have the members of the staff secured by means of a thumbscrew. It is very necessary that the chain-man be carefully drilled in the use of the staff before commencing operations. He should see that each length of the staff is drawn out to its proper length and the spring-clips are secure, and in carrying it he should be careful not to injure it by allowing it to strike the boughs of trees or buildings. In open country he may carry it with the lowest portion over his shoulder, but in woods, orchards, etc., he had better carry it trailing with the top joint in front of him; in crossing a ditch or brook he should either get some one to hold it whilst he gets over, or should lay it gently across with the bottom in the direction in which he is going, and upon no account use it as a jumping-pole. The staff must always be perfectly plumb, and the chain-man must hold it by standing behind it with his fingers on the top of the first joint, as in Fig. 266. When once upon a back- or fore-sight he must never move until so instructed by the surveyor, and to avoid any chance of an error in booking by reason of the staff not being exactly perpendicular, it is as well to gently wave the top backwards and forwards from *a* to *b*, as in Fig. 267. It is quite enough for an intelligent man to look after holding the staff and to obey the instructions of the leveller, and between them there should be a code of intelligible signals,

---

* Bourns, "Principles and Practice of Surveying," p. 220.

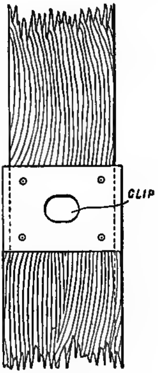


Fig. 264.

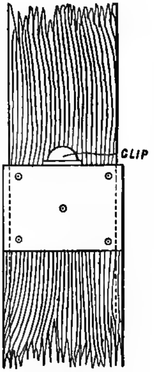
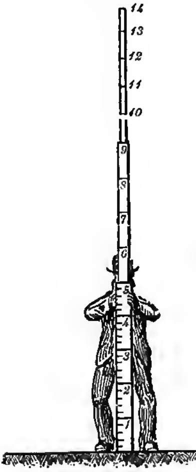
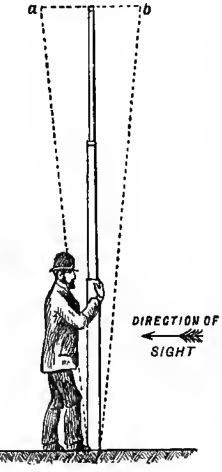


Fig. 265.



FRONT VIEW

Fig. 266.



SIDE VIEW

Fig. 267.

which at a distance or in windy weather will be extremely useful.

As to Distances.—After holding the staff upon the bench-mark and at such other points *en route* as may be necessary, at the commencement of the chainage or section, the staff-holder holds it on the surface of the ground, and when directed moves along the chain as required, or to the end. He should be well acquainted with reading the divisions on the chain, as it is sometimes necessary to book the distances from a point away, in which case you must trust to your chain-man, but *not* if you can possibly avoid it. Now across open ground there is little need for taking sights oftener than at the end of each chain, unless the ground be very undulating. In crossing a bank similar to that in Fig. 268, it is necessary to take the tops and bottoms—thus, 1140*a*, 1154*b*, the near bottom and top of slope, 1184*c* the foretop; whilst 1200 comes part of the way down the slope, the bottom of which *d* is 1215, but in a case of this kind it is not absolutely necessary to take a level at 1200, being so near to 1215. In the case of Fig. 269, in crossing a ditch and fence levels are required at *e*, *f*, *g*, and *h*, but distances must be taken at those points such as 1310, 1312, 1317, and 1319, and it is as well to make a sketch in the level-book similar to Fig. 269. In crossing a river, whose width admits of both banks

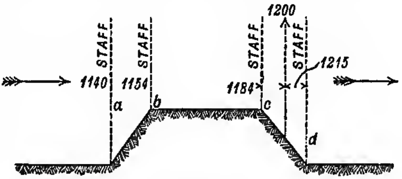


Fig. 268.

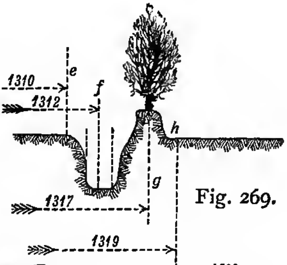


Fig. 269.

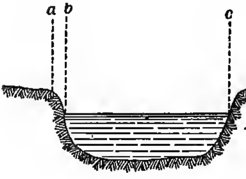


Fig. 270.

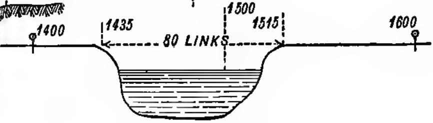


Fig. 271.

being observed from the same station, it is usual to take the edge of each bank and the impingement of the water on the shore, as *a*, *b*, *c*, *d* in Fig. 270; and if sufficiently shallow to allow the staff to be read with the bottom upon the bed, so much the better; if not, the depth of the surface of the water above the bed must be ascertained by sounding either with the levelling staff, or, if not long enough, with a line and lead.

Measuring across Streams.—If the river be too wide to measure with a chain, resort will have to be had to one or other of the methods of calculating the width described in Chapter IV. It sometimes happens, as in Fig. 271, that the end of a chain comes near to the edge of a river, whose width is too great to admit of a chain-peg being on the other side; in such a case it is unnecessary to resort to calculation, if the exact width is taken with the chain. Supposing it to be not wider than 100 links, by care it is possible to connect and to continue the chainings. In this case the near edge of the river is 1435, and the width to the opposite edge is 80 links, thus 1435 + 80 = 1515, and if 15 links is held at that point, then the end of the chain will be 200 links from the last arrow at 1400. In the case of a wide river, of say 3 or 4 chains' width, it is desirable to establish a bench-mark and send a man across with a staff and instruct him to hold the staff upon a bench-mark on the other side, then take a long-distance sight across and allow for curvature and refraction. This only as a test of the subsequent operation of levelling round by possibly a circuitous route as shown in Fig. 272, when it may be necessary to sight for upwards of 1½ mile round by a bridge; or across some convenient ford, in which case, having levelled from A to B, it will be absolutely imperative to check back from B to A before continuing the section. In taking the level of water of a tidal river it is necessary to ascertain the level of high and low water.

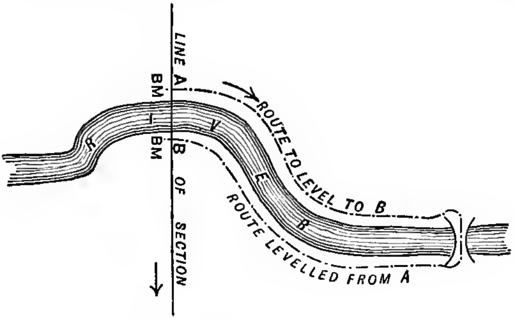


Fig. 272.

Providing for Curvature, &c.—It will have been noticed that, in speaking about curvature and refraction, I said it was seldom considered in modern practice, as by equalising the distance between the staff at each end and the instrument, the necessity for making the allowance would be obviated. If only back- and fore-sights are required it will not be difficult to arrange for the equidistance of the staff, but it does not necessarily follow that the instrument must be exactly in line with the staves. Always select some eligible position upon which to plant your level, so as to command as large a range of your work as possible consistent with the necessity to have the back- and fore-sights equidistant.

In Fig. 273 I give a simple illustration, which really deals with the whole question, however complicated. In the line of section from A to D it is assumed that we commence at A

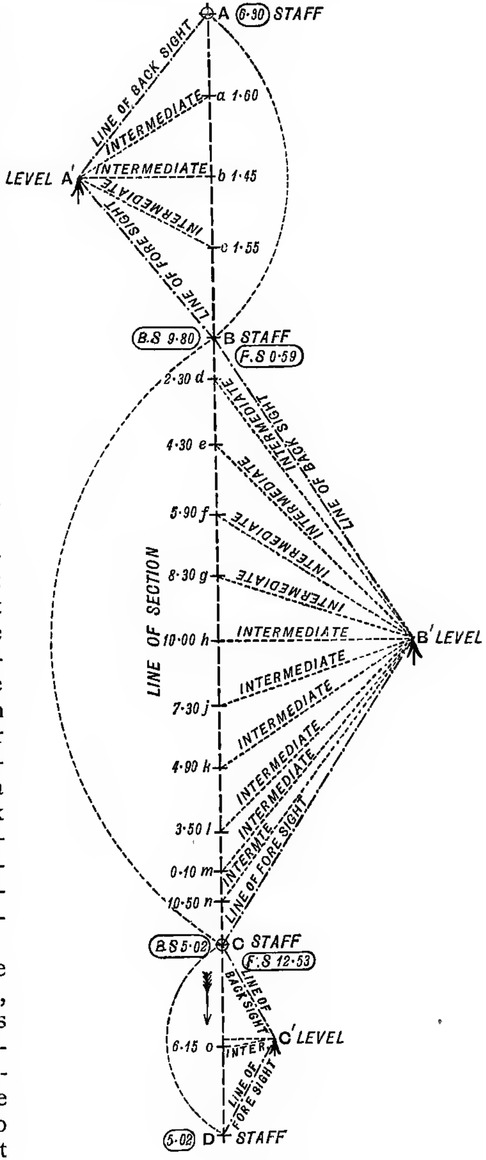


Fig. 273.

with the staff reading 6·30, the instrument being at A′: the staff is then held at *a* (1·60), at *b* (1·45), and *c* (1·55), all three intermediates, and finally at B for a fore-sight, the same distance (or thereabouts) as from A. Now by keeping A and B the same distance from A′ we have fulfilled the condition required by curvature and refraction, and if the instrument is in perfect adjustment the depths of the intermediates *a*, *b*, *c*, below the line of collimation A B, although of different radii to A and B, yet for all practical purposes will be sufficiently accurate. This will be possibly better understood by reference to Fig. 274. Here let me say that, whilst it is absolutely essential that the back- and fore-sights should be most accurately observed, because the difference of their sum will be the actual rise or fall from the commencement to the termination, yet for all practical purposes it is not necessary (except in the case of the level of water, existing railways, or road crossings) to read intermediates nearer than tenths. Thus 1·43 would be booked 1·40, and 1·47 would appear as 1·50. By so doing a great deal of unnecessary labour and complication in making up the book is avoided, and seeing that with even the largest scale in practice it is impossible to plot less than $\frac{10}{100}$th of a foot, it is a needless waste of time to observe so minutely in the field.

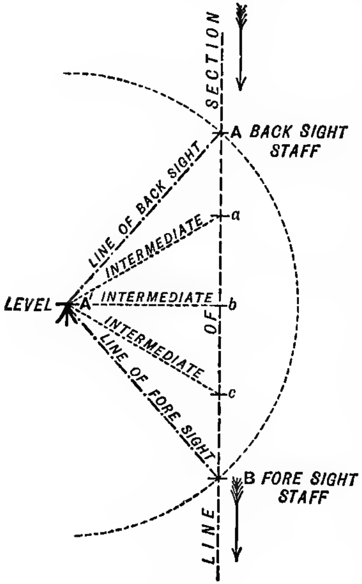


Fig. 274.

Passing back to consideration of Fig. 273: Having observed the fore-sight at B (0·59), and previously taken care that the staff is held upon some firm place, the face thereof being now turned towards B′, to which point the instrument has been transplanted, and duly adjusted, the reading of the back-sight at B is 9·80; and now follow the various points along the line, *d* (2·30), *e* (4·30), *f* (5·90), *g* (8·30), *h* (10·00), *j* (7·30), *k* (4·90), *l* (3·50), *m* (0·10), and *n* (10·50), all intermediates, whilst C (12·53) is the fore-sight. The same principle as previously explained equally applies, and so on *ad infinitum*, showing at the finish of the section—

Back-sight.	Fore-sight.
6·30	0·59
9·80	12·53
5·02	5·02
21·12	18·14
18·14	

2·98 rise from A to D.

This is only a very simple illustration, but it may be adopted either for a great length of section or for a few chains.

Instructions to Staff-holder.—It is desirable that the surveyor should direct the staff-holder as to the points at which it is necessary to take readings, especially for back- and fore-sights, and unless he has some trustworthy person to read the distances on the chain-line he should ascertain the longitudinal measurements himself; certainly he must personally superintend the establishment of bench-marks, and see that the staff is not only held on the highest point, but that it is the same place which is described in the "remarks" column.

Plenty of Information.—Another point is that the remarks should be in as much detail as possible, accompanied by neat and graphic sketches of any important features met with in the section, especially with regard to the bench-marks. A sight should certainly be taken at the end of every chain except under exceptional circumstances. It may be well here to explain, that it is not by any means necessary that there should be any longitudinal measurements at either a back- or a fore-sight, but if it be found convenient to change at a point on the line of section which is to be determined by measurement, then the distance will appear opposite the fore-sight and opposite the next back-sight (which represents the same spot). There will be no distance, but for facility in after work a dash should be drawn across the column. Thus, referring to Fig. 273: if with the level at A′ the surveyor had intended to take an intermediate at B, but found that the rise of the ground would hardly justify his continuing further; instead of entering 0·59 as an intermediate he would book it as a fore-sight, and put the distance upon the chain-line opposite, as in Fig. 275; and having moved the level to B′, in sighting the staff held at the same place (viz. B) would read and enter in the first column the back-sight 9·80, so that at 1·60 the distance was 1 chain (100 links), at 1·45 = 200, at 0·55 = 300, at F S 0·59 = 400, at B S 9·80 = 400, at 2·30 = 430 links, and so on. I should explain that in Fig. 273 the back- and fore-sights A, B, C, and D are, for particular illustration of a system, shown upon the line of section, but, if they are not needed as part of the section when plotted, and are kept equidistant from the instrument, they may be at any point right or left of the line. Then again, I have been frequently asked if the first back-sight is the commencement of the section? I say, no. The first back-sight must necessarily be upon a bench-mark, in as near proximity to the commencement of the section as possible; but as a general rule the zero of the chainage is an intermediate; and the same applies to the last fore-sight, which may be some distance from the termination of the section, involving a number of back- and fore-sights before the bench-mark is reached. And when this has been done, then the difference between the sum of the back-sights and fore-sights will represent (or should do) the difference between the levels of the first and last bench-mark.

Back-sight.	Intermediate.	Fore-sight.	Distance.
6·30			000
	1·60		100
	1·45		200
	1·55		300
		0·59	400
9·80			—
	2·30		430
	4·30		500

and so on.

Fig. 275.

Again, as the intermediate sights are the depths below the varying lines of collimation (which are regulated by back- and fore-sights alone), then so long as they have been accurately observed they are disregarded in making up the field-work, and are only affected in the rise and fall columns, as connected with the reduced levels. But let it be said that the accuracy of the section so far as its minor details are concerned depends entirely upon the care with which the intermediates are observed, especially in reading long distances, as a IX. may be easily taken for XI., which involves an error at this particular point on the section of two feet, but does not in any way affect the whole section. Patience and care will obviate such an unpardonable error.

Taking the Level of Water.—In taking the level of the surface of water, it is best to so place a stone on the fore-shore that it is only just covered with a film of water, and then hold the staff upon the stone. This applies only to standing water; but for a tidal stream the exact time of the observation should be chronicled, and, from a nautical almanack or by other means, the exact position of high and low water may then be determined.

Levelling with Theodolite.—Except under circumstances which are unavoidable, the use of the theodolite for levelling purposes should be confined to ascertaining inaccessible points or for the heights of mountain sides, for which the ordinary operations are inadmissible. In such cases the procedure is in accordance with the principles illustrated under "Heights and Distances" (pp. 147–153). When, however, a section has to be taken where, owing to steepness or loose shifting surface, a level cannot be properly planted, or where configuration of the ground would necessitate too many shifts of the instrument and sights of inconvenient shortness or objectionable inequality of length: the theodolite will be found useful. The operation presents varieties of detail too numerous to be all worked-out here; but the following example illustrates the mode of procedure, and, with due modification according to circumstances, will enable the student to solve all other cases.

The last point to which levelling by the ordinary means can be carried, is marked by a peg at H (Fig. 276). The theodolite is here planted, and the height of its axis A above the peg is measured. The points on the slope at which sights are to be taken, B, C, D, E, are then selected and marked, and the straight-line distances from point to point, on the slope, A B, B C, C D, D E, are measured: these measurements must be made with the utmost possible accuracy. A vane or mark is fixed on the levelling-staff

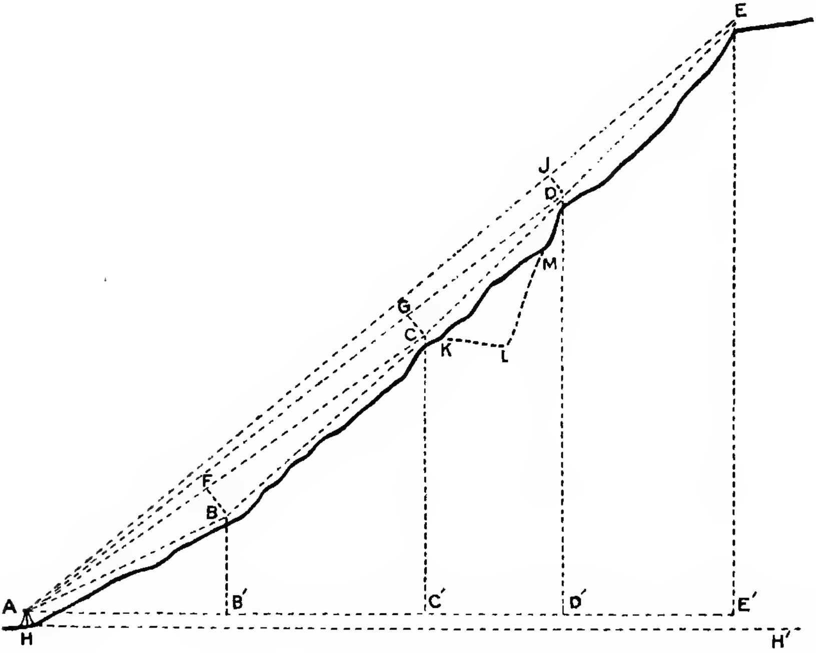


Fig. 276.

at the reading corresponding to the height A H of the axis above starting-peg; and the angles of elevation of this mark are observed at each selected point on the slope.

Let the data thus obtained be:—

Length on ground A B = 143·5; angle of elevation B A B′ = 26°
" " B C = 172·4; " " C A C′ = 34° 45′
" " C D = 129·0; " " D A D′ = 38° 10′
" " D E = 154·8; " " E A E′ = 40°

$$\text{Angle } CAB = 34^\circ\,45' - 26^\circ = 8^\circ\,45'$$
$$\text{,, } DAC = 38^\circ\,10' - 34^\circ\,45' = 3^\circ\,25'$$
$$\text{,, } EAD = 40^\circ - 38^\circ\,10' = 1^\circ\,50'.$$

The observations are then worked out as follows:—

From obtuse angle B let fall perpendicular B F on base A C, dividing triangle A B C into two right-angled triangles A F B, C F B.

From obtuse angle C let fall perpendicular C G on base A D, dividing triangle A C D into two right-angled triangles A G C, D G C.

From obtuse angle D let fall perpendicular D J on base A E, dividing triangle A D E into two right-angled triangles A J D, E J D.

In triangle A B′ B are given

$$\text{angle } B' = 90^\circ$$
$$\text{,, } A = 26^\circ$$
$$\text{side } b' = 143{\cdot}5$$

To find sides b and a.

$$b = \cos 26^\circ \times 143{\cdot}5 = 128{\cdot}9591$$
$$a = \sin 26^\circ \times 143{\cdot}5 = 62{\cdot}9063.$$

In triangle A F B are given

$$\text{angle } F = 90^\circ$$
$$\text{,, } A = 8^\circ\,45'$$
$$\text{,, } B = 90^\circ - 8^\circ\,45' = 81^\circ\,15'$$
$$\text{side } f = 143{\cdot}5$$
$$\text{then } \sin 90^\circ : 143{\cdot}5 :: \sin 81^\circ\,15' : b = 141{\cdot}8299$$
$$\text{and } \sin 90^\circ : 143{\cdot}5 :: \sin 8^\circ\,45' : a = 21{\cdot}8297.$$

In triangle C F B are given

$$\text{angle } F = 90^\circ$$
$$\text{side } f = 172{\cdot}4$$
$$\text{,, } c = 21{\cdot}8297$$
$$\text{then } 172{\cdot}4 : \sin 90^\circ :: 21{\cdot}8297 : \sin C = 9{\cdot}1025106$$
$$= \sin 7^\circ\,16'\,28''$$
$$\text{and } 90^\circ - 7^\circ\,16'\,28'' = 82^\circ\,43'\,32'' = B$$
$$\text{whence } \sin 90^\circ : 172{\cdot}4 :: \sin 82^\circ\,43'\,32'' : b = 171{\cdot}0124$$
$$\text{and } AC = AF + FC = 141{\cdot}8299 + 171{\cdot}0124 = 312{\cdot}8423.$$

In triangle A C′ C are given

angle $C' = 90^\circ$
„ $A = 34^\circ 45'$
side $c' = 312{\cdot}8423$
then $c = \cos 34^\circ 45' \times 312{\cdot}8423 = 257{\cdot}0458$
and $a = \sin 34^\circ 45' \times 312{\cdot}8423 = 178{\cdot}3190.$

In triangle A G C are given

angle $G = 90^\circ$
„ $A = 3^\circ 25'$
„ $C = 90^\circ - 3^\circ 25' = 86^\circ 35'$
side $g = 312{\cdot}8423$
then $\sin 90^\circ : 312{\cdot}8423 :: \sin 86^\circ 35' : c = 312{\cdot}2861$
and $\sin 90^\circ : 312{\cdot}8423 :: \sin 3^\circ 35' : a = 18{\cdot}6448.$

In triangle D G C are given

angle $G = 90^\circ$
side $g = 129{\cdot}0$
„ $d = 18{\cdot}6448$
then $129{\cdot}0 : \sin 90^\circ :: 18{\cdot}6448 : \sin D = 9{\cdot}1599581$
$= \sin 8^\circ 18' 36''$
and $90^\circ - 8^\circ 18' 36' = 81^\circ 41' 24'' = C$
whence $\sin 90^\circ : 129{\cdot}0 :: \sin 81^\circ 41' 24'' : c = 127{\cdot}6434$
and $AD = AG + GD = 312{\cdot}2861 + 127{\cdot}6434 = 439{\cdot}9294.$

In triangle A D′ D are given

angle $D' = 90^\circ$
„ $A = 38^\circ 10'$
side $d' = 439{\cdot}9294$
then $d = \cos 38^\circ 10' \times 439{\cdot}9294 = 345{\cdot}8898$
and $a = \sin 38^\circ 10' \times 439{\cdot}9294 = 271{\cdot}8549.$

In triangle A J D are given

angle $J = 90^\circ$
„ $A = 1^\circ 50'$
„ $D = 90^\circ - 1^\circ 50' = 88^\circ 10'$
side $d = 439{\cdot}9294$
then $\sin 90^\circ : 439{\cdot}9294 :: \sin 88^\circ 10' : d = 439{\cdot}7041$
and $\sin 90^\circ : 439{\cdot}9294 :: \sin 1^\circ 50' : a = 14{\cdot}0671.$

In triangle E J D are given

angle $J = 90^\circ$
side $j = 154{\cdot}8$
„ $d = 14{\cdot}0671$
then $154{\cdot}8 : \sin 90^\circ :: 14{\cdot}0671 : \sin E = 8{\cdot}9584343$
$= \sin 5^\circ 12' 50''$
and $90^\circ - 5^\circ 12' 50'' = 84^\circ 47' 10'' = D$
whence $\sin 90^\circ : 154{\cdot}8 :: \sin 84^\circ 47' 10'' : d = 154{\cdot}1595$
and $AE = AJ + JE = 439{\cdot}7041 + 154{\cdot}1595 = 593{\cdot}8636.$

In triangle A E′ E are given

$$\begin{aligned} \text{angle } E' &= 90^\circ \\ \text{,, } A &= 40^\circ \\ \text{side } e' &= 593{\cdot}8636 \\ \text{then } e = \cos 40^\circ \times 593{\cdot}8636 &= 455{\cdot}0369 \\ \text{and } a = \sin 40^\circ \times 593{\cdot}8636 &= 381{\cdot}7282. \end{aligned}$$

Where any considerable break occurs in the general ground-line between two selected points, as shown by the dotted line K L M, and the point L cannot be observed with the theodolite and levelling-staff, it may be determined either by measurement from the two nearest selected points C and D, or by separate observation with a hand-level or a clinometer.

The heights B B′, C C′, &c., added to the reduced level of peg H, will give the reduced height of the several selected points above datum.

Levelling with Aneroid.—The aneroid barometer has been fully described in Chapter III., and it is necessary only to explain its manipulation in the field. The larger the size, the more satisfactory the observations. The surveyor should provide himself with an accurate plan or map of the district through which he proposes to take the levels, and at the points of observation he should mark with a small dot, and place letters as A, B, C, &c., so that he may identify their relative positions from his note-book in which he records the readings. The temperature at starting should be noted, and the index or zero of the movable scale "should be set to where the hand of the instrument points." "On ascending a mountain the hand travels backward, and as each division represents 100 ft. (on the movable scale), an approximate indication of the ascent is thus readily obtained." The aneroid should be held perfectly horizontal, and gently tapped during an observation. "Subtract the reading at the lower station from that at the upper station; the difference is the height in feet."

Levelling with the Hypsometer.—The hypsometer is a portable instrument for ascertaining heights by the temperature at which water boils. The following description of it, and the tables for its use, compiled by Mr. Francis Galton, F.R.S., given in Appendix to the present volume, are by permission extracted from the Royal Geographical Society's "Hints to Travellers," vol. i.

"The boiling-point apparatus consists of a thermometer, A, generally graduated from 180° to 215°*; a spirit-lamp, B, which fits into the bottom of a brass tube, C, that supports the boiler, D;

---

* "When they are intended to be used at very great elevations, the thermometers will have to be specially constructed with extended scales."

and a telescope tube, E, which fits tightly on the top of the boiler. The thermometer is passed down the tube, E, from the top until within a short distance from the water, *which it should never touch*, and is supported in that position by an india-rubber washer, F. The steam passes from the boiler up the tube, E, and escapes by the hole, G. To pack this instrument for travelling, withdraw the thermometer, and put it into a brass tube, lined with indiarubber, having a pad of cotton-wool at each end; take off the tube, E, shut it up, and put the small end into the boiler, D, which it fits, then withdraw the spirit-lamp, B, screw the cover over the wick and replace it in C. The whole of this apparatus fits into a circular tin case, 6 inches long, and 2 inches in diameter.

"To use the boiling-point thermometer:—Take the apparatus to pieces, pour some water into the boiler, D, about one quarter full is quite sufficient; then put the instrument together, as shown in the drawing, taking care that the thermometer is just clear of the water, and light the spirit-lamp; as soon as the water boils, the steam ascending through the tube, E, will cause the mercury to rise; wait until the mercury becomes stationary, and then read the thermometer; at the same time, take the temperature of the air in the shade with an ordinary thermometer.

"If the traveller is visiting a region where the elevations are very great, he should, when purchasing this apparatus, see that the thermometers are capable of registering a greater height than those which are usually supplied, and that the lamp is large enough to hold a good supply of spirit, as it is a common fault to make it too small, and the tube carrying the wick should be long to prevent overheating the spirit. A screen, which may be made of tin to fold up, is most useful to place on the windward side, and at a very low temperature is almost indispensable, as the heat is otherwise carried off too rapidly for the water to boil properly."

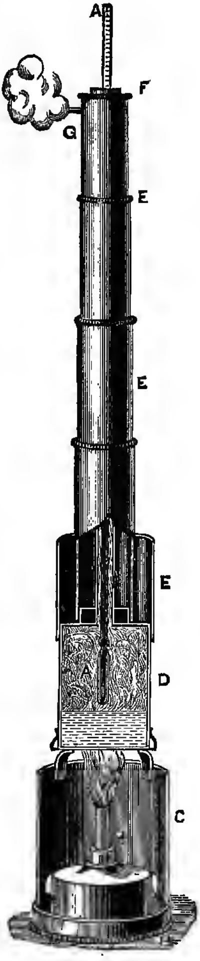


Fig. 277.

"Enter Table I., with the boiling-point at each of the two stations, and extract the numbers that stand opposite to them in the column headed ‘Altitude, &c.’ The difference between these numbers gives the difference of height between the two stations, supposing the mean temperature of the intermediate air to be 32° Fahr. The correction for the temperature of the air, when it differs from this value, is given in Table II. We take the mean of the thermometers (exposed in shade) at the upper and lower stations, and we enter Table II. with that mean value, and the number that stands opposite to it, in the column headed ‘Multiplier,’ must be multiplied with the results obtained from Table I. Thus:—

At station A the boiling-point = 195·1°, tabular number = 9040
B ” ” = 210·3°, ” ” = 887
Approximate difference of height = 8153 ft.

“To correct for temperature of intermediate air:—

At station A, temp. of air = 65° Fahr.
” B, ” = 73° ”
2)138
69 = mean temp. of intermediate air.

“In Table II. the multiplier corresponding to 69° is 1·082, and 1·082 × 8153 = 8821 (neglecting decimal fractions).

“In those rare cases where greater altitudes are dealt with than are included within the limits of the table, the traveller should allow 570 feet for the difference between 185° and 184°; 572 feet for that between 184° and 183°; 574 feet for the next interval, and so on.”

“When the boiling point at the upper station alone is observed by the traveller, he . . . [usually] has no option but to take the mean height of the barometer, reduced to the sea-level, in the district in which he is, and for the same season of the year, and to use this in the place of observations at a lower station. He will find what he wants in the maps of mean barometric pressure, reduced to sea-level, that are given in most of the physical atlases (‘Bartholomew’s Physical Atlas,’ Vol. III., is the most recent of these), and also in ‘Report on the Scientific Results of the Voyage of the Challenger, during the years 1873–76,’ ‘Physics and Chemistry,’ Vol. II.”

“Whenever the observations at the upper and lower stations are not strictly simultaneous, or when the mean barometer is taken in place of the lower station, the correction for diurnal variation must not be omitted, especially in the tropics, where, in other respects, the barometer is very steady. The mean amount of diurnal variation in different parts of the world is also given in

Berghaus' maps. An error of one or two hundred feet might often be caused by the neglect to allow for it."

Cross-sections.—Cross-sections in their general acceptance mean a line of levels taken at right angles to the longitudinal section at every chain, or oftener if necessary. Their length is regulated by circumstances; for railways from 1 to 5 chains on each side, at points right and left at all changes of contour. They are set out either with a cross-staff or preferably an optical square. The most satisfactory and accurate method is to treat the sections at each chain as consecutive members—0, 1, 2, 3, 4, &c., starting at the commencement of the longitudinal section—and, looking in direction of its termination, to treat all observations either of height or distance as being right or left of the centre line (or line of section), as in Fig. 278; and having set out three sight-lines, commence to measure from the centre, right and left in each separate case, noting any irregularity in the surface of the ground. These measurements should be personally made by the surveyor, who should be provided with a quantity of pieces of white paper (about 1½ in. square), upon which he writes the number of the cross-section, and the measurement in feet (all cross-sections should be measured in feet); and after these particulars have been carefully written upon the paper, it should be placed in a slit of a stick or twig, pointed at the other end, and stuck in the ground at the point to be observed. Thus, as in Fig 279, it will be observed that the cross-section is at $\overset{\text{M. CH.}}{0\cdot01}$ (no miles, 1 chain), and on the right-hand side there are five points,

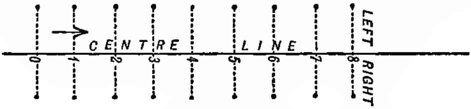


Fig. 278.

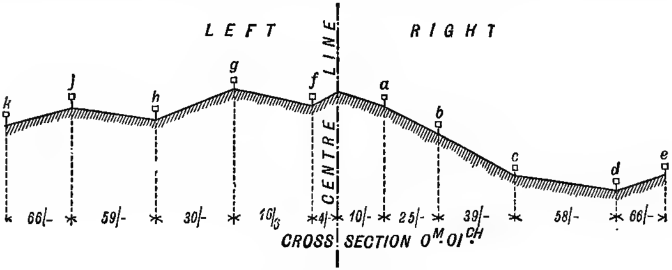


Fig. 279.

a, *b*, *c*, *d*, *e*, of 10 ft., 25 ft., 39 ft., 58 ft., and 66 ft. from the centre, whilst on the left there are also five points, *f*, *g*, *h*, *j*, *k*, of 4 ft., 16 ft. 6 ins., 30 ft., 59 ft., and 66 ft. respectively. Take the point *b* on the right and *g* on the left, they would be marked on the paper (as in Figs. 280 and 281), No. 1 section, 25 ft. right and 16 ft. 6 in. left. The chief advantage obtained by this process is, that not only does the surveyor personally superintend these preliminary operations, but after a series of eight or a dozen cross-sections have been set out and measured all the higher points of the series may be taken from one point, so that the change of instrument is minimised. The staff-holder, who should be properly instructed as to his duties, proceeds to each of the points, and holding the staff thereat, he picks up the ticket, and at a signal from the surveyor he reads out in a clear, loud voice, "Cross-section number one, 16 feet 6 inches left," the surveyor booking this repeats it, and if correct the ticket should be destroyed, so as not to be taken again.

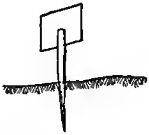


Fig. 280.

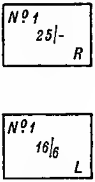


Fig. 281.

In conclusion, I recommend the surveyor to make his assistants thoroughly understand their duties and his requirements, and, by a code of signals mutually understood, a great deal of satisfactory work may be accomplished in almost dumb show.

Precise Levelling is a term applied to a system of levelling for the purpose of ascertaining, with the closest possible approach to absolute accuracy, the elevation of bench-marks pertaining to great territorial surveys such as the Ordnance Survey of the United Kingdom, the Government surveys of the United States of America, and similar works executed for national purposes. It is a highly complex affair of exceeding delicacy and minuteness of detail; and, with the exception of work required in certain vast hydraulic engineering undertakings, it lies wholly outside the sphere of ordinary surveying. It requires a course of special study and instruction which cannot be adequately set forth within the limits assignable to such a subject in the present volume. The whole matter is admirably treated in a work by Professor J. B. Johnson,* to which, and to the publications relating to the levelling of the Ordnance Survey, the student is referred for full information and details.

* "Theory and Practice of Surveying," 8vo, New York 1900.

CHAPTER X.

CONTOURING.

CONTOURING is the art of delineating upon a plan a series of lines which represent certain altitudes parallel with the horizon, or, in other words, "lines of intersection of a hill by a horizontal plane." The simplest illustration is the high and low water marks along the sea-shore, where the fringe of seaweed marks the extreme boundary of high water, and its zig-zag outline is due to the water finding out the inequalities of the level of the shore, so that whatever form this fringe may take, all round the coast of this "sea-girt island" will be found a line approximately parallel to the horizon.

Another and very primitive illustration: if varying quantities of different coloured liquids, commencing with the lightest colours in the largest quantities, were poured into some basin-shaped vessel whose sides would absorb some of the colours, so as to leave the mark of their highest level, and smaller quantities of colour of graduating darkness were successively poured in and emptied out, the defined lines made by those different colours would represent concentric circles on the sides of the basin, whose distance apart would be governed by the varying quantities of the different coloured liquids, and these lines would be the contours of the sides of the vessel.

Vertical Intervals and Horizontal Equivalents.—It is the province of the modern surveyor to practically show upon his plans these lines of contour. The known differences of height thereof are called the *vertical intervals*, and their distances apart upon the survey are termed the *horizontal equivalents*, as will be seen by Fig. 282. In Figs. 283 and 284 we have a simple illustration of contour lines upon the truncated cone (Fig. 283) at points A, B, C, D, E, F, G, H, which in plan are represented by the concentric circles in Fig. 284, so that in the former case the relative heights B over A, C over B, &c., represent the vertical intervals, whilst in Fig. 284 the distances B from A, C from B, &c., are the horizontal equivalents.

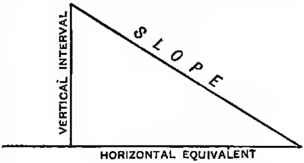


Fig. 282.

In Figs. 285 and 286 we have examples of the form contour lines will show on plan whose planes are projected from a section of irregularity. The contours will occur in smaller horizontal distance, in proportion to the steepness of the ground. The contour lines in Fig. 285, besides giving the relative altitudes

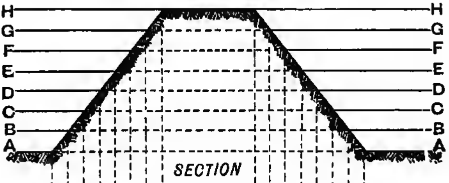


Fig. 283.

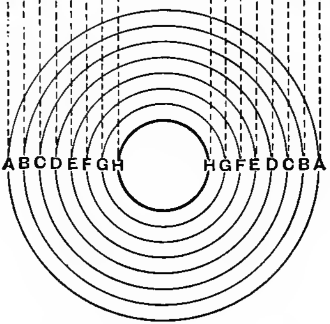


Fig. 284.

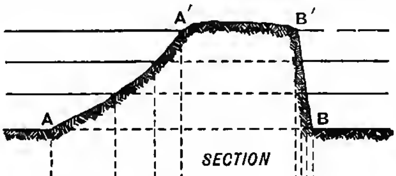


Fig. 285.

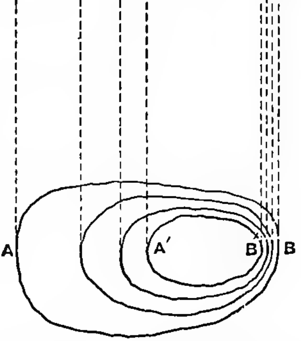


Fig. 286.

explain the form and flexure of every slope; thus A A′ and B B′ (Fig. 285) show the exact concavity and convexity of the slopes A A′, B B′ in Fig. 286.

Now these vertical intervals are to be determined by two methods; 1st by angular observations, 2nd by means of levelling.

As to the first of these: it has been shown, in the chapter on "Chain Surveying," that in chaining up or down a slope allowance for hypotenusal measurements can be made by observing its angle of elevation. Conversely, the difference of level between points on a slope may be calculated from that angle. If A = angle of slope, V the vertical interval between the contours, H the horizontal equivalent, and L the length of slope from contour to contour; then—

$$L = \cot A \times V,$$
$$H = \sec A \times L.$$

Fig. 287 shows the slope of a hill having in profile three different lines, A C, C D, and D E, their angles of elevation being respectively 10°, 35°, and 65°; whence, if H be put = 25, the horizontal equivalent of A C is 141·78 and its hypotenusal length 143·97. Of C D, these are respectively 35·70 and 43·59; and of D E, 11·66 and 27·58. The following table will facilitate computation.

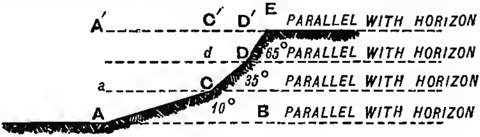


Fig. 287.

Table of Horizontal Equivalents of Various Angles of Slope for a Vertical Interval of 25.

< A	L cot A × V	H sec A × L	< A	L cot A × V	H sec A × L	< A	L cot A × V	H sec A × V
1°	1432·47	1432·25	16°	90·70	87·19	31°	48·54	41·61
2°	716·34	715·91	17°	85·51	81·77	32°	47·18	40·01
3°	477·68	477·03	18°	80·90	76·94	33°	45·90	38·50
4°	358·39	357·52	19°	76·79	72·61	34°	44·71	37·06
5°	286·84	285·75	20°	73·10	68·69	35°	43·59	35·70
6°	239·17	237·85	21°	69·76	65·13	36°	42·53	34·41
7°	205·14	203·61	22°	66·74	61·88	37°	41·54	33·18
8°	179·63	177·88	23°	63·98	58·90	38°	40·61	32·00
9°	159·81	157·84	24°	61·46	56·15	39°	39·73	30·87
10°	143·97	141·78	25°	59·16	53·61	40°	38·89	29·79
11°	131·02	128·61	26°	57·03	51·26	41°	38·19	28·76
12°	120·24	117·62	27°	55·07	49·07	42°	37·36	27·77
13°	111·14	108·29	28°	53·25	47·02	43°	36·66	26·81
14°	103·34	100·27	29°	51·57	45·10	44°	35·99	25·89
15°	96·59	93·30	30°	50·00	43·30	45°	35·36	25·00

For smaller vertical intervals, the tabular number divided by 25 and multiplied by the new V, will give the L and H value.

For sketch surveys this method is useful; and Figs. 288 and 289 show how sections thus obtained may enable contour lines (lines of equal altitudes) to be sketched-in.

Contouring by angle of slope, however, is not suitable for cases where much accuracy is required: for this purpose the work must be done by actual levelling, the two usual methods being that of cross-sectioning, and that of setting out the contour lines on the ground.

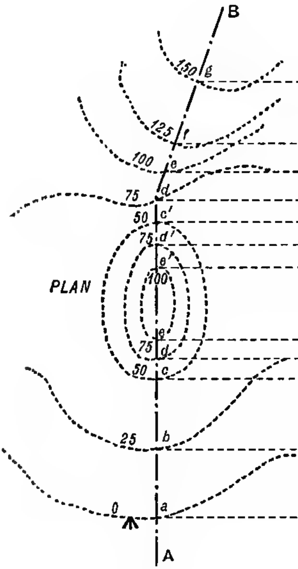


Fig. 288.

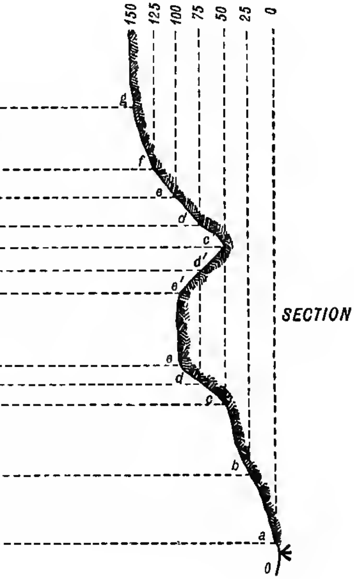


Fig. 289.

In the first of these, cross-sections are taken along lines normal (as nearly as can be judged) to the general curves formed in plan by principal salient and retiring features of the ground, as shown in Fig. 290 by the lines D E, F G, H J, K L. These lines being set out, levels are taken along them, from which, when plotted, points answering to the reduced levels of the intended vertical intervals are marked-off on the plan and determine the figure of the contour lines. Or, alternatively, the reduced level of the first contour line having been settled by reference to a bench-mark, the levelling-staff is shifted along the line of each cross-section in succession until it stands on a spot where its reading gives that reduced level: a peg is driven at each such spot, the positions of these pegs marking the several points of the contour lines.

In the second method, the reduced level of the first contour line having been determined as before, the staff is held at a salient or retiring feature in the estimated run of the contour line, and

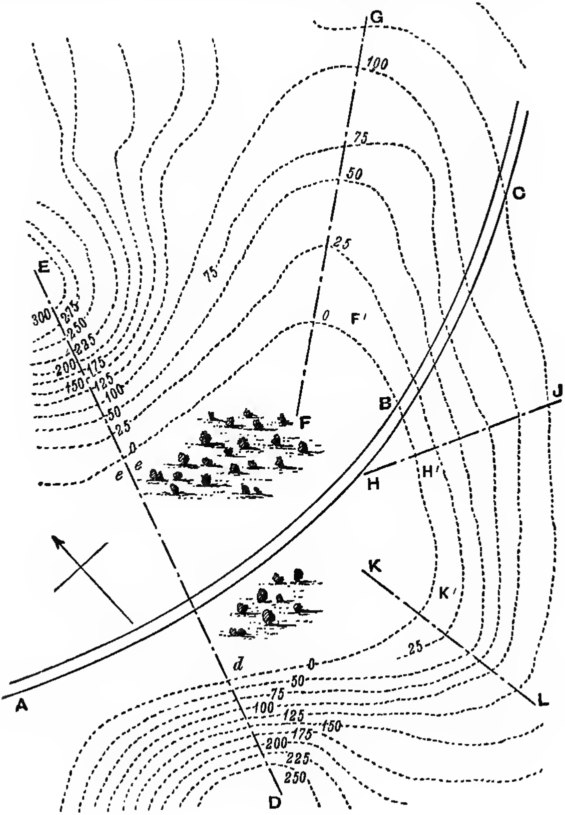


Fig. 290.

there shifted up or down the slope until the desired reading is obtained. A peg is driven at the spot: the staff is then removed to a suitable point further along the estimated contour line, and a peg driven where the reading is the same; and so on. The staff remaining at the furthest point along the contour line which the surveyor chooses to read with that set of the level, he enters the reading as a fore-sight, the level is then planted at a suitable place further in advance and a back-sight taken; after which the staff is taken forward to another selected feature of the ground, where readings agreeing with the last back-sight are taken and the places marked with pegs, as before.

In each of the above cases, the lines formed by the pegs whose reduced level is the same are surveyed, either by ordinary chain-surveying, or by traversing, as may be found most advisable, and plotted on plan.

The location and height of several points being known, their contour lines can be laid down intermediate between other known points, with more or less approximation to accuracy according as the slopes of the ground are more or less uniform. In Fig. 291, the points *b*, *c*, *d*, *e*, *f*, *h*, *a*, and *g* being known, and the contour lines *b a*, *d h*, and *f g* determined, the position of the intermediate ones may be plotted by proportional interpolation. The horizontal equivalent of *b f* being 640 ft. with a total rise of 40 ft. and a nearly uniform slope of 1 in 16, the horizontal equivalent of each contour is 160 ft. and its vertical interval 10 ft. The horizontal equivalent of *a g* is 360 ft. with a total rise of 40 and a slope of 1 in 6·75, the horizontal equivalent of each contour being 90 feet. A cross-section of the slope through the point *h* has a horizontal equivalent of 540 feet, and the contour lines there will be 135 feet apart. The completed contours in the figure are shown in full lines and the interpolated ones by dotted lines.

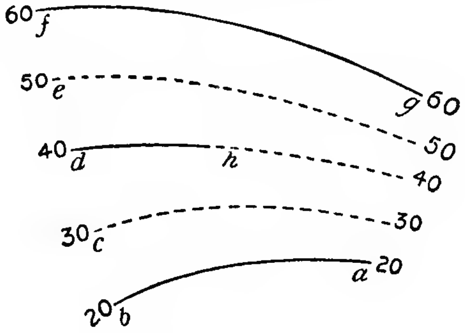


Fig. 291.

An ingenious mode of marking-off interpolated contours is given by Mr. N. Kennedy, M.Inst. C.E.*

---

* "Surveying with the Tacheometer." London: Crosby Lockwood & Son.

CHAPTER XI.

SETTING OUT CURVES.

PRACTICAL surveyors are nowadays required to perform so many more duties than heretofore, that any work upon the subject of their duties would be incomplete if it did not treat upon the setting out of curves. It does not necessarily follow that these curves are only for railway work, as in the development of property it is often requisite to lay out new roads and boundaries, which, for economical and other reasons, frequently are required to take the form of regular curves.

The most accurate and satisfactory method of laying out curves is by means of a theodolite, but for approximate results the operation may be performed by tangents and offsets, or chords and ordinates.

In most cases a curve is used to connect two straight lines, whose relative positions are such that one forming an angle with the other they intersect each other at some given point. In Fig. 292 it will be seen that the lines A *a* and C *c* intersect at the point

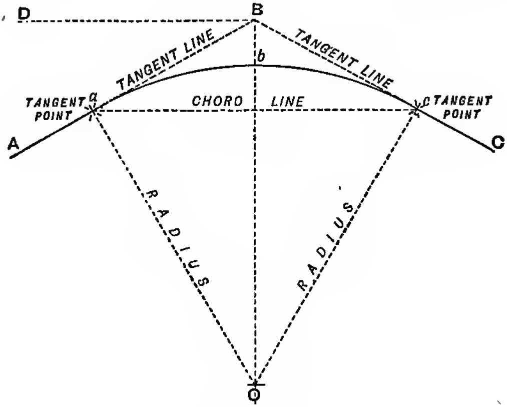


Fig. 292.

B. It matters not how acute or obtuse the angle of intersection may be, there is some curve, great or small, which will connect these two lines, to which they will be tangential.

In considering a railway, as an illustration, it simply consists of a series of straight lines, whose directions form angles with each other, whereby it is necessary to connect each with the other by means of curves, as is illustrated in Fig. 293, by the five lines A B, B C, C D, D E, E F, and four curves *a a'*, *b b'*, *c c'*, and *d d'*. Here we have the angles A B C, B C D, C D E, and D E F, without knowing the value of which it is impossible to set out the curves upon the ground.

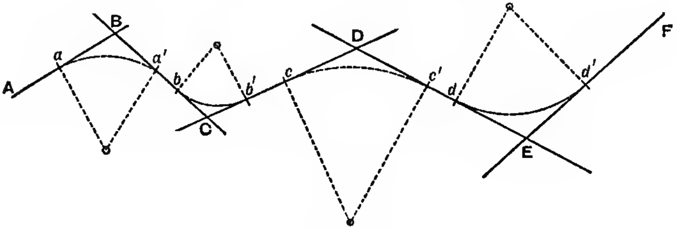


Fig. 293.

It may be well here to mention that for railway work it is better to lay out these straight lines and make them the base-lines of the survey. This may be done either by traversing or, preferably, by taking the included angles with the theodolite. It need hardly be explained that for the purpose of taking up the features on the right and left hand of these lines a complete system of triangulation must be adopted.

Having obtained an accurate record of the relative positions of these straight lines, which should be plotted to as large a scale as possible, together with the details of the survey, it will then be possible to determine the various radii of the connecting curves.

Limit of Radii.—In speaking of the radii of curves, I may say that curves of less than 12 chains' radius are not desirable for railway work. I have known less, but for many reasons sharp curves are to be avoided. It is a very mistaken theory that curves of small radius enable the engineer to economise in the design of his work, or in other words to avoid undue severance of property; and it is a very questionable policy, for against a small saving in the purchase of the necessary land (which is settled once for all) must be placed the constant wear and tear of the permanent way and rolling stock, which, if capitalised at a period of years, will prove a very formidable amount. Again, in these days of high speed it is absolutely out of the question to adopt sharp curves. There is no fixed rule to govern the limit of radius of curves, as so much depends upon local and other circumstances, which it is not the province of this work to consider.

Preliminary.—Now to take a simple illustration, we will assume that in Fig. 292 the ∠ of intersection A B C is 135°; bisect this = 67° 30′, which deducted from 90° = 22° 30′ the ∠ of deflection B *a c* = D *b a* = B O *a*. The line B O is at right angles to the line *a c*.

We will assume the radius of the curve = 30 chains, and it is required to find its centre. Multiply the natural secant of the ∠ of deflection (= 22° 30′) by the radius; then

$$\text{Nat. sec. } 22^\circ\ 30' = 1{\cdot}08239 \times 30 = 32{\cdot}4718 \text{ chains,}$$

which is the distance from the intersection B of the tangents to the centre O of the curve; and 32·4718 − 30 = 2·4718 chains = the distance B *b* from the point of intersection to the point *b* where the arc is bisected.

To determine the points of commencement and termination of the curve (the "tangent-points"), multiply the natural tangent of the ∠ of deflection by the radius; this gives the length of tangents B *a* and B *c*. Thus

$$\text{Nat. tan. } 22^\circ\ 30' = 0{\cdot}41421 \times 30 = 12{\cdot}4264 \text{ chains} = \text{B}\,a \text{ and B}\,c.$$

When the point of intersection is not accessible, the length of tangents is set out as follows (Fig. 294). Run any line E F from tangent to tangent; then

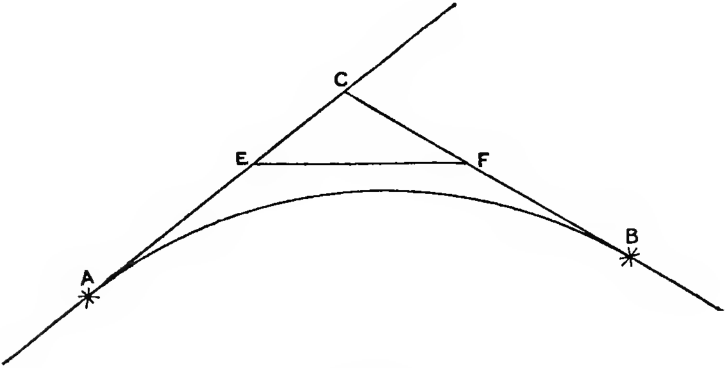


Fig. 294.

$$\angle \text{CEF} = 180^\circ - \angle \text{AEF}$$
$$\angle \text{CFE} = 180^\circ - \angle \text{BFE}$$
$$\text{therefore } \angle \text{ACB} = 180^\circ - (\angle \text{CEF} + \angle \text{CFE})$$
$$\text{and side EC} = \frac{\text{EF} \times \sin \text{CFE}}{\sin \text{ACB}}$$
$$\text{and side FC} = \frac{\text{EF} \times \sin \text{CEF}}{\sin \text{ACB}}.$$

When a portion of the curve itself is inaccessible (Fig. 295), a point B in it from tangent-point A is set out as follows. The chord of any arc = twice sine of half the ∠ subtended by that chord at centre of curve. Thus

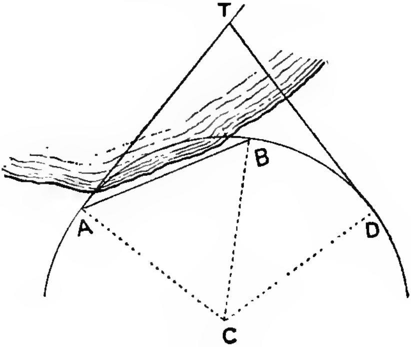


Fig. 295.

$$\angle \text{TAB} = \tfrac{1}{2} \angle \text{ACB}; \text{ whence AB} = 2 \sin \text{TAB} \times \text{radius}.$$

The tangents having been produced to their intersection B (Fig. 292), a stout peg is driven there, and the exact point of intersection marked by a spike driven into the top of the peg. The theodolite is adjusted over this mark; the ∠ of intersection A B C observed; the distance B *b* calculated; and the point *b* fixed by bisecting ∠ A B C and driving a peg lined-in by the theodolite on the line of bisection at the distance B *b*. The length of tangents is then set-off from B, a peg is lined-in by the theodolite and driven at their ends *a* and *c*, and distinguished by a peg driven at each side on a line transverse to the tangent (some prefer to drive four pegs, as shown in Figs. 296, 297, 298).

The data assumed and calculated as above described, being adopted for illustration, we are now prepared to set out the curve by one or other of several methods, the most useful being:—By Tangential Angles; by Offsets from Chords produced; by Offsets from Tangents; and by Ordinates from Chords.

The symbols and formulas are as follows:—

R = radius of curve
I = half ∠ of intersection
F = ∠ of deflection (= half ∠ at centre of curve)
D = distance from centre of curve to intersection of tangents
X = external secant of F
T = length of tangents
L = length of curve
C = tangential ∠ in minutes and decimals for each chord of same denomination as radius
N = number of chords

Then

$$F = 90^\circ - I$$
$$D = \sec F \times R$$
$$X = D - R$$
$$T = \tan F \times R$$
$$L = {\cdot}000582\ R\ (5400 - I \text{ in minutes and decimals})$$
$$N = \frac{5400 - I \text{ in minutes and decimals}}{C}$$
$$C = \frac{1718{\cdot}873387}{R}.$$

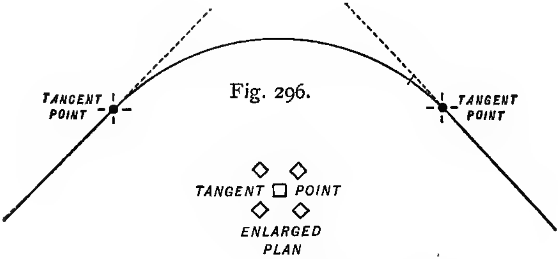


Fig. 296.

Fig. 297.

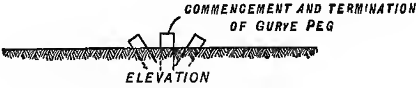


Fig. 298.

By Tangential Angles.—Plant theodolite over first tangent-point, clamp at 180°, and sight upon a pole in the straight line backwards. Unclamp upper plate, and reverse by bringing the reading to zero, when, if the work has been correctly set out, the cross-wires should sight the spike in the intersection peg; and, with the vernier set to ∠ of deflection, the cross-wires should sight upon the second tangent-point. The radius of curve being

30 chains, the tangential ∠ for a 1-chain chord = 57′ 17·7468″. With one end of chain at the tangent-point, a pin at the other end, ranged to that ∠, marks the first point on the curve. Point

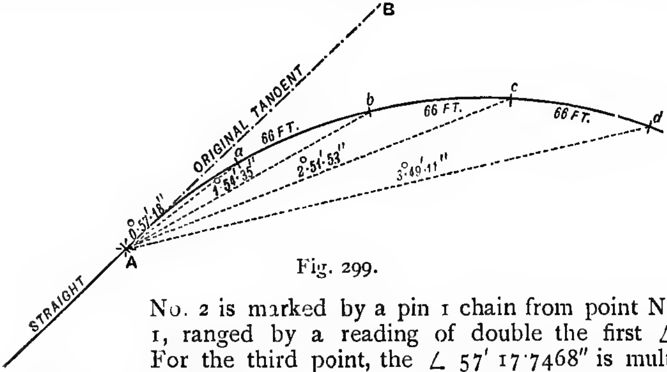


Fig. 299.

No. 2 is marked by a pin 1 chain from point No. 1, ranged by a reading of double the first ∠. For the third point, the ∠ 57′ 17·7468″ is multiplied by 3, and a pin 1 chain from point No. 2 is ranged at that ∠, and so on (Fig. 299): the record of the several angles at the points being as follows :—

1st	tangential ∠ =	57′ 17·7468″ =	57′ 18″ =	57·296′
2nd	„ „ =	1° 54′ 35·4936″ =	1° 54′ 35″ =	1° 54·592′
3rd	„ „ =	2° 51′ 53·2404″ =	2° 51′ 53″ =	2° 51·887′
4th	„ „ =	3° 49′ 10·9872″ =	3° 49′ 11″ =	3° 49·183′
5th	„ „ =	4° 46′ 28·7340″ =	4° 46′ 29″ =	4° 46·479′

In the above example, the fractions of seconds are given to four places of decimals to show the reason of the apparent irregularity in the additions. As theodolites for ordinary railway and survey work seldom read to smaller angles than 20 seconds = one-third of a minute, tables of tangential angles are rarely carried to smaller subdivisions. Excess or defect in any one ∠ is too small to be of any practical account in setting out a curve, and, being in the table adjusted to the nearest half or one-third of a minute, it is not cumulative. Decimals are easily converted into thirds by the following table :—

From	0·0000	to 0·1666	both	inclusive	= $\frac{0}{3}$
„	0·1667	„ 0·4999	„	„	= $\frac{1}{3}$
„	0·5000	„ 0·8333	„	„	= $\frac{2}{3}$
„	0·8334	„ 1·0000	„	„	= $\frac{3}{3}$.

It is not often the case that a curve commences or terminates at even chainage; and, the initial or the terminal chord, or both, being thus less than the others, the tangential ∠ must be modified accordingly. Retaining the data already employed in illustration, we will suppose that a curve commences at 6 miles 27·32 chains, and is 23·56 chains in length. The first chord on it will have a tangential ∠ corresponding not to 1 chain but to 100 − 32 = 68 links $= \frac{68}{100} \times 57·296' = 38·96'$; and the last chord will be determined thus:—

	Chains.	Chains.
Length of curve =		23·56
1 chord of 68 links =	00·68	
22 chords of 100 links =	22·00	22·68
Last chord =		00·88

Its tangential ∠ will be $\frac{88}{100} \times 57·296 = 50·53'$; and the curve ends at 6 miles 50·88 chains.

Before commencing to set out, it is advisable to make a complete list of the tangential angles of the curve, from the first tangent-point to the last: it saves much trouble in the field, where the surveyor has his mind occupied and his hands full. For the curve we have taken as an illustration, the list of tangential angles will be as follows:—

1st chord = 0° 38·96′
2nd " = 1° 36·26′
3rd " = 2° 33·55′
4th " = 3° 30·85′
5th " = 4° 28·14′
6th " = 5° 25·44′
7th " = 6° 22·74′
8th " = 7° 20·03′
9th " = 8° 17·33′
10th " = 9° 14·62′
11th " = 10° 11·92′
12th " = 11° 9·22′
13th chord = 12° 6·51′
14th " = 13° 3·81′
15th " = 14° 1·10′
16th " = 14° 58·40′
17th " = 15° 55·69′
18th " = 16° 52·99′
19th " = 17° 50·29′
20th " = 18° 47·58′
21st " = 19° 44·88′
22nd " = 20° 42·17′
23rd " = 21° 39·47′
24th " = 22° 30.00′

If the curve is to the left, the tangential angles above given would be deducted from 360° and the pegs set accordingly. Thus for the first chord the instrumental reading would be 360° − 38·96′ = 359° 21·04′, for the second 360° − 1° 36·26′ = 358° 23·74′, for the third 360° − 2° 33·55′ = 357° 26·45′; and so on.

Curves of less radius than 15 chains should be set out in half-chain chords, for which the tangential angles of whole-chain chords of curves double the radius can be used.

Save in curves of radius exceeding 30 chains, it is not desirable to set out more than from five to eight chords from the same station, because the tangential ∠ becomes too large to ensure the placing of the pegs exactly on the line of curve. There may thus be one or more shifts of the theodolite; and sometimes trees, buildings, or other obstacles may prevent even that number being set.

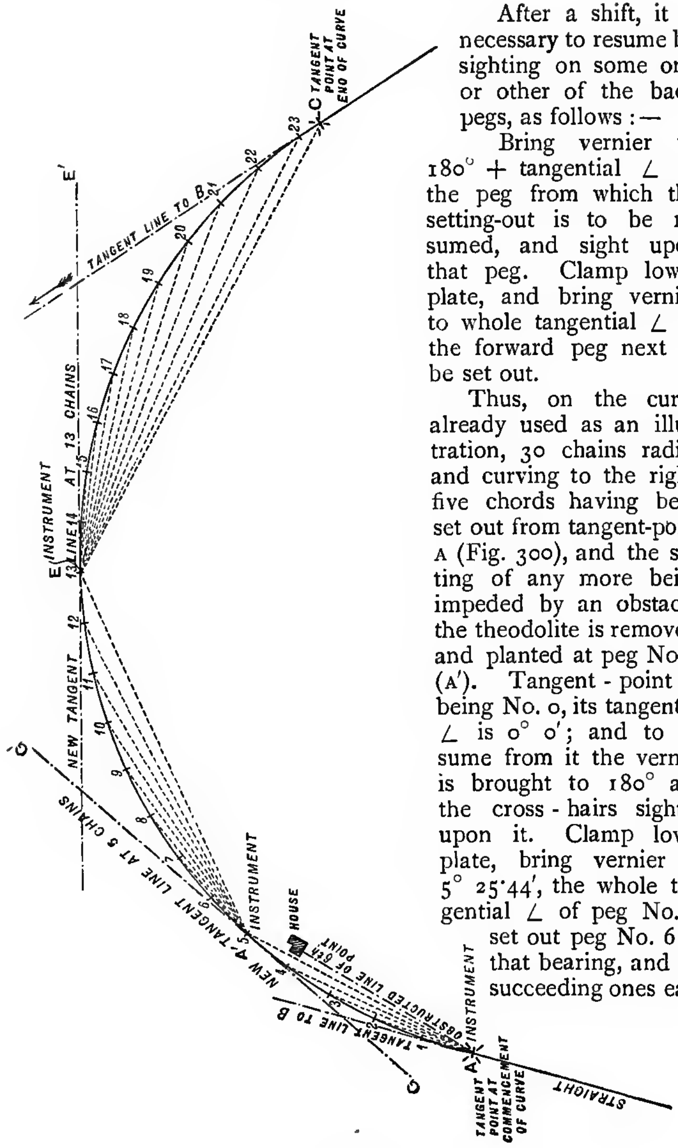


Fig. 300.

After a shift, it is necessary to resume by sighting on some one or other of the back pegs, as follows:—

Bring vernier to 180° + tangential ∠ of the peg from which the setting-out is to be resumed, and sight upon that peg. Clamp lower plate, and bring vernier to whole tangential ∠ of the forward peg next to be set out.

Thus, on the curve already used as an illustration, 30 chains radius and curving to the right, five chords having been set out from tangent-point A (Fig. 300), and the setting of any more being impeded by an obstacle, the theodolite is removed, and planted at peg No. 5 (A′). Tangent-point A being No. 0, its tangential ∠ is 0° 0′; and to resume from it the vernier is brought to 180° and the cross-hairs sighted upon it. Clamp lower plate, bring vernier to 5° 25·44′, the whole tangential ∠ of peg No. 6, set out peg No. 6 on that bearing, and the succeeding ones each on its own whole tangential ∠, *i.e.* peg No. 7 on 6° 22·74′, No. 8 on 7° 20·03′, &c. When peg No. 13 is set out, perhaps another obstacle may oblige another removal of theodolite. Plant it at peg No. 13 as a fresh station. Bring vernier to 180° + tangential ∠ of previous station (peg No. 5) = 180° + 4° 28·14′, sight on peg No. 5, and clamp lower plate. Bring vernier to read 13° 3·81′ the whole tangential ∠ of peg No. 14, and set out that peg; and so on.

Observe that in a curve to the left the vernier is to be brought to 180° − tangential ∠ of the peg from which the setting-out is to be resumed.

As a check upon the work as it proceeds, it is well to occasionally bring the vernier to 180° + (or − as the case may be) the tangential ∠ of the furthest visible back peg, and sight upon that peg, clamp lower plate, and bring vernier to whole tangential ∠ of second tangent-point if visible; the cross-hairs should then cut this point. If they fail to do so, some error has crept into the setting-out.

At the close of the work, the tangential ∠ of the final chord ought to cut upon the second tangent-point.

It is best not to drive the stumps until the whole curve has been set out; not only because chain-pins can be more accurately set out at first, but because they may need shifting, and also because driving the stumps may shake the theodolite.

Setting-out Curves with Two Theodolites.—This is in some respects the most satisfactory instrumental method: the points on the curve may be found without measurement, and it is especially suitable in cases where a river, a part of a lake, or other obstacles prevent the use of the chain; also in very hilly ground, where the measurement of the chord-lines would be attended not only with difficulty but also with liability to inaccuracy.

Fig. 301 is an illustration of this method. The straight lines if produced to B would intersect in the bay, and it is required to set out the points of the curve at 1, 2, 3, 4, 5, and 6. By the method explained on pp. 239 and 240, and illustrated by Figs. 294 and 295, the ∠ of intersection may be obtained and the tangent-points A and C fixed. At each of these points a theodolite should be planted and adjusted to its tangent-line A B or C B.

In this example we will assume the curve to be to the right, the radius 8 chains, the ∠ A B C 92° 30′ and the chords 2 chains each. The ∠ of deflection is 43° 45′, the tangential ∠ for each of the 2-chain chords 7° 9·72′, the length of the curve 12·24 chains, and the number of chords 6·11.

Bring vernier of theodolite at A to 180° and sight a mark on the straight line A D backwards towards D. Clamp lower plate, and bring vernier to 7° 9·72′, the tangential ∠ of 1st chord.

The number of chords being 6·11, and the bearing of the first of them being thus set off from A, the tangential ∠ from C corresponding to peg No. 1 of the curve will be (6·11 − 1) = 5·11 × 7° 9·72′ = 36° 35·86′. Bring vernier of theodolite at C to 180° and sight a mark on the straight line C E forwards towards E. Clamp lower plate and bring vernier to 360° − 36° 35·86′ = 323° 24·14′; and at the intersection of this bearing with the bearing from A, put down the pin for first point of the curve.

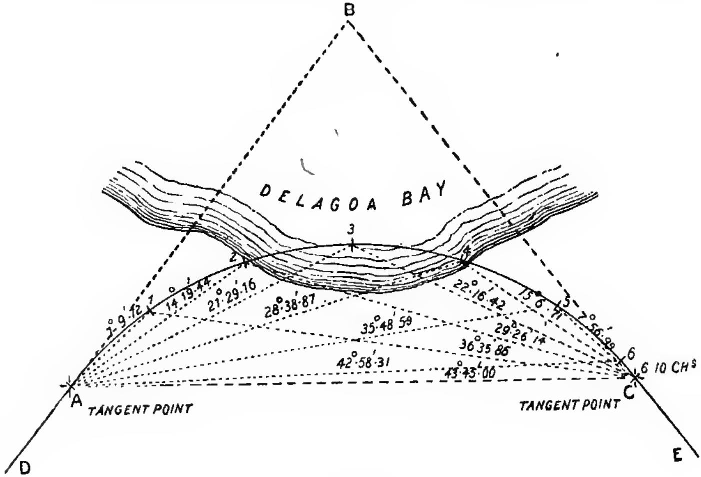


Fig. 301.

The tangential ∠ at A for chord No. 2 is 14° 19·44′, and the corresponding ∠ at C is 360° − (36° 35·86′ − 7° 9·72′) = 360° − 29° 26·14′ = 330° 33·86′, and so on; the complete list of tangential angles being as follows:—

Chord No.	Theodolite at A.		Theodolite at C.		
	From Tangent A B.		From Tangent C B.		Reading on Theodolite.
1	∠ B A 1	7° 9·72′	∠ B C 1	36° 35·86′	323° 24·14′
2	,, B A 2	14° 19·44′	,, B C 2	29° 26·14′	330° 33·86′
3	,, B A 3	21° 29·16′	,, B C 3	22° 16·42′	337° 43·58′
4	,, B A 4	28° 38·87′	,, B C 4	15° 6·71′	344° 53·29′
5	,, B A 5	35° 48·59′	,, B C 5	7° 56·99′	352° 3·01′
6	,, B A 6	42° 58·31′	,, B C 6	47.27′	359° 12·73′
7	,, B A C	43° 45′			

The ∠ B A C, if calculated according to the tangential ∠ for a 2-chain chord, would be 43° 45·58′, or nearly 35 seconds in excess of the ∠ of deflection, owing to the difference between the true length of the curve and the sum of 6·11 chords of 2 chains each. A difference of this nature always exists in the case of a curve set out by chords; but in all ordinary cases the difference is so small as to be immaterial. When, however, the radius of the curve is small relatively to the length of the chords, the difference becomes so great that, in order to avoid confusion by suggestion of error in the work, the final tangential ∠ should be so modified as to make up the total ∠ of deflection; as is done in the foregoing example.

Curves of Different Radii.—It may happen that whilst for good reasons it may be desirable to traverse a certain portion of the ground by a curve of say 60 chains radius, yet an obstruction may occur which involves either a change in the radius of the curve, or (what is frequently done) the stoppage of the original curve at some point, and after a short length of straight line the adoption of a curve of different radius in order to avoid the obstruction. Thus in Fig. 302 we see that after setting out a certain distance

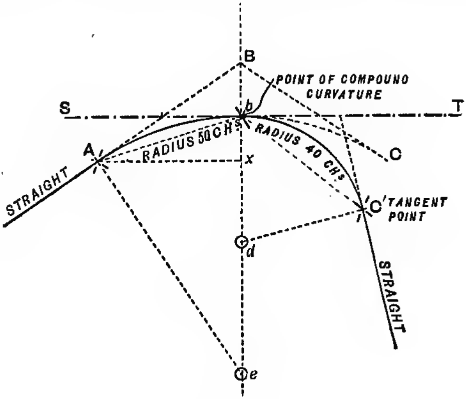


Fig. 302.

from A to *b* with a radius of 50 chains, that from this latter point it is necessary to reduce the radius to 40 chains. Now, assuming that we have set out 8 chords from A, then the tangential angle B A *b* will be 4° 35′. Remove the theodolite from A to *b* and set the vernier at 335° 25′ (being 360° − 4° 35′, as we are now working the upper plate from right to left), clamp the two plates, direct the telescope on to A, clamp the lower and unclamp the upper plate, fix the latter at zero, and we obtain a tangent-line

S *b* T common to the two curves, and from *b*, which is termed *the point of compound curvature*, we may now proceed to set out the tangential angles for the curve whether of greater or smaller radius than the first one.

Curves of Contraflexure.—" Reverse " curves or curves of contraflexure, as Fig. 303, are set out by establishing a common tangent-line F G by the same process as just described, and setting out tangential angles from right to left from C, in which case each angle for one chord must be consecutively deducted from 360 deg.

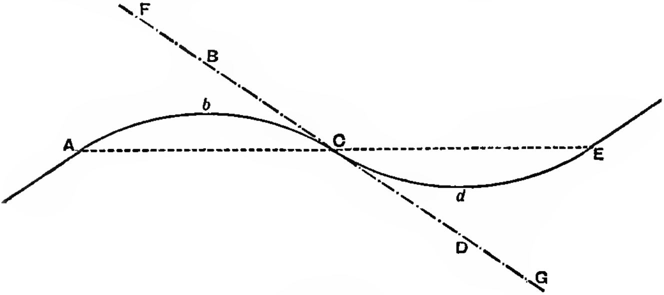


Fig. 303.

It should here be stated that a length of straight line, usually two chains, should always intervene between any curves, whether similar or reversed, as it is under very exceptional circumstances—at least as far as English practice is concerned—that one curve proceeds directly from another. Upon the Continent it is, I am aware, customary to use parabolic curves, but a whole library of scientific reasoning and deductions will not supersede the result of our own practical experience; and seeing that we have express trains running at more than double the highest speed of the Continental railways, I think we may fairly assume that the principles which govern our own system are well founded.

Setting out Curves by Offsets.—I shall very briefly con-

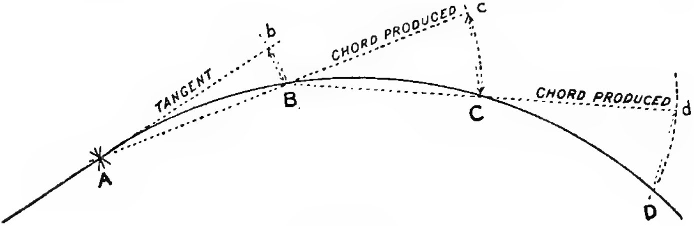


Fig. 304.

sider these methods, experience having proved that they can be used in cases only where accuracy is not of much importance.

The most usual system is by means of an offset from the tangent-line, at the first point on the curve; and from the chord produced, for each subsequent point (Figs. 304, 305).

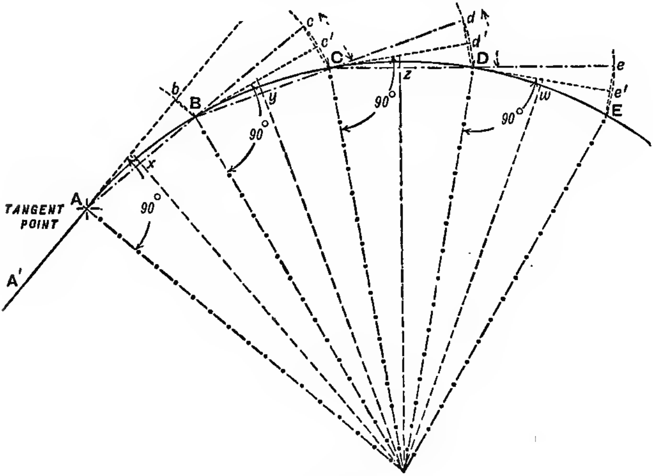


Fig. 305.

L = length measured in units of radius: from tangent-point along tangent, for first offset; and from the several points of curve set out, along their chords produced, for the second and subsequent offsets.

O = offset from the end of this length and at right angles to the line on which it has been measured.

R = radius of curve.

The formulas for calculating the length of offsets are as follows:—

For 1st offset (*b* B) from tangent,

$$O = R - \sqrt{R^2 - L^2}$$

For 2nd and succeeding offsets (*c* C, *d* D, &c.) from chords produced,

$$O = 2\,R\,(R - \sqrt{R^2 - L^2})$$

the offset from chord being double that from tangent.

Thus if $L = 1$ chain, and $R = 20$ chains,

O for 1st offset (*b* B)

$$= 20 - \sqrt{20^2 - 1^2} = 1{\cdot}65 \text{ foot} = 1 \text{ foot } 7\tfrac{3}{4} \text{ inches,}$$

O for 2nd and succeeding offsets (*c* D, *d* D, &c.)

$$= 2\left(20 - \sqrt{20^2 - 1^2}\right) = 3{\cdot}3 \text{ feet} = 3 \text{ feet } 3\tfrac{1}{2} \text{ inches.}$$

If L does not exceed $\frac{R}{20}$, the formula $O = \frac{L^2}{2R}$ for first offset, and $O = \frac{L^2}{R}$ for succeeding ones, is practically correct.

Should the curve terminate at the end of a whole chord, as at D (Fig. 305), the offset must be the same as that for the first point, viz. *b* B, and should exactly reach to the straight line D *e*′ beyond the curve. If the curve terminates on a broken chord, the length of the offset must bear the same proportion to the first one as the fractional length of the chord does to the length measured along the first tangent.

To set out a curve by this or the next following method requires the very greatest care, any error, even if slight, having a tendency to accumulate throughout the work. I have always used an offset-staff (Fig. 306), made of a lath of good hard wood a little longer than the longest offset, having on one edge, at the exact length of the tangential and the chord offsets, two notches to admit the arrow by which the point of the curve is to be marked. The length A *b* having been measured along the tangent, and the end of the offset-staff set at the arrow there fixed, the chain is laid from A to B and held at the first notch, and an arrow there put in. The chord-line A B is then produced to *c* and measured, becoming the line from which the second offset is to be measured. Here the same operation is repeated, but the chain is now laid from B to the second notch, and an arrow put in; and so on.

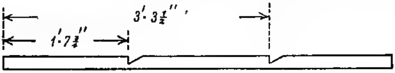


Fig. 306.

Setting out Curves from Same Tangent.—Another method of setting out a curve by offsets is from the same tangent (Fig. 307), the offsets being all at right angles thereto. In this system the first offset is found by the same rule as in the preceding method; and the subsequent offsets are this result multiplied by the square of the number of points. Thus for a 20-feet curve—

	Inches.		Ft.	In.
1st offset	19·81	=	1	7¾
2nd „	19·81 × 4	=	6	7
3rd „	19·81 × 9	=	14	9¾
4th „	19·81 × 16	=	26	4¾
5th „	19·81 × 25	=	41	3

&c., &c., &c.

Owing to the great length of the offsets, this method is less desirable for use than even the one last described.

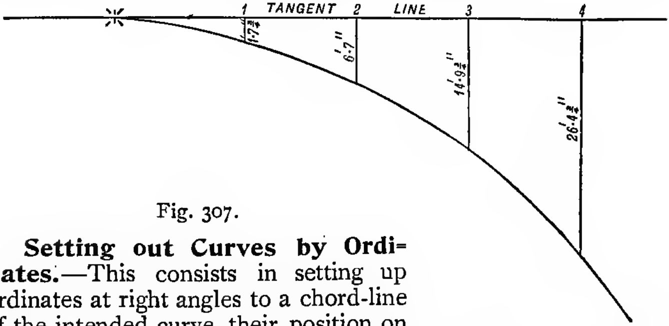


Fig. 307.

Setting out Curves by Ordinates.—This consists in setting up ordinates at right angles to a chord-line of the intended curve, their position on the chord, and their length, determining the several points of the curve. In Fig. 308 the reference of the letters is thus:

c = Chord,
v = Versed sine,
R = Radius of curve,
x = Distance of ordinate from centre of chord,
o = Length of ordinate.

All dimensions are in the same unit of measure. The formulas are these:—

$$v = R - \sqrt{R^2 - (\tfrac{1}{2}\,c)^2}$$
$$o = \sqrt{R^2 - x^2} - (R - v).$$

It is not only a very accurate and simple method, but also the one most suitable for use in a forest country or one much obstructed with bush and coppice-wood, the clearing required being but slight. The calculation of the ordinates involves a certain amount of time and trouble rendering it cumbersome for use in the field if they have to be made on the spot. But with a set of simple tables, such

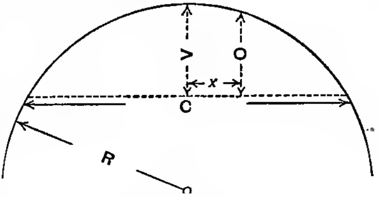


Fig. 308.

for example as Kröhnke's "Kurven," it is easy and extremely expeditious.*

Finally, as regards curves in general, it is to be borne in mind that the permanent chain-stumping of the curve should always be set out by careful measurement after the setting-out (by whatever method performed) is completed.

---

* Of its merits I can speak from experience, having employed it in Sweden in the survey and setting out of some 120 miles of railway line, much of which lay through forest land, where any other system with which I am acquainted would have necessitated the felling of a great deal of large timber. A 12mo, handy pocket-size translation of Kröhnke has since then been published by Kegan Paul, Trench, Trübner & Co., Limited, London, 1896.—A.B.

CHAPTER XII.

OFFICE WORK.

NEXT to proficiency in all field operations, office work is of great importance. A man may be ever so clever a surveyor, and even renowned for his accuracy, but unless he can portray the results of his observations graphically, so that the least initiated can easily comprehend their meaning, his work will be deprived of a very considerable amount of merit. He may be an excellent draughtsman in some ways, yet fail utterly to give adequate expression to the result of days or even weeks of patient labour, if he cannot in a minimised form give a true reproduction of his operations.

Necessity for System.—System is a very potent element in all branches of surveying, especially draughtsmanship. The beautiful Ordnance plans, in various scales, are the result of accuracy in the field and methodical elaboration in the office. Take even the 1-inch map, and it seems to speak for itself; whilst the larger scales enable the authorities, by their perfect administration, to delineate the most minute features, of which these plans are faithful representations.

George Stephenson, in the early days of railway enterprise, was wont to express the opinion that a map or detailed drawing should be so executed as to enable either to be read "like a book;" and there is no reason whatever why a survey should not be so as well.

To this end, I wish to give a few preliminary hints which may be of service to the student.

Roughly plot the Survey-lines.—1st. Roughly plot the chief lines of your survey to see what form it will take, so that you may arrange it symmetrically upon the paper upon which you intend to plot it.

Let the Paper be well seasoned.—2nd. Provide a piece of well-seasoned paper—Whatman's double-elephant, cold-pressed, is the best—and the paper should be mounted upon holland.

Draw a Scale on Paper before commencing.—3rd. Before commencing to plot your survey draw the scale upon the paper, so that you may apply your boxwood scales from time to time to ascertain whether the paper has been affected by temperature.

Boxwood Scales best.—4th. Boxwood scales are preferable to ivory.

Plot Survey North and South.—5th. Always plot your survey looking north, so that the top, bottom, left, and right respectively represent north, south, west, and east.

Paper Perfectly Flat.—6th. Keep your paper perfectly flat, and endeavour not to move it from the drawing-table during the process of plotting.

Laying down the Survey-lines on Paper.—7th. Having made a rough plan of your principal lines, proceed to lay them down carefully upon the permanent paper, commencing with your principal base-lines.

Check Measurement.—8th. Measure each line from left to right (using a pricker) upon a faint pencil-line, and check back from right to left and test its accuracy.

Marking Stations.—9th. Mark round the puncture representing a station with a pencil-ring thus ⊙, and opposite each station in faint pencil enter the distance, thus ——⊙——.

Straight-edge.—10th. Having plotted your principal base and survey-lines with a steel straight-edge (the longer the better), proceed to draw in these with a fine red line * (carmine or crimson lake), being specially careful that the lines are drawn accurately between the points only.

Never plot from Pencil Lines.—11th. Under no circumstances plot your offsets or any detail lines from pencil chain-lines.

As to plotting Long Lines.—12th. If the base or any other lines are longer than your straight-edge, do not seek to produce the line hand-over-hand-wise, but take a silk thread and stretch it tightly between the extreme ends, and with a pricker (held perfectly vertical) make punctures at frequent intervening points, then you may apply the straight-edge, and be sure you have as true a line as is possible.

Plot all Survey-lines first.—It is much better to plot all the survey-lines previous to commencing details, as any error, if detected, may be adjusted by re-measurement upon the ground, which might seriously affect the position of certain points of offset.

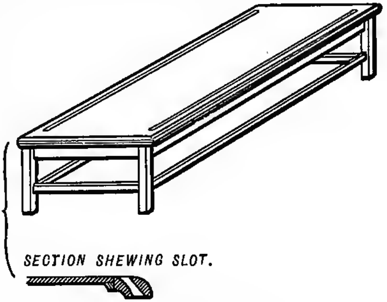
Plot each Day's Work as soon as possible.—Generally speaking, it is better to plot each day's work at once. I do not say the same evening, for arduous duties in the field (often upon a very meagre meal) and a heavy feed on one's return from work are not conducive to the patience, clearness of brain, or energy required for the purpose. On a large survey I recommend alternate days for field and office work, or using fine weather for the former, for say two or three days, and devoting wet days to office work.

---

* A good surveyor need never be afraid of leaving the survey-lines upon his plan.

Equipment of Office.—Now as to the equipment of an office. I differ entirely from those who argue that a surveyor who may have to take up temporary quarters at an hotel or inn, near the scene of his field operations, should plot his work under the very inconvenient circumstances often attending his sojourn. I am not speaking of a small survey, which may be plotted almost anywhere, and it is certainly preferable to do plotting in close proximity to the work rather than at a distance, in case of any mistakes in the chaining. But on a large survey it would be next to impossible to expect at an inn such facilities for plotting the work as are necessary, unless a room be specially engaged and fitted up for the purpose. This, however, must entirely depend upon circumstances, and no general rule can be laid down. Assuming, however, that arrangements of a satisfactory nature can be made, it is necessary for us to consider the needful equipments of the office.

Drawing-tables.—The drawing-table is of great importance. It should be made of well-seasoned timber and free from all imperfections, such as knots, &c.; it should be perfectly joined and clamped, and planed to an even surface. A convenient size is 8 ft. long by 4 ft. wide, and it should be supported upon a substantial under-framing with legs, *not trestles.* The edge all round should have a bull-nose from 3 to 4 in. deep, and it is better to have a slot lengthwise on each side, well rounded on the inner edge, so that the paper, if longer than the board, may pass through, and thus be protected from creasing during the process of plotting (see Figs. 309 and 310).



Figs. 309 and 310.

The paper should be held down by lead weights, 3″ × 2″ × 1″ (weighing about 2½ lbs.), covered with cloth or, preferably, wash-leather, and care should be observed in resting them, even so covered, by placing them on pieces of waste paper, in case of any defect in the covering, or dirt. I have already stated that a steel straight-edge should be provided, as long as possible (say 6 ft.), having a bevelled edge. This straight-edge should, when done with each day, be carefully wiped, as the moisture of the hand is productive of rust, and be placed either in a specially constructed case lined with green baize, or hung up in a dry place, encased in wash-leather or brown paper, to protect it from damp.

Scales.—A box of six boxwood scales, 12 in. long, with the accompanying offset scales, is indispensable. These scales are, one, two, three, four, five, and six chains to one inch on one side and corresponding feet on the other side—that is to say, the full length of the 1-chain scale of 12 in. represents 12 chains on one side and 792 ft. on the other; the 2-chain scale, 24 chains and 1,584 ft.; the 3-chain, 36 chains or 2,376 ft.; the 4-chain, 48 chains or 3,168 ft.; the 5-chain, 60 chains or 3,960 ft.; and the 6-chain, 72 chains or 4,752 ft. The offset scales are 2 in. long, representing 2, 4, 6, 8, 10, and 12 chains, or 132, 264, 396, 528, 660, and 792 ft. Boxwood scales are more reliable than ivory, and I prefer them to vulcanite. Always wipe them well before and after use, as the moisture of the hands encourages them to collect dirt.

Pricker.—All surveys should be plotted with a pricker with as fine a point as possible, and care should be taken to avoid making either too many or too large punctures, and round those required for further reference I always mark lightly with a pencil thus ⊙.

Pencils.—Only the best quality of lead should be used to plot work. HHH or HHHH are the best; and don't lean too hard upon the pencil, as by so doing you make an indentation as well as a line.

Points of Pencils.—As to the best form of point for a pencil, I cannot say that I am very much enamoured of the chisel-shape. It certainly marks well against the straight-edge, and for mechanical drawing is much the best; but for plotting a survey, if (as it always should) the pencil be held perfectly vertical and a fine point kept, I think it is easier and better to manipulate.

Protractors.—The best form of protractor is circular, of as large a diameter as possible. Electrum or brass protractors are best, of which there are various kinds. Figs. 311, 312, and 313

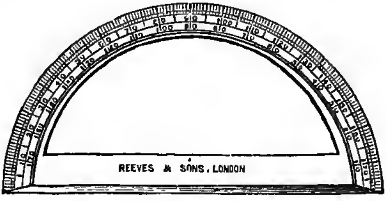


Fig. 311.

represent the simplest types, but for extensive work there are protractors having arms, at the end of each of which is a very fine pricker, and the instrument is so arranged that the centre of the protractor being adjusted to the point of intersection, the arms are in line on either side with this centre, and may be fixed upon the line (Fig. 314). It has a glass disc in the centre, with lines at right angles to each other, thus enabling the instrument to be adjusted to any point on the survey-line. An arm *b*, working from a collar attached to the centre, is governed by a slow-motion screw *f* which actuates the arms *a a*; these when not in use are folded over as shown. Another form of protractor which makes its appearance at all times is what is called the "ivory" or "military" protractor,

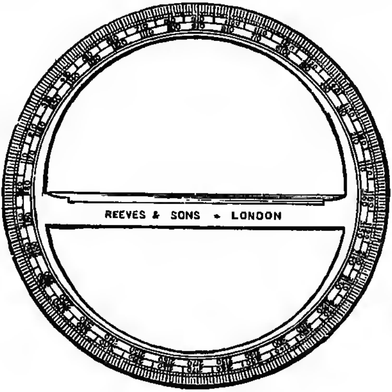


Fig. 312.

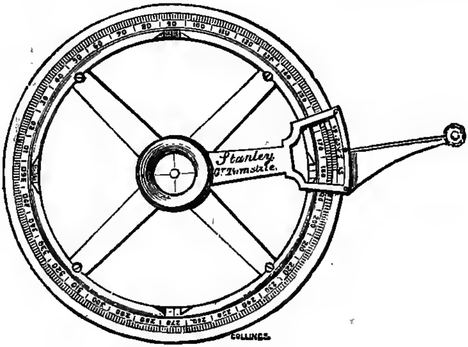


Fig. 313.

Fig. 315. It is a wonderful combination, and for portability and general utility (except for the purpose for which it is made) it is to be commended. For plotting a survey I should say do not use it except as a ruler for inking-in the boundaries, &c.

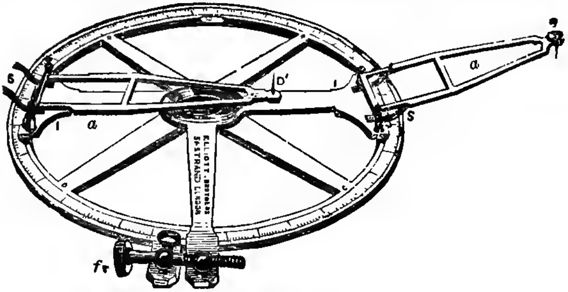


Fig. 314.

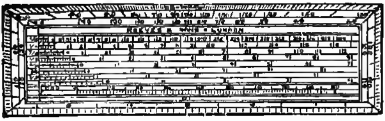


Fig. 315.

Beam Compasses.—For striking arcs of large radius such as are often required in plotting a chain-survey, ordinary compasses are useless even with the lengthening bar. For such purposes these arcs should be described by means of beam compasses or trammels (see Fig. 316). This excellent instrument consists of two brass boxes, each having a movable plate parallel with its vertical side, which is actuated by screws *a a*, so that it can be clamped tight against the mahogany * beam A. One of these brass boxes has a slow-movement screw D which enables the point C to be slightly moved at pleasure, whereby it may be adjusted to a hair's-breadth. The points may be removed at either end, and a pen or pencil one substituted.

How to use the Beam Compass.—The best way to manipulate the beam compass is to draw a pencil line, and upon this to carefully measure the required length with a scale, and then to apply the compass by moving the boxes approximately along the beam so that the points are near the mark, then clamp the screws *a a*, and with the slow-motion screw D get the exact position.

---

* These beams are made in any fair length of well-seasoned mahogany, having a "T" head to stiffen them.

Great Care in striking an Arc.—Great care is required in striking an arc with beam compasses, as at first, until one is accustomed to their use, they appear clumsy. Place the point of one end upon the station, holding the box lightly with the left hand, whilst with the right you guide the other box in the direction required, taking care to keep vertical the arm carrying the points, and not to press heavily upon the box. Thus if upon the line A B (Fig. 317), which is 1,260 links long, we wish to determine the point C, we must measure on a pencil line the length A C = 1,430 links, and placing the point at A describe an arc at C. And again with the length B C adjusted in the compasses, viz. 1,825 links, we describe an arc intersecting the other arc at C, and from A and B we draw the lines A C, B C respectively. Should there be a check- or tie-line, as from A to D, on B C we must strike the arc whose radius is 1,115 links, corresponding with the distance which the station D is from B, and draw a line A D, which when scaled should correspond with our measurements in the field, viz. 1,040 links.

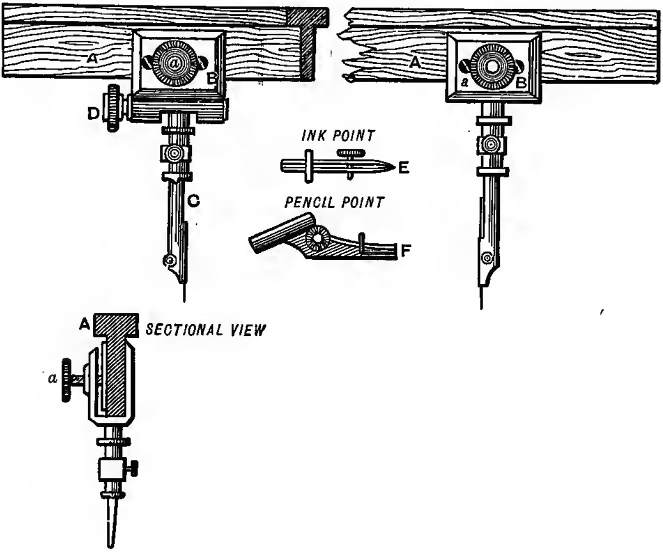


Fig. 316.

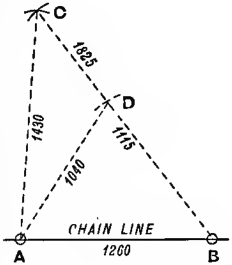


Fig. 317.

Pricker or Needle-holder.—No survey should be plotted without a pricker or needle-holder, as the finest puncture is all that is necessary to mark a point, and in a small-scale survey the thickness of even a very hard pencil would represent several links. Fig. 318 illustrates the usual type of pricker, in the absence of which, however, a very useful tool may be made with a halfpenny pen-holder and an embroidery needle heated in a candle and driven in eye-ways. I have one by me now whose total cost was under a penny, which I have used for years.

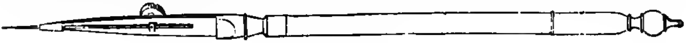


Fig. 318.

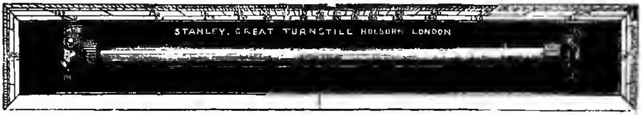

STANLEY, GREAT TURNSTILE HOLBORN LONDON

Fig. 319.

Parallel Rules.—Parallel rules are exceedingly useful in plotting a survey, and for traverse work they are indispensable. Those made to work upon rollers (as in Fig. 319) are the most reliable, and should be from 15 to 24 in. long, brass or gun-metal being far preferable to ebony.

Set-squares, &c.—For setting out right angles and to facilitate plotting, vulcanite or mahogany set-squares are necessary, similar to those illustrated in Figs. 320 and 321, those in Fig. 321 being framed in mahogany and edged with ebony: the former are less liable than the latter to get "out of square," but are more apt to soil the paper.

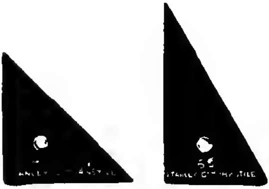


Fig. 320.

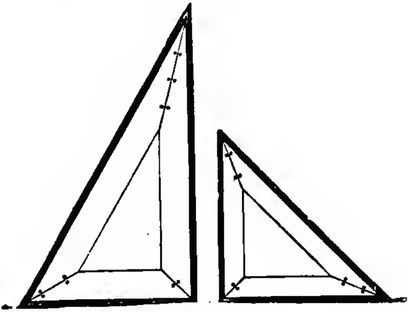


Fig. 321.

An extremely serviceable set-square is a skeleton one resembling what is shown in Fig. 321, made of electrum or other not readily tarnishable metal, having on each face three tiny ivory knobs which enable it to run frictionless on the paper, and allow of its use on parts of the drawing where lines already inked-in are not yet dry—the latter feature rendering it in many cases a welcome time-saver to the draughtsman.

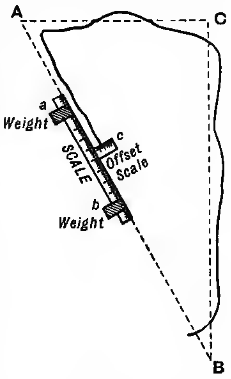


Fig. 322.

Offsets.—In plotting offsets or any of the features of a survey the greatest care is requisite. Place the edge of the scale accurately on the line, as in Fig. 322, and place two weights on *a* and *b*, then gently draw the offset scale C along the edge of the other scale to the point where it is required to make a lateral measurement, and prick off the length of the offset. It will be seen that a portion of a triangular field has been already plotted.

Curves.—No office should be without a box of curves, such as Fig. 323, which are made of pearwood, and are of regular radii from $1\frac{1}{2}$ to 150 in.

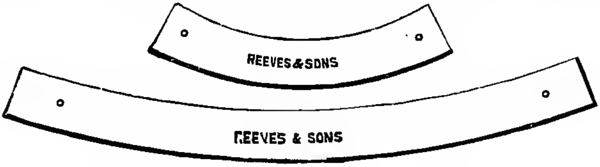


Fig. 323.

French curves are also very useful for drawing irregular curved figures.

Drawing-pens.—A survey should be distinguished by good draughtsmanship, equally with accuracy in execution.

The various boundaries, fences, streams, buildings, &c., should be neatly drawn in ink, for which a good drawing- or ruling-pen is indispensable; and the survey-lines—the basis of the whole work—require to be drawn with a clear but fine line.

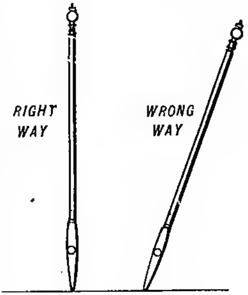


Fig. 324.

A good drawing-pen will with care last for years. I have one of Swiss make that I had in 1862, and I am in the habit of using it at the present time. Much depends upon the way in which a pen is used and the care that is taken of it. Fig. 324 illustrates the right and wrong way of holding a drawing-pen. In the former case not only do you wear the point equally, but you have perfect command over the pen, whilst in the latter you wear the points at one angle, and you cannot manipulate the pen with the same facility or neatness as if held vertical. The various types of drawing-pens are shown in Fig. 325. A is the ordinary pen; B has a hinged nib *a* which enables it to be cleaned better than A, and also is easier to sharpen; C is a double or road pen, its chief advantage being assumed to be the possibility of drawing lines straight or curved parallel to each other at one stroke. But I am bound to confess that I have only used one upon one single

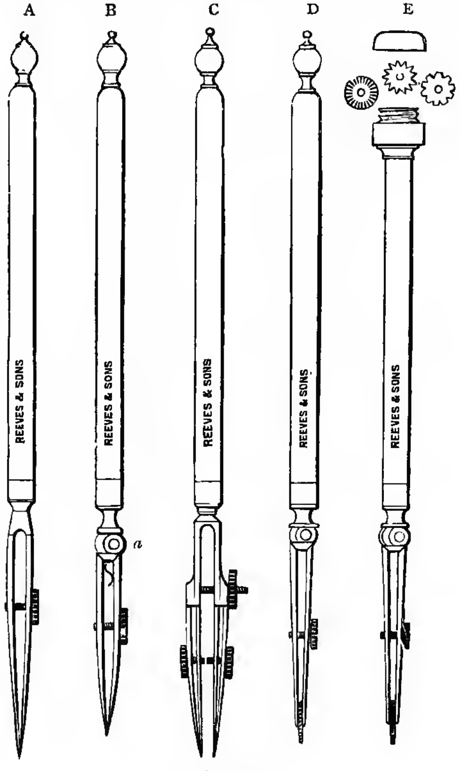


Fig. 325.

occasion, and found it to be not only a great nuisance but such a heavy tax upon my equanimity, that I have not tried one since. An instrument-maker would strongly recommend it; I don't. D and E are dotting- or wheel-pens, the latter of which has at the head a small receptacle for wheels of different lengths of dot. These instruments are neat as pieces of workmanship, but, without great care, are apt to make a smeared instead of a dotted line. If you are the draughtsman you should be—and there is no possible excuse why you should not—you can draw parallel and dotted lines far more neatly and effectively without such contrivances than you can with them.

Dividers.—Fig. 326 illustrates the usual form of dividers. A is the ordinary sector type, as is B, only with double joints, which for

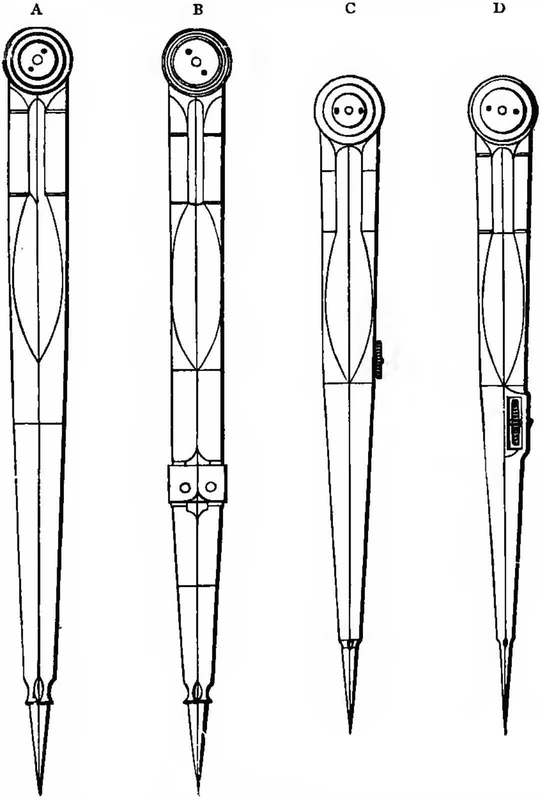


Fig. 326.

purposes required in plotting and surveying are not to be recommended, as even with the best instruments their joints in time get loose. C and D are hair-dividers, with outside and inside screws respectively. These instruments will be found exceedingly useful for accurate measurements. And let me here warn the student against applying the points of the dividers upon the scale for the purpose of measuring on a plan; it is wrong and slovenly, and spoils the scales. Mark off the distance you require on paper, and apply your dividers thereto.

Spring-bows.—Needle spring-bows (Fig. 327) are indispensable for plotting a survey: the other kind make too large holes in the paper.

The equipment of a surveyor would be quite incomplete without a set of ordinary drawing instruments such as is shown in Fig. 328. A is the ordinary cheek compass; at *a*, the point may be removed, and in the slot may be substituted either the pencil or ink point B, or if the sweep is not sufficiently long a lengthening bow may be made to intervene.

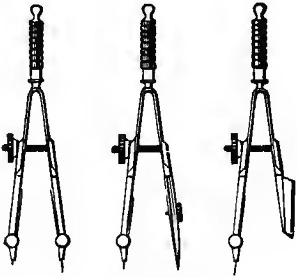


Fig. 327.

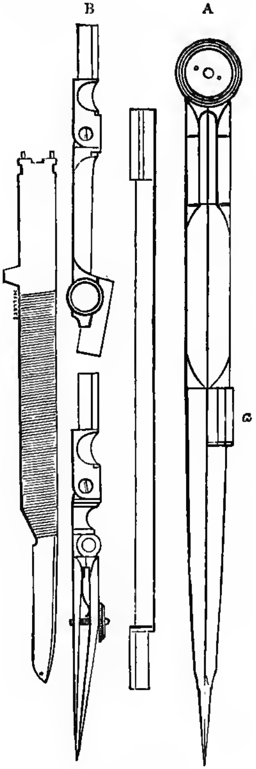


Fig. 328.

Proportional Compasses. — For enlarging and reducing plans, of which I shall have something to say presently, I now mention the proportional compass, of which Figs. 329 and 330 are illustrations—the former when closed, the latter when open for use. On the one face of the divider (as in Fig. 330), on the left of the groove, is a scale of lines, whilst 'on the right side is one of circles; and on the other face (see Fig. 329), on the left side of the groove is a scale of plans and on the right one of solids.

To set the instrument, it must first be accurately closed (as in Fig. 329), so that the two legs appear but as one; the nut *c* being then unscrewed, the slider may be moved, until the line across it coincides with any required division upon any one of the scales. Now tighten the screws and the compasses are set.

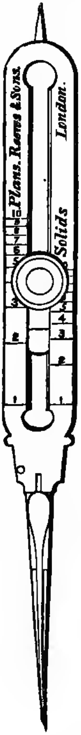


Fig. 329.

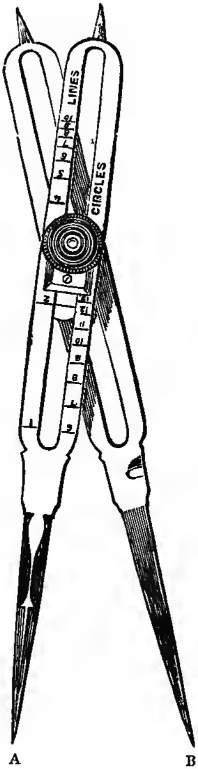


Fig. 330.

To use the Proportional Compasses.—To enlarge or reduce a plan, once, twice, thrice, or up to ten times, bring the line on the slider, opposite the scale of lines to a mark represented by 2, 3, 4, 5, 6, 7, 8, 9, or 10, and at the short end you will have that much less than the other, and *vice versâ.* But of this I shall say more presently.

Horn Centre.—A thin transparent disk of horn about $\frac{9}{16}$ in. in diameter, having three small needle-points to keep it steady. Placed over the centre from which an arc is to be struck with compasses, it prevents their point from making a hole in the paper.

India Rubber.—This useful aid to erasure should be resorted to as little as possible, for good work and workmanship should not require to be obliterated. Yet, if it is necessary at times—and it must be, of course—the best kind is the soft white vulcanized rubber; only use it gently, taking care not to damage the surface of the paper, or you will regret it when you commence putting the tints on your plan.

Indian Ink.—For all purposes of draughtsmanship the best is the only ink to be used, and the extra cost of good quality, as compared with that of inferior, is so slight as to be hardly worth discussing. Indian ink should be used quite fresh each day, and should be kept covered up. To mix it properly, place sufficient water in the saucer, and rub the ink round until it adheres to the sides. Never use either a brush or a pen for filling the drawing-pen, but dip the nib gently into the ink, and with a piece of wash-leather rub off the superfluous.

For mixing up indian ink or any large quantity of colour, the nest of saucers (Fig. 331) is most useful as fitting one on the other. They virtually keep the colour hermetically sealed. For colouring plans in great variety the round slant and basin (Fig. 332) is extremely useful, as you may have occasion to wash your brush frequently, whilst for ordinary variety of tints the ordinary straight slant (Fig. 333) is convenient.

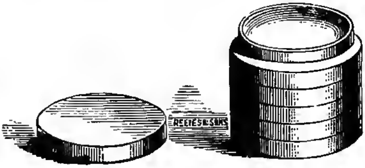


Fig. 331.

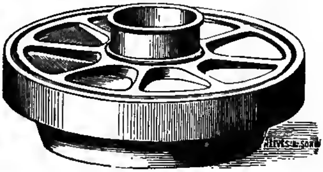


Fig. 332.

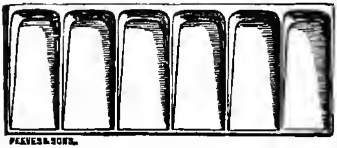

REEVES & SONS.

Fig. 333.

Colours.—For colouring plans, I prefer the cake to the pans, as in mixture you get a better tint without risk of foreign matter getting in, which can hardly be avoided by using a brush with the pans. Of course, in the case of mixture, each colour must be separately rubbed up, and the incorporation must take place afterwards.

The following is a list of the chief colours required by the surveyor :—

Brown Madder
Burnt Sienna
,, Umber
Carmine
Chinese White
Cobalt Blue
Crimson Lake
Chrome Yellow
Emerald Green
French Blue
Gamboge
Hooker's Green
Indian Red
,, Yellow
Indigo
Neutral Tint
Payne's Grey
Prussian Blue
Raw Sienna
,, Umber
Scarlet Lake
Sepia
Vandyke Brown
Venetian Red
Vermilion
Yellow Ochre
Ultramarine

Conventional Signs and Colours.—The following are some of the conventional colours used to illustrate the principal features of a survey. Fences are shown by a firm line; post and rail thus:

—|—|—|—|—; walls by parallel lines; paled fences thus: — — — — — — — —.

Roads are tinted in light burnt sienna. Footpaths of macadamised roads by a darker tint of the same colour. Pavements by neutral tint.

Buildings are variously tinted lake, whilst outbuildings are shown by light indian ink. In some cases existing buildings are shown by neutral tint or light indian ink, whilst new or proposed buildings are tinted lake. Churches or public buildings are generally delineated by some special method, such as hatching.

Water is shown by Prussian blue or ultramarine. There are various ways of doing it, the most effective being by what is termed rippling; or it may be coloured dark at the edge, and led off by a fairly dry brush, called shading. Trees are either sketched in indian ink or are coloured. Pasture-land is tinted green, or if uncoloured is marked *Pas.*, in distinction to *Ara.* for arable land. Marsh-land and heath or gorse are shown as on page 159.

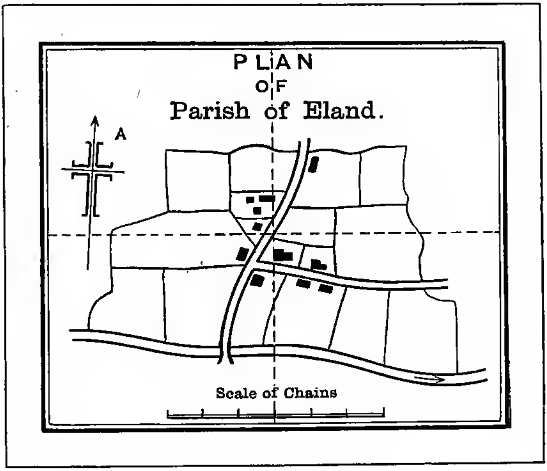


Fig. 334.

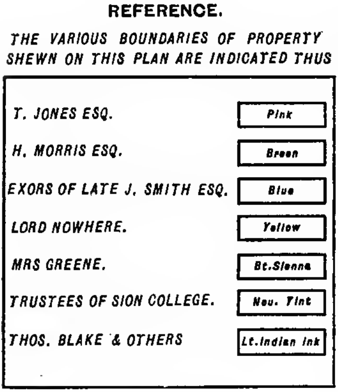
All buildings when inked-in and coloured should be back-lined on the right-hand side and bottom, bearing in mind that light falls over the left shoulder at an angle of 45 deg. And here let me say that, if possible, a plan should not be coloured for at least twenty-four hours after it has been inked-in, as a preventive against the ink running.

The addition of a few drops of photographer's solution of bichromate of potash, well mixed with the indian ink after rubbing up, has been found to set it so quickly that colour may be used within a very short time after it dries. This expedient may be of considerable service to the draughtsman when time is of much importance; but if adopted, care must be taken to frequently change the ink in the drawing-pen, as this soon thickens and interferes with a free flow.

Commence Inking-in from Top.—In commencing to ink-in a plan, it is always best to begin working from top to bottom, taking care to keep the lower part well covered over, so as to prevent dirt or grease getting on the paper.

Always work from Left to Right.—In all operations, field or office, it will be found most convenient to work from left to right, and in all cases the top and bottom, left and right sides of the paper, should represent north, south, west, and east.

Place Work in Centre of the Paper.—Great care should be taken so that the plan is in the centre of the paper, from the sides, leaving as much space as possible for the title, which should always be at the top, and should any of the ground be irregular in shape, as in Fig. 334 at A, it is as well to place the north point in such a spot as will keep the plan symmetrical.



REFERENCE.

THE VARIOUS BOUNDARIES OF PROPERTY SHEWN ON THIS PLAN ARE INDICATED THUS

T. JONES ESQ.	Pink
H. MORRIS ESQ.	Green
EXORS OF LATE J. SMITH ESQ.	Blue
LORD NOWHERE.	Yellow
MRS GREENE.	Bt.Sienna
TRUSTEES OF SION COLLEGE.	Neu. Tint
THOS. BLAKE & OTHERS	Lt.Indian Ink

Fig. 335.

Boundaries of Different Properties. — Boundaries of different property may be shown by an edging of different colours; if for one only, lake or green is most usual; but when there are a variety of owners, the boundaries are generally indicated by lake, green, blue, yellow, burnt sienna, neutral tint, light indian ink, with a schedule of colours as reference in the corner, as in Fig. 335. And where I have written the name of the colour it should be tinted in the block to correspond with the edging of the boundaries.

Paint Brushes and Pencils.—With regard to paint brushes, or pencils, as they are properly called, I need hardly say that the best are the cheapest, and if taken care of will last a lifetime. To leave brushes in water, or to neglect to cleanse them after use, is unpardonable.

Precautions in Colouring.—In colouring take care to mix sufficient, never mix more than is wanted, but a less quantity makes it sometimes difficult to match. Colours should be mixed light, as, if the tints are not dark enough, they can be easily strengthened by an extra coat, whereby blotched colouring is avoided. It is best to colour towards you, taking care not to go over the same place a second time if possible; the colour in parts wants to be floated towards the draughtsman. Do not take too much colour in your brush, and always have a small clean brush handy to finish-off an edge. It is most convenient to have a piece of clean white blotting-paper to rest the wrist on when colouring, also to take up colour that oversteps the boundary. Be very careful not to go over the edge, as it makes a plan look very ragged. Colouring is best done by a slow and regular stroke, extra care being observed at boundaries. For shading, a brush at each end of the handle is requisite, the one to put the colour on, and the other clean and slightly moistened to lead off the colour. The process is best done from left to right. Sable brushes are preferable to camel's hair.

North Points.—North points are shown in various ways, some ornamental and others quite plain, of which types are here given. I have seen the plans of noblemen's and gentlemen's estates got up with such elaboration that they were almost pictures. For instance, the north points were painted to represent lilies of

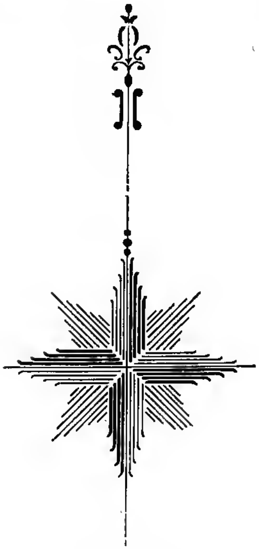


Fig. 336.

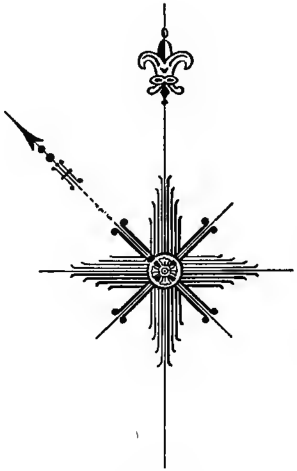


Fig. 337.

the valley and other beautiful flowers, evidence of the artistic skill of the draughtsman; but the practical surveyor of to-day has no time or inclination to adorn his plans with out-of-place decorations, and I recommend the adoption of a neat and simple figure, such as the examples in Figs. 336, 337, and 338. In all cases the magnetic north should be shown by a dotted line.

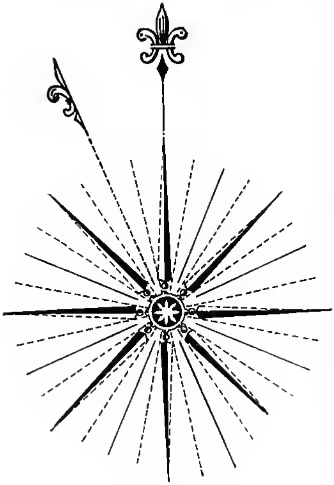
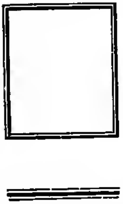


Fig. 338.

Borders.—Every plan should have a border round it, with a margin of from 1 to 2½ inches. A simple line is very neat for an ordinary plan, and where greater elaboration is necessary, then either a thin line on the top and left, with a thick line bottom and right, as in Fig. 339, or as in Fig. 340, with a thick line in the midst of two fine lines. Sometimes a very fine and large plan, the size of which say is 16 feet square, will bear a



Figs. 339 and 340.

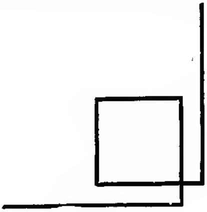
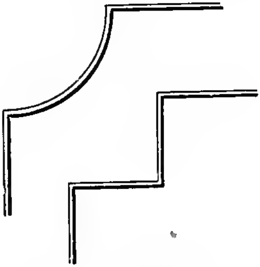


Fig. 341.



Figs. 342 and 343.

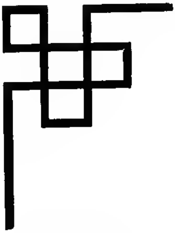


Fig. 344.

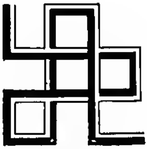


Fig. 345.

line of neutral-tint, say three-eighths thick, and strongly black-lined in indian ink.

Some plans are finished with ornamental corners, such as are shown in Figs. 341, 342, 343, 344, and 345, which are as simple and effective as possible; for I need hardly say that a good survey does not require much adornment, and the neater it is finished off the better it will commend itself.

Printing and Writing on Plans.—One of the last and most important things in connection with a plan is the writing, to which too much attention cannot be paid. For a plan may be perfect so far as draughtsmanship and colouring are concerned, but entirely spoilt by reason of bad writing. Here again simplicity should govern the work. There is nothing neater than block letter, either vertical or on the slant, but with a very little extra time the letters may be made effective by using tints. Now there is a strong prevailing idea that any kind of printing will do on a plan, and a great fancy is expressed for stencil-plates. This is decidedly wrong, as the neater the writing the more effective the plan. Stencil-plates are convenient for marking sacks or the address of *voyageurs* upon those clean deal boxes one sees outside the trunk manufacturer's, but in the drawing office (except of course where work is done at so much an hour) they are out of place.

The title of a plan should be carefully set out from a centre line, and the letters, especially the large ones, pencilled faintly, for which the template, Fig. 346, will be found very useful, giving as it does the angle of the slanting portions of the various letters.

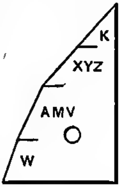


Fig. 346.

Scales.—The best kind of scales for plotting are divided into chains and tens of links on one side, and equivalent feet on the other, so that the mark of two chains would be 132 feet on the feet scale, and the same applies to the offset scale.

I do not suppose the scale-maker could offer any other explanation why 2-chain, 3-chain, and other such scales should be marked 20, 30, 40, &c. True it is they are sometimes used by engineers to plot work to 20, 30, 40 feet to an inch, but it is well to bear in mind that the scales marked 10, 20, 30, 40, 50, and 60 are really 1, 2, 3, 4, 5, and 6 chains to an inch, and the subdivisions are each ten links, and equally on the "feet" side, the 1, 2, 3, 4, &c., represent 100, 200, 300, 400, &c., feet, the greater subdivisions 10 and the lesser 5 feet each.

Enlarging and reducing Plans.—It is often necessary to enlarge or reduce either whole or portions of surveys. For reliable purposes, the most satisfactory method is to replot the work to a larger or smaller scale from your field notes. But this may not always be possible, consequently in these days of "labour saving," we have appliances for expeditiously accomplishing these results. As this work would be incomplete without a description of the pantagraph and eidograph, I have elected to quote from an excellent authority upon the subject *—an author to whom I have already referred (*ante*, pp. 73, &c.). But although I do so, it must not be inferred that I entirely approve of either instrument, against the use of which I have somewhat of a prejudice, added to which I do not consider their great cost always justifies their adoption.

Pantagraph.—"The Pantagraph (Fig. 347) consists of four rulers, A B, A C, D F, and E F, made of stout brass. The two longer rulers, A B and A C, are connected together by, and have a motion round, a centre at A. The two shorter rulers are connected in like manner with each other at F, and with the longer rulers at D and E; and, being equal in length to the portions A D and A E of the longer rulers, form with them an accurate parallelogram, A D F E, in every position of the instrument. Several ivory castors support the machine parallel to the paper, and allow it to move freely over it in all directions. The arms, A B and D F, are graduated and marked $\frac{1}{2}$, $\frac{1}{3}$, &c., and have each a sliding index, which can be fixed at any of the divisions by a milled-headed clamping-screw, seen in the engraving. The sliding indices have each of them a tube, adapted either to slide on a pin rising from a heavy circular weight

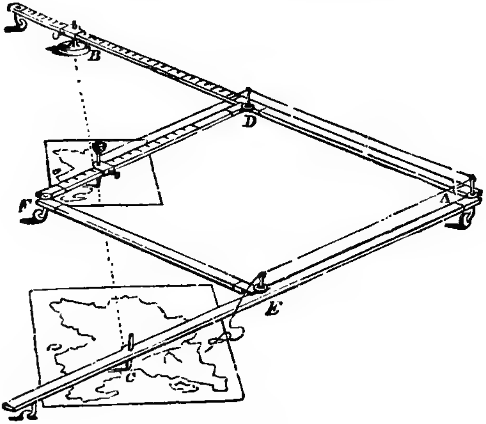


Fig. 347.

---

* "Drawing and Measuring Instruments," p. 65, by J. F. Heather, M.A. Crosby Lockwood & Son, London.

called the fulcrum, or to receive a sliding holder with a pencil or pen, or a blunt tracing-point, as may be required.

"When the instrument is correctly set, the tracing-point, pencil, and fulcrum will be in one straight line, as shown by the dotted line in the figure, and which may be proved by stretching a fine string over them. The motions of the tracing-point and pencil are then each compounded of two circular motions, one about the fulcrum, and the other about the joints at the ends of the rulers upon which they are respectively placed. The radii of these motions form sides about equal angles of two similar triangles, of which the straight line B C, passing through the tracing-point, pencil, and fulcrum, forms the third side.

"The distances passed over by the tracing-point and pencil, in consequence of either of these motions, have then the same ratio, and, therefore, the distances passed over in consequence of the combination of the two motions have also the same ratio, which is that indicated by the setting of the instrument.

"Our engraving (Fig. 347) represents the pantagraph in the act of reducing a plan to a scale of half the original. For this purpose the sliding indices are first clamped at the divisions upon the arm marked ½; the tracing-point is then fixed in a socket at C, over the original drawing; the pencil is next placed in the tube of the sliding index upon the ruler D F, over the paper to receive the copy; and the fulcrum is fixed to that at B, upon the ruler A B. The machine being now ready for use, if the tracing-point at C be passed delicately and steadily over every line of the plan, a true copy, but of one-half the scale of the original, will be marked by the pencil on the paper beneath it. The fine thread represented as passing from the pencil quite round the instrument to the tracing-point at C, enables the draughtsman at the tracing-point to raise the pencil from the paper, whilst he passes the tracer from one part of the original to another, and thus to prevent false lines from being made on the copy. The pencil-holder is surmounted by a cup, into which sand or shot may be put, to press the pencil more heavily on the paper, when found necessary.

"If the object were to enlarge the drawing to double its first scale, then the tracer must be placed upon the arm D F, and the pencil at C; and if a copy were required of the same scale as the original, then, the sliding indices still remaining at the same divisions upon D F and A B, the fulcrum must take the middle station, and the pencil and tracing-point those on the exterior arms, A B and A C, of the instrument."

The Eidograph.*—"The pantagraph just described requires four supports upon the paper, and from this cause, and from its numerous joints, its action is apt to be unsteady. An instrument to avoid these defects was invented by Professor Wallace in 1821.

---

* Heather's "Drawing and Measuring Instruments," p. 70.

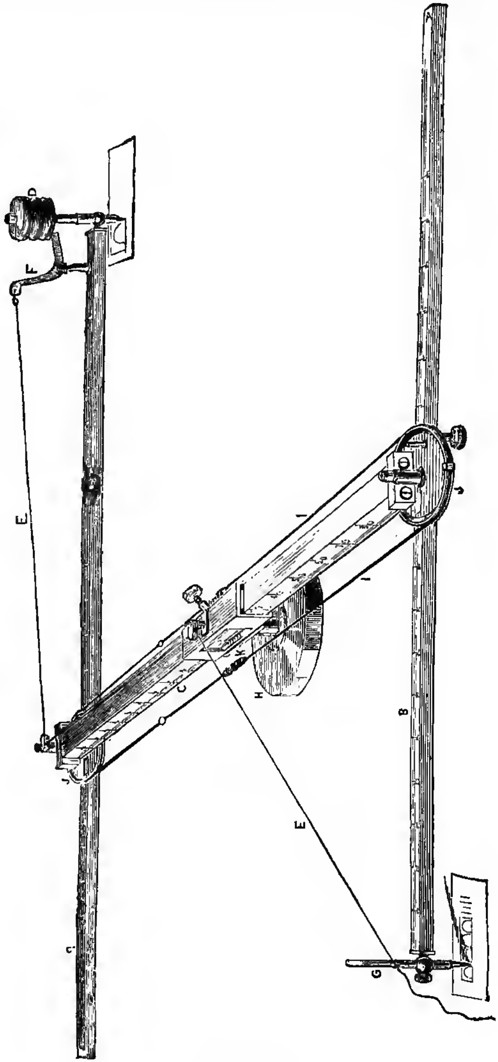


Fig. 348.

This instrument (Fig. 348), called the eidograph, is more regular in its action than the pantagraph, as will be readily understood from the following description of its construction, by which it will be seen that there is only one point of support upon which the entire instrument moves steadily and regularly; and the joints, if we may so term them, consist of fulcrums fitting in accurately ground bearings, the motion round these fulcrums being capable of adjustment for regularity as well as accuracy. It also possesses the further advantage over the pantagraph, that it may be set with equal facility to form a reduced copy bearing any proportion whatever to the original, while the pantagraph can only be set to vary the relations between the original and the copy in the few proportions which are specifically marked upon it.

"The point of support of the eidograph is a heavy weight, H, formed exteriorly of brass and loaded internally with lead, and having three or four small needle-points to keep it steady on the paper. The pin, forming the fulcrum upon which the whole instrument moves, projects from the centre of this weight on its upper side, and fits into a socket attached to a sliding-box, K. The fulcrums are ground to fit very accurately. The centre beam, C, of the instrument fits into and slides through the box K, and may thus be adjusted to any desired position with respect to the fulcrum, and then fixed by a clamping-screw attached to the box. Deep sockets are attached to each end of the centre beam, into which are accurately fitted the centre pins of the two pulley-wheels J J. These pulley-wheels are made most exactly of the same diameter, and have two steel bands, I I, attached to their circumference, so that they can move only simultaneously, and to exactly the same amount. By means of screw adjustments these bands can have their lengths regulated so as to bring the arms of the instrument into exact parallelism, and, at the same time, to bring them to such a degree of tension as shall give to the motions of the arms the required steadiness, which forms one of the advantages of the instrument over the pantagraph. The arms, A and B, of the instrument pass through sliding boxes upon the under side of the pulley-wheels, these boxes, like that for the centre beam, being fitted with clamping-screws, by which the arms can be fixed in any desired position. At the end of one of the arms is fixed a socket with clamping-screw, to carry a tracing-point, G, and at the end of the other is a socket for a loaded pencil, D, which may be raised when required by a lever, F F, attached to a cord which passes over the centre of the instrument to the tracing-point. The centre beam C, and the arms, A, B, are made of square brass tubes, divided exactly alike into two hundred equal parts, and figured so as to read one hundred each way from their centres, and the boxes through which they slide have verniers, by means of which these divisions may be subdivided into ten, so that with their help the arms and beam may be set to any reading containing not more than three places of figures. A loose leaden weight is supplied with the instrument to fit on any part of the centre beam, and keep it in even balance when set with unequal lengths of the centre beam on each side of the fulcrum.

"The pulleys, J J, being of exactly equal size, when the steel bands I I are adjusted so as to bring the arms of the instrument into exact parallelism, they will remain parallel throughout all the movements of the pulleys in their sockets, and thus will always make equal angles with the centre beam. If, then, the two arms and the centre beam be all set so that the readings of their divisions are the same, a line drawn from the end of one arm across the fulcrum to the end of the other arm will form with the beam and arms two triangles, having their sides about equal angles proportionals, and being, therefore, similar; hence any motion communicated to the end of one arm will produce a similar motion at the end of the other, so that the tracing-point being moved over any figure whatever, an exactly similar figure will be described by the pencil."

To adjust the Eidograph, and examine its Accuracy.—"Set the indices of all three verniers to coincide with the zero divisions on the centre beam and arms, and make marks at the same time with the tracer and with the pencil; then move the pencil-point round until it comes to the mark made by the tracer, and if the tracer at the same moment comes into coincidence with the mark made by the pencil, the arms are already parallel, and the instrument consequently in adjustment; but if not, make a second mark with the tracer in its present position, and bisecting the distance between this mark and the mark made by the pencil, bring the tracer exactly to this bisection by turning the adjusting screws on the bands. The instrument being now in adjustment, if the zero division be correctly placed on the arms and beam, the pencil-point, tracer, and fulcrum will be in the same straight line, and they will still remain so when the instrument is set to give the same readings on the three scales, whatever those readings may be, if the dividing of the instrument be perfect.

"The instrument being adjusted we have next to set it so as to make the dimensions of a copy, traced by its means, bear the desired proportion to the original. It must be borne in mind that the divisions on the instrument are numbered each way from the centres of the beam and arms up to 100, and that the verniers enable us to read decimals or tenths of a division; so that if the indices of the verniers were a little beyond any divisions, as 26, and the third stroke of the verniers coincided with the divisions marked 29, the reading would be 26·3. Now suppose it were required to set the instrument so that the proportion of the copy to the original should be that of one number, a, to another number, b. Suppose x to represent the reading to which the instrument should be set, then the centre beam and arms are each divided at their fulcrums into portions whose lengths are $100 - x$ and $100 + x$ respectively, and consequently $\frac{100 - x}{100 + x} = \frac{a}{b}$, from which we find that the required reading $x = \frac{100\,(b - a)}{b + a}$; thus if the proportions are as 1 to 2, we have $x = \frac{100\ (2 - 1)}{2 + 1} = \frac{100}{3} = 33{\cdot}3$, and the instrument must be set with the third divisions of the verniers beyond the indices on the third divisions of the instrument beyond the 33rd. We have, therefore, the following simple rule: Subtract the lesser term of the proportion from the greater, and multiply it by 100 for a dividend, add together the two terms of the proportion for a divisor, and the quotient will give the reading to which the instrument is to be set.

"The following readings are thus obtained:—

Proportions.	Readings.	Proportions.	Readings.
1 : 2	33·3	2 : 3	20
1 : 3	50	2 : 5	42·9
1 : 4	60	3 : 4	14·3
1 : 5	66·7	3 : 5	25
1 : 6	71·4	4 : 5	11·1

"When the copy is to be reduced, the centre beam is to be set to the reading found, as above, on the side of the zero next to the arm carrying the pencil-point, and this arm is also to be set to the same reading on the side of its centre or zero nearest the pencil-end, while the tracer-arm is to be set with the reading furthest from the tracer. When the copy is to be enlarged, these arrangements must of course be reversed: thus 50 being the reading for the proportion 1 : 3, Fig. 349 will represent the setting to make a copy having its linear dimensions three times those of the original; where p represents the position of the pencil-point, t that of the tracer, and F the place of the fulcrum. Fig. 350 represents in the

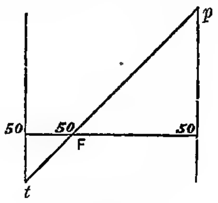


Fig. 349.

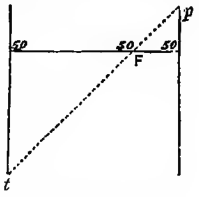


Fig. 350.

same way the setting to make the linear dimensions of the copy one-third of those of the original."

Enlarging and reducing by Squares.—Failing the replotting of the work for the purpose, the only satisfactory and accurate method of enlarging and reducing plans is by means of squares and proportional compasses. This method is illustrated by the following example:—

Let Fig. 351 represent the plan of an estate which it is required to copy on a reduced scale of one-half. The copy will therefore be half the length and half the breadth, and consequently will occupy but one-fourth of the space of the original. Take a sheet of tracing-paper and draw two lines at perfect right angles to each other, as O J, O 9, at the top and left of the sheet; now very accurately and carefully divide these lines into spaces of some convenient length, say, $1\frac{1}{2}$ to 2 ins., as *a*, *b*, *c*, *d*, *e*, *f*, *g*, &c., and 1, 2, 3, 4, 5, 6, &c., and draw the squares formed by the intersections in fine blue lines. Now place this piece of tracing-paper over the plan to be enlarged or reduced, and fasten it well down with drawing-pins. Then take another piece of tracing-paper and divide it into squares larger or smaller according to the proportions required: in Fig. 352 they are half the size, consequently whatever the divisions O *a*, *o b*, O 1, O 2, &c., are (Fig. 351), those in Fig. 352 will be half. Beside the plan to be reduced, on the right-hand side lay down a piece of drawing-paper, upon which shall be laid a piece of transfer-paper, and upon this is laid the sheet of smaller squares, all of which are then firmly secured by weight or drawing-pins. In the proportional compasses fix the line across the slides to be coincident with the line opposite the 2 on the left side of the groove (Fig. 330), by which means A B is twice C D, to test which

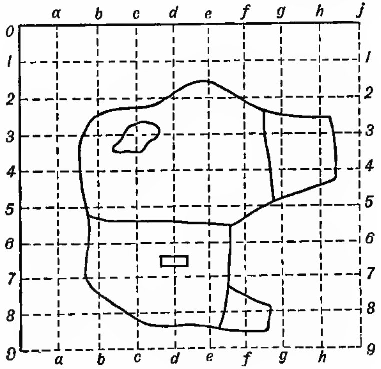


Fig. 351.

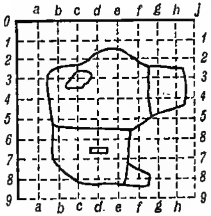


Fig. 352.

upon a line pick off any length A B, then if the points C D accurately bisect this length you have the right proportion. And as a further test, try your squares in the same way, A B being fixed at one of the subdivisions in Fig. 351, then if the sheets of squares have been accurately drawn, C D will exactly measure the length on the reduced sheet of squares. To reduce the plan, mark those points on the large squares where the fences, &c., intersect, and measure vertically and horizontally the distance from the nearest intersection of the horizontal and vertical lines with the A B end of the compass, and at similar points on the small squares mark the same distances with the C D end of the compasses and make marks, then if with a fine pencil you draw the lines connecting these points, you will not only have a record of the work you have accomplished, but it will be transferred to the paper beneath.

Copying a Plan.—To copy a plan it has been recommended to place it over a sheet of clean paper, and to prick-through all the fences, buildings, &c., and then to connect the punctures by drawing the lines first in pencil and then in ink. Such a system is to be condemned: first, because it spoils both the plan and the copy by the prick-marks; secondly, there is a liability of the plan becoming shifted, in which case there is no possibility of readjusting it; and thirdly, it takes just twice the necessary time to accomplish; added to which, there is always a liability of error.

The method I recommend is to make a neat tracing of the plan, and to place this upon transfer-paper over a sheet of drawing-paper. Then place a clean sheet of tracing-paper over the whole, and retrace the plan, by which means you have an accurate record of how much of the work you have accomplished, and no injury is done to the paper upon which the plan is to be copied.

When working over transfer-paper, a clean sheet of stiff enamelled metal, having two knobs for lifting when shifting it, should be laid upon that part of the work whereon the hand or anything else rests; otherwise pressure is apt to set off from the transfer-paper grimy marks on the paper underneath.

General Hints.—In plotting a survey the following hints may be useful:—

1. Dust your table, and well cover that part of the paper upon which you are not working.
2. Do not wear your watch in your waistcoat pocket.
3. Do not have an inkstand or your colour-pans on the same table.
4. Always clean your scales, protractor, set-squares, straight-edge, &c., before use.
5. Rule-in your survey lines in lake or carmine before you commence to plot your details.

6. Always use fresh ink every day, and (if possible) do not colour over work recently inked-in.

7. Before commencing to plot, draw a scale on the paper, and also a north point.

8. Do not make calculations upon slips of paper, but always have a foolscap scribbling-book at hand, in which enter all your calculations and the dates upon which they are made.

9. Keep a separate field-book for each survey, and be careful to enter the dates of each day's work.

CHAPTER XIII.

LAND QUANTITIES

THE surveyor has not performed all his duties when he has plotted and finished his plan, for a matter of the greatest importance next to an accurate survey is to have a true record of the areas of the various properties shown upon the plan.

There are so many works which deal more or less exhaustively with the subject of computation of areas and quantities, that I do not propose to do more than briefly consider the various methods which may be adopted for the purpose, and to endeavour to apply them practically for the information of those who may not have had an opportunity of perusing such books, or to whom possibly the meaning of all that was contained therein has not been made sufficiently clear.

The following are the items of superficial measure of chief importance to the surveyor :—

Sq. Links.	Sq. Feet.	Sq. Yards.	Sq. Perches.	Sq. Chains.	Roods.	Acres.
2·296	1					
20·661	9	1				
625	272¼	30¼	1			
10,000	4,356	484	16	1		
25,000	10,890	1,210	40	2½	1	
100,000	43,560	4,840	160	10	4	1

1 Mile a chain wide = 8 Acres.

1 Square Mile { = 640 Acres.
= 3,097,600 Square Yards.
= 27,878,400 „ Feet.

To convert Acres into Square Miles multiply by 0·0015625.
To convert Square Yards into Square Miles multiply by 0·000000323.

A strip of land 10 chains long and 1 chain wide is 1 acre; 10 chains = 1 furlong; there are 8 furlongs to a mile; and consequently if 10 sq. chains = 1 acre, then 8 furlongs, 1 chain wide, will give the result of 8 acres per lineal mile.

Suppose we have a piece of ground which measures 23¼ chains long and 6½ chains wide, then

$$23{\cdot}25 \times 6{\cdot}5 = 151{\cdot}125 \text{ square chains.}$$

Now if we divide 151·125 by 10 we get 15·1125 acres, the decimal part of which should be multiplied by 4 to reduce it to roods, and the decimal part of the remainder by 40 to reduce it to perches, thus—

A.		A.	R.	P.
15·1125	=	15	0	18.

Averages in Fence Lines.—One of the first things necessary to be perfectly understood is, how to determine the averages of uneven fences or boundaries. I mean that it is simple enough with a piece of ground whose boundaries form a regular figure, such as a square or rectangle; but in practice this is seldom if ever the case, and the fences or boundaries being uneven and irregular, it is necessary to adjust them so that the inequalities may be accounted for. Fig. 353 is a simple illustration of what I mean.

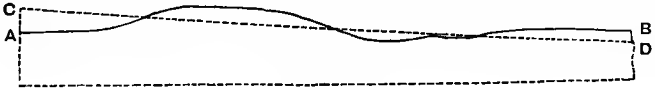


Fig. 353.

The boundary fence A B curves in and out, so that it is necessary to establish a mean line that will represent fairly the average. To do this we resort to what is termed a "give-and-take line," as C D; by which those portions of the ground on the top side of C D are ignored, as their area is considered to be equivalent to that of those portions below the line, which are really out of the property.

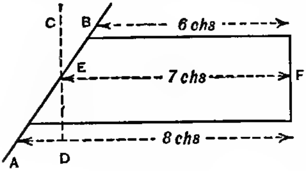


Fig. 354.

The same principles apply in the case of a slanting boundary, whence it is necessary to measure to get the mean length between two parallel boundaries, as in Fig. 354. Here, on the left of the property, is a fence running diagonally, whose length on the top side is 6 chains, and on the bottom side 8 chains. To get the mean length of course we can say $\frac{6+8}{2} = 7$ chains, but in practice a little judgment will enable one to arrive at a fairly accurate result.

By Triangles.—The most simple, and indeed most satisfactory, method of computing areas is by means of triangles. Thus, if upon the plan to be measured a sheet of tracing-paper is spread and securely fastened, then, with a fine pencil, let the whole area be divided into triangles, each of which (beginning at the top) should be consecutively numbered, and at the boundaries let the indentations of the fence be carefully treated on the give-and-take principle. This being done, lines perpendicular to the longest sides of each triangle should be dotted, and these, together with the longest sides, should now be accurately measured, and the dimensions scheduled as in the following example, Fig. 355. Here we have a property—the internal fences being purposely left out—the area of which it is necessary to compute. It will be seen that it has been divided into eleven triangles, the sides of some of which have been arranged so as to "give and take" the inequalities of the boundaries. The dotted lines show the triangulation, whilst

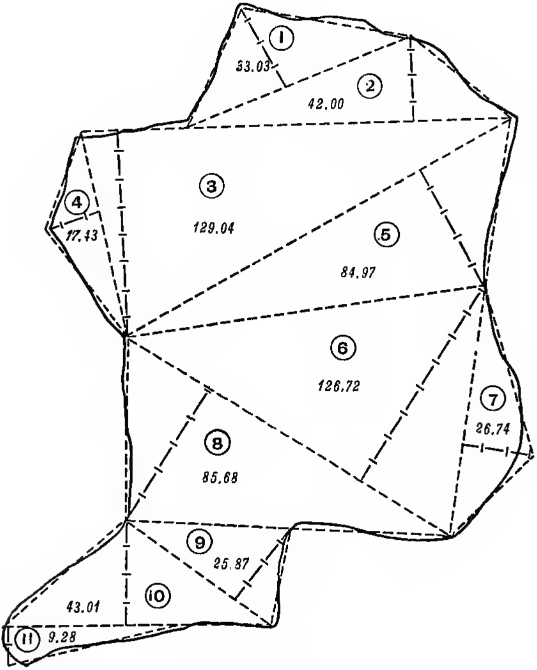


Fig. 355.

the perpendiculars are delineated by a dot and cross-stroke. The following is the schedule:—

Triangle No. 1. 9·05 × 3·65 = 33·03 sq. chains.
" 2. 12·00 × 3·50 = 42·00 "
" 3. 16·05 × 8·04 = 129·04 "
" 4. 8·30 × 2·10 = 17·43 "
" 5. 16·50 × 5·15 = 84·98 "
" 6. 14·40 × 8·80 = 126·72 "
" 7. 9·62 × 2·78 = 26·74 "
" 8. 14·40 × 5·95 = 85·68 "
" 9. 6·90 × 3·75 = 25·88 "
" 10. 9·82 × 4·38 = 43·01 "
" 11. 5·80 × 1·60 = 9·28 "
Divide by 2 and by 10)623·79 "
31·1895 acres.
4
0·758
40
30·32

A. R. P.
Area = 31 0 32.

It is always better to take the measurements in chains and decimals, to multiply them together, and divide the sum of the whole triangles by 2, to get the area.

Another example, Fig. 356, will serve a double purpose, viz. how the area may be determined as readily upon the ground, and without plotting, as from a plan. The figure is somewhat in the form of a boot, and by laying out a large triangle A B E, and another D C B, we are able by triangles to get the area of the greater portion of the field without much trouble. Upon the line A E of the larger triangle set up ordinates *a b*, *c d*, *e f*, *g h*, and *j k*. Then the area of each space 3 to 8 may be obtained as follows:—

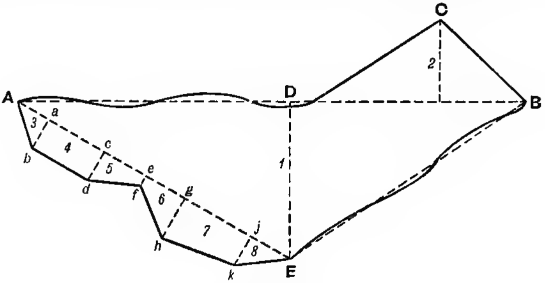


Fig. 356.

3. A *a* = 1·40
× *a b* = 1·20
1·68 area.

4. *a b* = 1·20
+ *c d* = 1·30
2·50
× *a c* = 2·50
6·25 area.

5. *c d* = 1·30
+ *e f* = 0·40
1·70
× *c e* = 1·83
3·111 area.

6. *e f* = 0·40
+ *g h* = 1·80
2·20
× *e g* = 1·60
3·52 area.

7. *g h* = 1·80
+ *j k* = 1·40
3·20
× *g j* = 2·95
9·44 area.

8. *j k* = 1·40
+ *j* E = 1·75
2·45 area.

All the foregoing are double areas, 3 and 8 being triangles, the sides A *a* and *j* E are respectively multiplied by *a b* and *j k*. The areas 4, 5, 6, and 7 have their two ends *added* together, and the sum multiplied by the distance apart. They may be tabulated as follows:—

No. 3 = 1·68
„ 4 = 6·25
„ 5 = 3·111
„ 6 = 3·52
„ 7 = 9·44
„ 8 = 2·45
26·451 sq. chains.
Add double area of No. 1 triangle = 87·120 „
„ „ 2 „ = 25·428 „
Divide by 2 and by 10)138·999
6·94995 acres area.
4
3·7998
40
31·9920

Total area, 6 A. 3 R. 32 P.

The double area of No. 1 triangle is 14·40 × 6·05 = 87·12; and No. 2 is 8·15 × 3·12 = 25·428.

Ascertaining Areas on Ground.—In Fig. 357 is illustrated Simpson's method of computing the area of an irregular piece of ground, either with or without plotting. In this case the line A B should be measured as near as possible in the middle of the plot, and marks should be left in the ground at the end of each chain: lines at right angles should be drawn through these points, and should be measured.

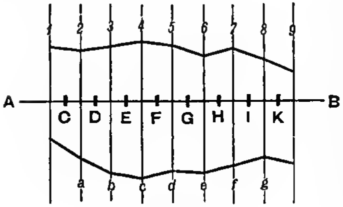


Fig. 357.

The following rule applies in this case:—

1st. The first and last lengths should be added together separately.

2nd. Now add the 2nd, 4th, 6th, and 8th lengths together, and multiply the result by 4.

3rd. Take 3rd, 5th, and 7th lengths, and multiply their sum by 2.

4th. Collect all these sums together, multiply by the common distance, or 100 links, and one-third of the product will be the area.

Example:—

A = 300 links.
B = 300 „
600 „
6,200 „
2,460 „
9,260 „
100
3)926,000 „
308,666 „ = 3 A. 0 R. 13·87 P.—*Ans.*

a 2 = 350 links.
c 4 = 450 „
e 6 = 400 „
g 8 = 350 „
1550 „
4
6200 „

b 3 = 400 links.
d 5 = 430 „
f 7 = 400 „
1230 „
2
2,460 „

Another and simpler way, but at the same time somewhat approximate, is to mark every half-chain, so that an imaginary line through C, D, E, F, G, H, I, K, will give a mean length of the strips, 1 2, 2 3, 3 4, 4 5, 5 6, 6 7, 7 8, 8 9. If these lengths are added together and the result multiplied by 100, we shall have the area, as follows:—

Example C = 325
" D = 375
" E = 425
" F = 440
" G = 415
" H = 400
" I = 375
" K = 325

3075
100

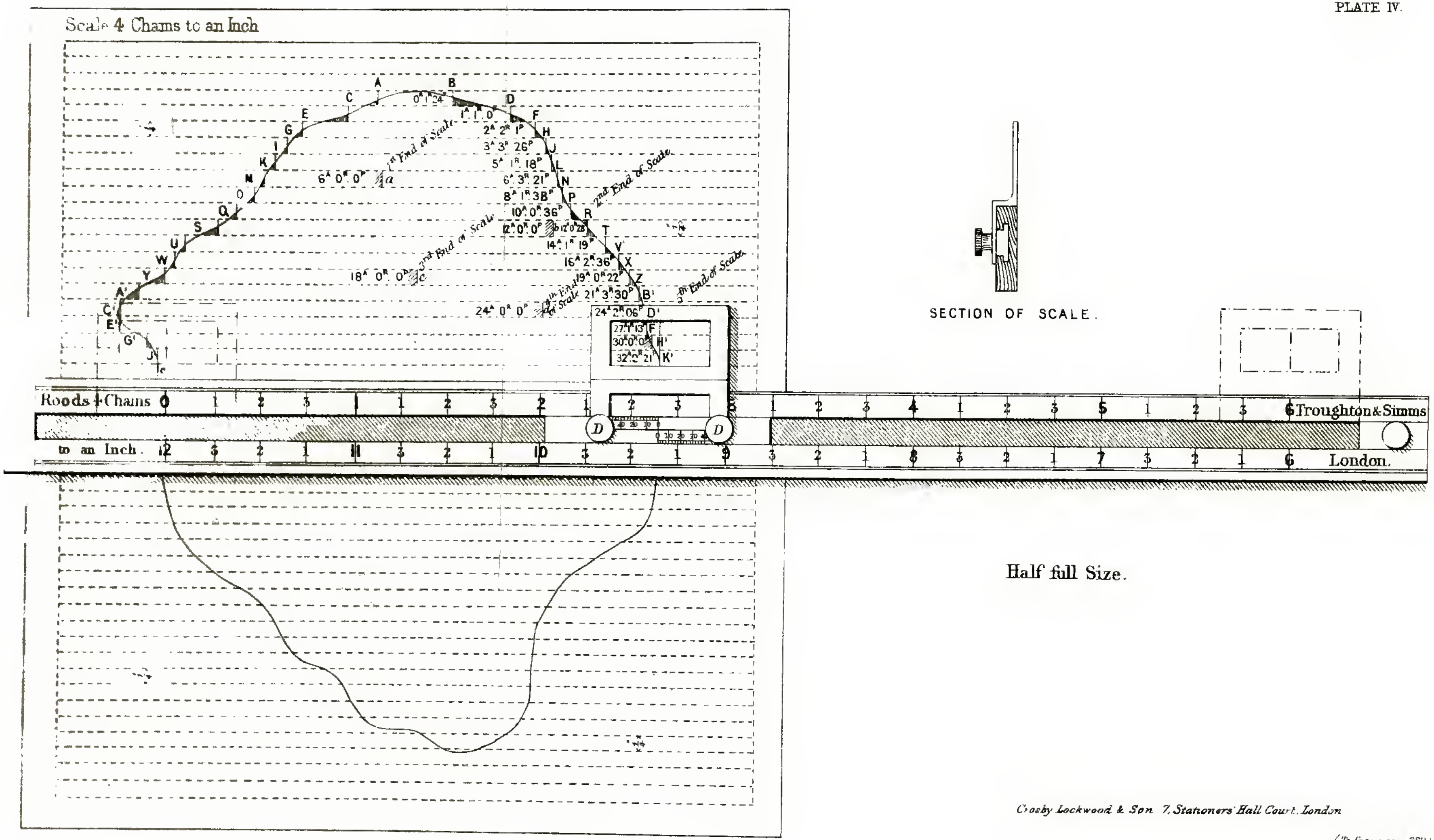
307,500 = 3 A. 0 R. 12 P.—*Ans.*

The slight discordance between this result and that gained in the same example above, shows the necessity of adhering to the previous and more accurate method, although it must be noted that neither of these is so simple or so satisfactory as the method of computing areas by means of triangles.

Computation Scale.—This last example serves as an excellent introduction to the computation scale, for the principles involved are precisely the same. For this, it is customary to prepare a piece of tracing-paper with horizontal lines a certain distance apart, drawn in blue. This distance between the lines is so arranged that a scale divided especially for the purpose, and moved from left to right between any two lines, shall record the area of the strip according to the length traversed. Thus, as a simple illustration, suppose we have spans of one quarter of an inch, and use a scale of four chains to an inch, the span would thus represent one chain. If we apply the scale to the left end of the span, and read ten chains on our scale, we shall have obtained an area of one acre; and supposing we were to measure the whole length of a 12-inch scale, which would give 48 chains, then we should record 4 acres and $8\frac{1}{10}$ths of another acre, or 4 A. 3 R. 8 P.

Now, what is done is to place the sheet of tracing-paper upon the plan to be computed and carefully fasten it down, taking care that one of the parallel lines cuts the most extreme point of the top of the plan; then, as each span will pass through the boundaries of the property, so may the area be computed.

Plate IV. (p. 289) is a practical illustration of the *modus operandi* of ascertaining the acreage by means of the computing scale. It represents a plan of an estate, drawn to a scale of 4 chains to an inch, over which is placed (and fastened down with drawing-pins) a sheet of tracing-paper, upon which have been carefully drawn blue



SECTION OF SCALE.

Half full Size.

Crosby Lockwood & Son 7, Stationers' Hall Court, London

(To face page 280)

are placed, those on the left side of the index to read with the divisions of the scale on the upper side of the groove, and those on the right of the index to read with the divisions on the under side of the groove.

In some scales the frame carries a piece of thin horn on which are ruled two lines parallel to the rule, at a distance apart which represents a chain, and the centre of this enclosure being determined by the intersections of its diagonals, a line L L, called the index line, is drawn through this centre at right angles to the parallel lines, and in the same straight line with the index on the brass tongue. But many scales are made with small holes pierced at L L, through which a piece of fine wire or thread is passed and held tightly in position by means of screws. The scale shown in the plate is arranged on this principle, and the index wire or line is shown to have passed from left to right, from zero to 2 acres and past 2 roods, whilst the index on the tongue records on the left side 21 perches (of course reading from right to left) so that the area of the space between the lines from J′ to K′ is 2 A. 2 R. 21 P. The dotted outline of the index frame on the left shows the position at the commencement, whilst that on the right shows its position at the end of the scale, so that the arm, having only traversed about one-half the length of the scale from J′ to K′, the scale must be carefully taken up and adjusted so that the index line cuts the "give-and-take" line of the next span from L′ to M′, and so on until the full length of the scale has been run. Referring to the plate, it will be seen that the progress of the index frame from A to B was 0 A. 1 R. 24 P., and having been moved to C it reads 1 A. 1 R. 0 P. at D, 2 A. 2 R. 1 P. at F, 3 A. 3 R. 26 P. at H, 5 A. 1 R. 18 P. at J, and we arrive at the extent of the scale before we can reach L, consequently when the index is at 6 A. 0 R. 0 P. as at *a*, we mark the point with a fine pencil-line.

Here I would pause to say that in this, as in all surveying operations, I strongly advocate working *always* from left to right, and consequently I should prefer the lower portion of the scale to be divided from 6 to 12, working left to right, instead of the way in which it is shown. It will be seen that I have used it in this case, as I advocate, instead of retracing our steps from 6 to 12, to do which I have added the readings on the upper scale to 6, 12, 18, 24, and 30 acres as the case has been, so that from *a* to N the scale recorded 0 A. 3 R. 21 P., therefore 6 A. + 0 A. 3 R. 21 P. = 6 A. 3 R. 21 P., and so on until *b*, *c*, *d*, and *e*. Thus, in the position of the scale at J′ K′ we have had *five* changes of *six* acres, and a length from *e* to K′ of 2 A. 2 R. 21 P., or a total area from A B to J′ K′ of 32 A. 2 R. 21 P.

Various Kinds of Computing-scales.—There are numerous types of computing-scales, some of a universal character, and others so constructed that instead of the frame working upon a tongue, the groove is made to receive strips of very thin box-wood, upon which are divided scales of from 1 to 6 chains to an inch, and the various Ordnance scales.

Areas by Different Scales to Plan.—The scale illustrated in the plate is of so simple and reliable a character that it commends itself; and whilst it is desirable, in an office where computation on a large scale is carried on, to have computing-scales of the various scales in vogue, yet it is quite possible to arrive at an accurate estimate of the area of property drawn to a different scale from that of the computer. For instance, suppose we have a plan 5 chains to an inch, the area of which it is desired to ascertain, but our computing-scale is 3 chains to an inch. As an example, we will assume that the operation of computation gives a result of 6 A. 2 R. 0 P. with the scale. Now, as 5 chains to an inch is much smaller than 3 chains, then the area will necessarily be greater, so that if we treat it as a rule-of-three sum we shall get the correct result. In examinations, I regret to say, this question has been a source of trouble and embarrassment to many students, who, even if they are happy in thinking of the proportion, quite forget that it will not be as three to five; but, as they are dealing with areas, it is as the square of three is to the square of five, so is the known area to that required. So that, having the area with the 3 chain scale of 6 A. 2 R. 0 P., we proceed as follows: $3^2 : 5^2 :: 6$ A. 2 R. 0 P. : 18 A. 0 R. 8 P. 26 YDS. 8 FT. = area of the plan drawn to a scale of 5 chains to an inch.

Planimeter.—There is another method of ascertaining the areas of a plan by what is known as the planimeter, invented by J. Amsler, Professor of Mathematics at Schaffhausen. But it is a very delicate instrument, and the slightest dirt or rust will throw it out of gear. "It consists essentially of two arms jointed together, so as to move with perfect freedom in one plane, and a wheel which is attached to one of the arms, and turning on this arm as an axis, records by its revolutions the area of the figure traced out by a point on the arms to which it is attached, while a point on the other arm is made a fixed centre, about which the instrument revolves." For a full description of its various parts, and of the method of using it, I cannot do better than refer the reader to Heather's "Drawing and Measuring Instruments," p. 80.* Like all instruments the object of which is to save labour, the planimeter, from the very delicacy of its construction, has to be used with the greatest care; and for ordinary practice it is hardly advisable to adopt it, on account of its great liability to injury. For myself, I much prefer to take off the quantities of land either by triangles or with a computing-scale.

---

* Crosby Lockwood & Son, London.

CHAPTER XIV.

COLONIAL LICENSING REGULATIONS.

In most of the British Colonies, surveyors enjoy a "Protection" unknown to the profession in the United Kingdom; the practice of surveying being lawful to licensed persons only, licences being granted on condition of passing certain examinations and complying with regulations from time to time promulgated by the various Colonial Governments, the latest dates whereof appear to be as follows:—

New Brunswick, 1877; Manitoba, 1883; Prince Edward Island, 1884; Quebec, 1889; British Columbia, 1897; Natal, 1899; Ontario and Queensland, 1900; New South Wales and New Zealand, 1901; Cape Colony, Orange River, and Southern Rhodesia, 1902; Tasmania, Transvaal, South Australia, Victoria, and Western Australia, 1903.

To qualify a candidate for examination for a licence, the majority of the above Colonies prescribe preliminary conditions: some, preliminary conditions and entrance examination: others one set of conditions and a single examination. In the following list (Section I. relating to the two former cases, and Section II. to the latter, and to the final examination in cases where more than one is prescribed) are set forth, under the titles of the Colonies, the latest particulars of those conditions and of the subjects of examination. The terms of the conditions and examinations for the Australasian Colonies and the Transvaal are compiled from the official documents themselves: those for the other African Colonies, and for Canada, from abstracts given in the Emigrants' Information Office "Professional Handbook" and other sources, and they are less specific in details than the former, though equally correct so far as they go. It may be added, that attainment of the age of 20 or 21 years, and production of good-character certificates, are in most cases stipulated for: these items are not here inserted.

In cases where there appears to exist a distinction between Ordinary land-surveyors and "Government," "Departmental," or "Dominion" land-surveyors, it is to the former only that the present chapter relates; but the distinction (if any) is not always clearly expressed nor easy to make out.

It will be seen that, in all save one Colony, the conditions are more or less modified in favour of candidates in respect of their professional or other status. Cape Colony stands as the sole exception, expressly declaring that "Admission as surveyors in Cape Colony is granted only to those who have qualified themselves for this profession in the Cape Colony, no foreign diploma or certificate being recognised."

I.—PRELIMINARY CONDITIONS AND EXAMINATION.

CANADA.

British Columbia.—No person may be admitted as an articled pupil to a surveyor unless he has passed an examination in British Columbia in penmanship, orthography, arithmetic, algebra, Euclid, plane trigonometry, and use of logarithms. He must then serve for three years with a surveyor in British Columbia, or for one year only if (1) he has served three years elsewhere, or (2) has had two years' course of study in surveying in any college or university.

Manitoba.—An ordinary candidate must pass an examination in English and elementary mathematics, and then be articled to a surveyor in the province for three years.

Any one who has after two years' study in any college or university received a diploma of competency in mathematics and surveying; or who is a duly licensed surveyor of Great Britain and Ireland and has served three years with a surveyor there—need serve only one year with a surveyor in Manitoba.

Persons who have served with a Dominion land-surveyor are entitled to go up for the final examination.

New Brunswick.—Certificate from some licensed surveyor that the candidate has had three months' practice in actual field-work.

Nova Scotia.—Certain [unspecified] qualified civil engineers must serve for a year in Canada before going up for final examination.

A qualified surveyor of the United Kingdom must serve in Canada for a year, and must pass the prescribed examinations.

For persons without qualification, an entrance examination, to be followed by a three years' service under articles.

The subjects for the entrance examination include penmanship and orthography, arithmetic, algebra, plane geometry, plane and spherical trigonometry, mensuration of superficies, and the use of logarithms.

Ontario.—A candidate must have served three years as apprentice to a surveyor in Ontario; and no person is admitted as an apprentice unless he has passed an examination in orthography, certain [unspecified] parts of mathematics, geography, and Canadian history.

Any one who has been practising as a surveyor elsewhere in His Majesty's dominions where the qualifications are similar to those required in Ontario, need not serve as an apprentice, or only for a certain [unspecified] limited time, before undergoing the final examination or such portions of it as the Board thinks necessary.

Graduates of the Royal Military College, Kingston, or of the Ontario School of Practical Science, or of the McGill College, Montreal, or of the School of Mining, Kingston, in civil engineering or mining engineering, have the privilege of a shortened term of apprenticeship; and they are also exempted from the apprenticeship examination.

Prince Edward Island.—None specified; the imposition of preliminary examination upon a candidate being in the discretion of the Board of Examiners and depending upon circumstances. A candidate producing, from a well-known College or University, certificates of his experience in surveying and thorough acquaintance with the use of the several instruments, would probably be exempted.

From the wording of Sect. 7 *of the Colonial Act* 47 *Vict. cap.* 5, *it would appear that a person who has been in the actual practice of a land-surveyor for a period of at least five years before the passing of that Act; or who is qualified under the provisions of a statute of the Canadian Parliament, is ipso facto qualified to practise.*

Quebec.—No person is admitted to study surveying until he has passed an examination in English and French, general geography and that of Canada in particular, history of Canada, arithmetic, elements of geometry, use of logarithms, and algebra up to and including quadratic equations. A candidate must have served for three years under a surveyor of the province, and had one year's practice in the field.

A candidate who has studied the above subjects for two years in some University or College in the province; or if he belongs to the Province of Quebec and has followed a complete course oı study at the Royal Military College, and has received from such University or College a diploma as civil engineer or surveyor; may be received as a student by a surveyor, and one year's service with him, instead of three, is sufficient.

AUSTRALASIA.

New South Wales.—(*a*) A candidate shall have served under Articles, or other similar agreement, with some qualified surveyor * or surveyors, for a period of four years, three of which must have been in the field; and shall produce satisfactory evidence of having received such education as will qualify him for the proper exercise of his profession.

(*b*) Or he shall have passed at any University recognized by an Australasian University, the matriculation examination, including English, geography, arithmetic, geometry, algebra and physics, or such other examination as shall, in the opinion of the Board, be equivalent thereto; and shall have been professionally employed with some qualified surveyor or surveyors for not less than three years, two of which must have been in the field.

(*c*) Or shall have taken the degree of Bachelor in Engineering at any University recognized by an Australasian University, and have been professionally employed in the field for a period of two years with a qualified surveyor or surveyors.

(*d*) In addition to evidence of service, a candidate shall produce a certificate from a qualified surveyor or surveyors, with whom he has served, that he is competent to undertake surveys.

(*e*) A candidate shall produce his original field-notes and plan plotted by him therefrom, of the survey of an area of not less than 40 acres, one of the boundaries of which shall be a watercourse or other natural feature or an irregular road. The field-notes, plan, and survey must be certified by the candidate to be entirely his own work.

New Zealand.—(*a*) *Same as New South Wales* (*a*), *but field service* two *instead of* three *years.*

(*b*) *Same as New South Wales* (*b*), *adding* trigonometry.

(*c*) Or shall have completed a course of the degree of Bachelor in Engineering at a recognized University, and have been professionally employed in the field for a period of two years with a qualified surveyor or surveyors.

(*d*) *Same as New South Wales* (*d*).

(*e*) Every candidate shall forward a plan and particulars of the measurement of a base-line not less than 80 chains in length connected by a series of at least three triangles with the triangulation of the district: a plan of at least 100 acres of rural land, contained within not less than ten sides: a plan of an area, having buildings thereon, in illustration of a town survey under the Land

---

* "A 'qualified surveyor' shall mean any person entitled to practise as a land surveyor in any part of the British Empire where the standard of examination is, in the opinion of the Board, equivalent to that prescribed by these Rules."

Transfer Act; and the original field-books, tabulations, and all calculations connected with the above surveys. The plans must be drawn and plotted on paper 18 in. by 18 in. from surveys made in accordance with the Survey Regulations, and shall be signed, dated, and certified as the work of the candidate.

Queensland.—(*a*) *Same as New South Wales* (*a*), *and definition of* "qualified surveyor" *the same.*

(*b*) *Same as New South Wales* (*b*).

(*c*) *Same as New South Wales* (*c*), *with the following addition:*— Provided always that at least one year of such field service shall have been served in one of the Australasian colonies.

(*d*) *Same as New South Wales* (*d*).

(*e*) *Same as New South Wales* (*e*).

South Australia.—(*a*) *Same as New South Wales* (*a*).

(*b*) *Same as New South Wales* (*b*).

(*c*) Or shall have completed a course of the degree of Bachelor in Engineering at any University recognized by an Australasian University, and have been professionally employed in the field for a period of two years with a qualified surveyor * or surveyors. Provided always that at least one year of such field service shall have been in any part of the British Empire or the United States of America.

(*d*) A candidate may be required to produce his original field-notes and plan plotted by him therefrom of the survey of an area of not less than 40 acres, one of the boundaries of which shall be a water-course or other natural feature, or an irregular road. The field-notes, plan, and survey must be certified by the applicant to be entirely his own work.

Tasmania.—(*a*) *Same as New South Wales* (*a*): *definition of a* "qualified surveyor" *being the same as that for South Australia* (*c*).

(*b*) *Same as New South Wales* (*b*).

(*c*) *Same as New South Wales* (*c*).

(*d*) *Same as New South Wales* (*d*).

(*e*) A candidate shall produce his original field-notes and plan plotted by him therefrom, of the survey of an area of not less than 40 acres, one of the boundaries of which shall be a water-course or other natural feature or an irregular road, and of an area having buildings thereon in illustration of a town survey. The field-notes, plan, and survey must be certified by the candidate to be entirely his own work.

---

* "A qualified surveyor shall mean any person entitled to practise as a land surveyor in any part of the British Empire or the United States of America where the standard of examination is, in the opinion of the Board, equivalent to that prescribed by these regulations."

Victoria.—(*a*) *Same as New South Wales* (*a*)*: definition of a* "qualified surveyor" *being the same as that for South Australia* (*c*).

(*b*) *Same as New South Wales* (*b*).

(*c*) Or shall have taken a degree in Civil Engineering at any University recognized by an Australasian University, and have been professionally employed in the field for a period of two years with a qualified surveyor or surveyors.

(*d*) *Same as New South Wales* (*d*).

(*e*) *Same as Tasmania* (*e*).

Western Australia.—(*a*) *Same as New South Wales* (*a*)*: definition of a* "qualified surveyor" *being the same as that for South Australia* (*c*).

(*b*) *Same as New South Wales* (*b*).

(*c*) *Same as Victoria* (*c*).

(*d*) *Same as New South Wales* (*d*).

(*e*) *Same as Tasmania* (*e*).

AFRICA.

Cape Colony.—(*a*) Admission as surveyors in Cape Colony is granted only to those who have qualified themselves for this profession in the Cape Colony, no foreign diploma or certificate being recognized.

(*b*) All persons passing the examinations are termed 'Government' Land Surveyors, even though practising privately.

(*c*) The first examination is held by the University, and is purely theoretical, and embraces the following subjects :—Arithmetic ; Algebra ; Geometry ; Plane and Spherical Trigonometry ; Astronomy, and the elements of Geometrical Optics, and Expansion under Heat ; Co-ordinate Geometry, Mensuration, and the elements of Geodesy.

Natal.—*None specified save* attainment of the age of 21 years. *See Section II.*

Orange River.—*None specified save in so far as may be implied in a somewhat vaguely-expressed clause* (b. 2) *in Section II.*

Southern Rhodesia.—A candidate shall have passed the Cape University examination in the theory of land surveying.

Transvaal.—*See Section II.*

II.—SOLE OR FINAL CONDITIONS AND EXAMINATION.

CANADA.

British Columbia.—(a.) A candidate must pass an examination in plane and solid trigonometry; spherical trigonometry so far as it includes the solution of triangles; the use of logarithms; measurement of areas; dividing or laying-off lands; elements of practical astronomy; the solution of some elementary problems; and the system of survey and registration of places in British Columbia.

(b.) A properly qualified surveyor of any other Province of Canada shall receive a commission as surveyor without serving articles, or passing any examination other than as regards the system of surveys of the lands of British Columbia.

(c.) A land-surveyor in any of His Majesty's dominions other than Canada, in which the standard of requirements is similar in theory and practice to that prescribed for this province, need not pass any examination other than as regards the system of surveys of lands of the province, and need not serve articles.

(d.) Any one who has passed the examination for and served two years in any of His Majesty's surveys, or as an officer in the Royal Engineers, or has for two years served under articles with a surveyor in any of His Majesty's dominions, or who has been in the active employ of any chartered railway company in British Columbia as a surveyor and engineer for five years, shall be admitted as provincial land-surveyor on passing the examination referred to in (d.) * above, without serving further apprenticeship.

(e.) Any British subject who is a qualified civil engineer, and who holds credentials as such from any British or Canadian University, or Chartered Institute of Engineers, or from any British or Canadian civil engineer, or firm of British or Canadian civil engineers of repute, may make surveys incidental to works of construction on which he is employed, and plans of such surveys are admissible to public record; and he may practise as a provincial land-surveyor on presenting such credentials to the Board or otherwise satisfying them that he is qualified to practise as a civil engineer in some part of His Majesty's dominions, and is possessed of a proper knowledge of the provincial system of land surveys, and has resided in the Province for the year preceding his application.

Manitoba.—(a.) An ordinary candidate must be articled to a surveyor in the province for three years after passing the entrance examination; and then pass a final examination in mathematics and surveying.

---

* *Corresponds to reference in* "Prof. Handbk." *Qy.* (b.) or (c.)?

(b.) Persons who since October 1875 have been admitted as surveyors in any Province of Canada, where the qualification required is similar to that in Manitoba, and when the province grants reciprocal privileges to Manitoba surveyors, are entitled to be admitted as surveyors in Manitoba without any examination other than one in the survey of land and the law as to registration of plans in Manitoba.

New Brunswick.—A candidate must pass an examination in arithmetic; algebra; geometry; mensuration; trigonometry; use of logarithms; surveying and dividing lands; plotting and map-drawing; spherical trigonometry and astronomy so far as will enable him to lay down a true meridian line and ascertain the latitude; and practical use of instruments.

Nova Scotia.—(a.) The subjects of examination are:—Euclid, first four books, and propositions first to twenty-first of sixth book; plane trigonometry so far as it includes solution of triangles; use of logarithms; mensuration of surfaces, including calculation of area of right-lined figures by latitude and departure; dividing and laying-off of land; rules for solution of spherical triangles, and of their application to surveying in the following elementary problems of practical astronomy, viz.:—to ascertain the latitude of a place from an observation of a meridian altitude of the sun or of a star: to obtain the local time and the azimuth from an observed altitude of the sun or a star: from an observed altitude of a circumpolar star, when at its greatest elongation from the meridian, to ascertain the direction of the latter.

(b.) A candidate must be familiar with surveying operations and capable of reporting intelligently thereon; conversant with the keeping of field-notes and their plotting on plans of survey; the describing of land by metes and bounds for title; the adjustments and methods of use of ordinary surveying-instruments; the Dominion system of survey, and the standing instructions for Dominion land surveyors from time to time published by authority.

Ontario.—(a.) *Same as Nova Scotia* (a.).
(b.) *Same as Nova Scotia* (b.).

Prince Edward Island.—(a.) *Same as Nova Scotia* (a.).
(b.) *Same as Nova Scotia* (b.), *the clauses following the words* use of ordinary surveying-instruments *being altered to* the Dominion and Provincial systems of survey.

Quebec.—The subjects of examination are:—Geometry; rectilinear and spherical (theoretical and practical) trigonometry; practical astronomy; linear and topographical drawing; levelling and all other questions relating to practical surveying; the use and theory of instruments; geology; mineralogy; the forest flora of Canada; the mode to be pursued in establishing boundary lines; the investigation of titles to property; and finally all fundamental questions of law connected with the measurement of land.

AUSTRALASIA.

New South Wales.—(a.) Trigonometry, plane and spherical; geometry and algebra. These subjects will be treated so far as they are applicable to surveying.

(b.) Reduction of traverses; computation connected with the setting out of roads and curves; adjustment of discrepancies in surveys; computation of areas, including such as have irregular and curved boundaries.

(c.) Detail of field practice, including the keeping of field notes; topographical surveying; setting out of areas; redetermination of boundaries; laying out of roads; setting out curves; plotting from field-notes; stadia surveying; barometric and other measurement of heights; surveying under Real Property Act; writing descriptions of boundaries.

(d.) Levelling; measurement of earthworks.

(e.) Principles of construction, adjustment, and use of the following instruments:—Theodolite, plane-table, sextant, telemeter, level, compass, clinometer, barometer, thermometer, and steel band.

(f.) Determination of time, latitude, and azimuth; reduction of star places; elementary geodesy, including spherical excess and convergence of meridians; declination of the magnetic needle.

(g.) Plan-drawing; compilation of plans; projection of maps and charts.

(h.) Elementary physics; elementary geology.

(i.) Candidates producing satisfactory evidence of having been matriculated, or of having passed the senior examination at a University, or of having completed a course of the degree of Bachelor in Engineering at any University recognized by an Australasian University, may be exempted from working out papers in elementary geometry, trigonometry, and algebra, and will be credited with 66 per centum of the maximum marks allotted those papers.

(j.) Certificates entitling to practise are issued without examination to duly qualified surveyors of other Australasian States.

New Zealand.—(a.) *Same as New South Wales* (a.).

(b.) *Same as New South Wales* (b.), *with addition of* triangulation, including polygonal and ray-trace computations.

(c.) *Same as New South Wales* (c.), *with substitution of* cutting off given areas *instead of* setting out of areas, *and* Land Transfer Act 1885, *instead of* Real Property Act; *and omitting* writing descriptions of boundaries.

(d.) *Same as New South Wales* (d.).

(e.) *Same as New South Wales* (e.).

(f.) *Same as New South Wales* (f.).

(g.) *Same as New South Wales* (g.), *with addition of* description-writing.

(h.) *Same as New South Wales* (h.), *with addition of* survey regulations.

(i.) *Same as New South Wales* (i.), *with addition after the words* senior examination at a University, *of:* or the Senior Civil Service Examination of New Zealand.

(j.) The examination in each subject may be *vivâ voce* or in writing as the Board may from time to time determine. In the event of a candidate failing to pass at the first examination, the Board may, at its discretion, allow him to come up at the next examination, and may determine the special subjects in which he must pass on the second occasion.

(k.) *Same as New South Wales* (j.)

Queensland.—(a.) *Same as New South Wales* (a.).

(b.) *Same as New South Wales* (b.).

(c.) *Same as New South Wales* (c.).

(d.) *Same as New South Wales* (d.).

(e.) *Same as New South Wales* (e.).

(f.) *Same as New South Wales* (f.).

(g.) *Same as New South Wales* (g.).

(h.) Elementary optics and light in relation to the construction and use of surveying instruments; elementary stratigraphical geology.

(i.) *Same as New South Wales* (i.).

(j.) *Same as New South Wales* (j.).

South Australia.—(a.) *Same as New South Wales* (a.).

(b.) *Same as New South Wales* (b.), *with addition of* triangulation.

(c.) *Same as New South Wales* (c.), *substituting* cutting off given areas *instead of* setting out of areas.

(d.) Levelling; grading; measurements of earthworks; including practical tests.

(e.) *Same as New South Wales* (e.), *with addition of* tacheometer.

(f.) *Same as New South Wales* (f.), *with addition of:* With practical tests.

(g.) *Same as New South Wales* (g.), *with addition of:* drawing of sections and contours.

(h.) Physics and geology (elementary); names and description of Australasian timbers of economic value (*vivâ voce*).

(i.) Candidates producing satisfactory evidence of having matriculated or passed the senior examination at a University, or the senior Civil Service examination of New Zealand, may be exempted from working out the papers in elementary geometry, trigonometry, and algebra, and will be credited with 66 per centum of the maximum number of marks allotted to those papers.

(j.) Candidates producing evidence of having taken a degree in Civil Engineering at any University recognized by an Australasian University, may be further exempted from working out the papers in physics and geology.

(k.) Certificates entitling to practise are issued without examination to duly qualified surveyors of the other Australasian States or of any part of the British Empire or of the United States of America.

Tasmania.—(a.) *Same as New South Wales* (a.).

(b.) *Same as South Australia* (b.).

(c.) Detail of field practice, including the keeping of field notes; topographical, trigonometrical, subdivisional, and other surveys; setting out of areas; redetermination of boundaries; laying out of roads; setting out curves; plotting from field-notes and from co-ordinates; stadia surveying; barometric and other measurement of heights; surveying under Real Property Act; writing descriptions of boundaries. (Candidates may be required to make actual surveys.)

(d.) *Same as South Australia* (d.).

(e.) *Same as South Australia* (e.).

(f.) *Same as South Australia* (f.).

(g.) *Same as South Australia* (g.).

(h) *Same as South Australia* (h.).

(i.) *Same as South Australia* (i.).

(j.) *Same as South Australia* (j.).

(k.) *Same as New South Wales* (j.).

Victoria.—(a.) *Same as New South Wales* (a.).

(b.) *Same as South Australia* (b.).

(c.) *Same as Tasmania* (c), *substituting* Transfer of Land Acts *instead of* Real Property Act.

(d.) *Same as South Australia* (d.).

(e.) *Same as South Australia* (e.).

(f.) *Same as South Australia* (f.).

(g.) *Same as South Australia* (g.).

(h.) *Same as South Australia* (h.).

(i.) *Same as South Australia* (i.).

(j.) *Same as South Australia* (j.).

(k.) *Same as New South Wales* (j).

Western Australia.—(a.) *Same as New South Wales* (a.).

(b.) *Same as South Australia* (b.).

(c.) *Same as Tasmania* (c.), *substituting* Transfer of Land Act *for* Real Property Act.

(d.) *Same as South Australia* (d).

(e.) *Same as South Australia* (e.).

(f.) *Same as New South Wales* (f.), *adding* With practical tests.

(g.) *Same as South Australia* (g.).

(h.) *Same as South Australia* (h.).

(i.) *Same as South Australia* (i.).

(j.) *Same as South Australia* (j.).

(k.) Certificates entitling to practise are issued, with or without examination, to licensed surveyors of the other Australasian States.

AFRICA.

Cape Colony.—(a.) The second examination is held in the Surveyor-General's office, and is of a practical nature, embracing the following subjects:—

(b.) Drawing, including topography, copying plans, and lettering. Neatness as well as accuracy in this branch is essential.

(c.) Use and adjustments of theodolites, and use of scales and planimeter.

(d.) Measurement of a base-line, and the different methods employed for securing extreme accuracy. Candidates may be required to measure a base-line.

(e.) Co-ordinates and their practical application to land-surveying. Candidates may be required to determine the co-ordinates of a number of given points, and to fix in the field their positions.

(f.) Overcoming difficulties in the field, such as observation of angles the vertices of which are inaccessible, and reduction of observations when there are small known errors in the positions of the observed signals.

(g.) Re-survey, under Land Beacons Act or otherwise; subdivision of private property; construction of plans and diagrams according to the regulations in force.

(h.) Astronomical determination of the latitude of a place, and of the true azimuth of one or more lines of a triangulation. Candidates may be required to determine, by astronomical observations, the direction of the meridian with reference to given points, and to determine the variation of the compass.

(i.) The above examination is to be followed by a Trial Survey. Candidates may be required to frame a plan of the survey, and to lay down on it accurately rivers, roads, etc., or other marked features of the ground within the limits of the survey.

Natal.—(a.) English history; arithmetic; elementary geometry and algebra.

(b.) Mathematics, embracing: geometry; algebra; elementary trigonometry; co-ordinate geometry of the straight line; statics and dynamics.

(c.) Theory, embracing: plane and spherical trigonometry; practical astronomy and geodesy, including determination of time, latitude, longitude, and azimuth; determination of heights by angular measure, spirit-level, and barometer.

(d.) Practical subjects, embracing: use and adjustment of theodolite, level, sextant, and prismatic compass; subdivision of land; setting-out curves; measuring-up earthworks; preparing plans; showing longitudinal and cross sections of earthworks; taking-out quantities.

(e.) Persons who hold diplomas from any recognized examining body are exempted from examination in the subjects covered by such diplomas.

Orange River.—(a.) Any person entitled to practise as a Government Land Surveyor* in Cape Colony may practise as a Land Surveyor in Orange River Colony.

(b.) Any person admitted as a Government Land Surveyor in Great Britain or Ireland or in any British Possession shall be entitled to be admitted as a Government Land Surveyor in Orange River Colony on proof to the satisfaction of the Deputy Administrator thereof:—

(1) That he has been admitted to practise as a Government Land Surveyor in any such country as aforesaid, and at the date of his application is still entitled to practise and has actually practised as such therein for not less than three years;

(2) That he has passed an examination or examinations prior to such admission, equivalent to the examinations required to be passed in the Cape of Good Hope for admission as a Government Land Surveyor.

Southern Rhodesia.—An examination in the practice of land-surveying, conducted by the Surveyor-General. [*See* Cape Colony (a. *to* i.).]

Transvaal.—No person shall be admitted to practise as a Licensed Surveyor until he has worked in the field for a period of not less than two years with a qualified Government Surveyor, and has passed:—

---

* *See* Cape Colony (*b*) *in Section I.*

(a.) In this colony such examination or examinations as may be prescribed by the Board of Examiners and approved by the Lieutenant-Governor in Council; or

(b.) In some other part of His Majesty's Dominions such examination or examinations as the Board of Examiners with the approval of the Lieutenant-Governor in Council may deem to be equivalent to the examination or examinations prescribed under sub-section (a.). If examination or examinations passed elsewhere shall be deemed to be equivalent in part only, the applicant shall be required to pass such supplementary examination or examinations as the Board of Examiners may prescribe; and

(c.) In addition to such examination or examinations prescribed in sub-section (a.) or (b.), such examination in the methods and procedure of surveying and in the Land Laws of the Colony as the Surveyor-General shall prescribe.

APPENDIX

TABLES

HYPSOMETER TABLES.

TABLE I.

Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.	Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.	Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.
185·0	14698	17·048	188·6	12660	18·432	192·2	10644	19·910
·1	14641	17·085	·7	12603	18·472	·3	10588	19·952
·2	14584	17·122	·8	12547	18·512	·4	10533	19·995
·3	14528	17·160	·9	12490	18·552	·5	10477	20·037
·4	14471	17·197	189·0	12434	18·592	·6	10422	20·080
·5	14414	17·235	·1	12377	18·632	·7	10366	20·123
·6	14357	17·272	·2	12321	18·672	·8	10310	20·166
·7	14300	17·310	·3	12265	18·712	·9	10255	20·208
·8	14244	17·348	·4	12209	18·753	193·0	10199	20·251
·9	14187	17·385	·5	12153	18·793	·1	10144	20·294
186·0	14130	17·423	·6	12096	18·833	·2	10088	20·338
·1	14073	17·461	·7	12040	18·874	·3	10033	20·381
·2	14017	17·499	·8	11984	18·914	·4	9978	20·424
·3	13960	17·537	·9	11928	18·955	·5	9923	20·467
·4	13903	17·575	190·0	11872	18·996	·6	9867	20·511
·5	13857	17·614	·1	11816	19·036	·7	9812	20·554
·6	13790	17·652	·2	11760	19·077	·8	9757	20·598
·7	13733	17·690	·3	11704	19·118	·9	9701	20·641
·8	13676	17·729	·4	11648	19·159	194·0	9646	20·685
·9	13620	17·767	·5	11592	19·200	·1	9591	20·729
187·0	13563	17·806	·6	11536	19·241	·2	9536	20·773
·1	13506	17·844	·7	11480	19·283	·3	9481	20·817
·2	13450	17·883	·8	11424	19·324	·4	9426	20·861
·3	13394	17·922	·9	11368	19·365	·5	9371	20·905
·4	13337	17·961	191·0	11312	19·407	·6	9315	20·949
·5	13281	18·000	·1	11257	19·448	·7	9260	20·993
·6	13224	18·039	·2	11201	19·490	·8	9205	21·038
·7	13167	18·078	·3	11146	19·532	·9	9150	21·082
·8	13111	18·117	·4	11090	19·573	195·0	9095	21·126
·9	13054	18·156	·5	11034	19·615	·1	9040	21·171
188·0	12998	18·195	·6	10978	19·657	·2	8985	21·216
·1	12942	18·235	·7	10922	19·699	·3	8930	21·260
·2	12885	18·274	·8	10867	19·741	·4	8875	21·305
·3	12829	18·314	·9	10811	19·783	·5	8820	21·350
·4	12772	18·353	192·0	10755	19·825	·6	8765	21·395
·5	12716	18·393	·1	10699	19·868	·7	8710	21·440

Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.	Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.	Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.
195·8	8655	21·485	200·5	6095	23·697	205·2	3574	26·096
·9	8600	21·530	·6	6041	23·746	·3	3521	26·149
196·0	8545	21·576	·7	5987	23·795	·4	3468	26·202
·1	8490	21·621	·8	5933	23·845	·5	3416	26·255
·2	8435	21·666	·9	5879	23·894	·6	3363	26·309
·3	8381	21·712	201·0	5825	23·943	·7	3310	26·362
·4	8326	21·751	·1	5771	23·993	·8	3256	26·416
·5	8271	21·803	·2	5717	24·042	·9	3203	26·470
·6	8216	21·849	·3	5663	24·092	206·0	3151	26·523
·7	8161	21·895	·4	5609	24·142	·1	3098	26·577
·8	8107	21·941	·5	5556	24·191	·2	3045	26·631
·9	8052	21·987	·6	5502	24·241	·3	2992	26·685
197·0	7997	22·033	·7	5448	24·291	·4	2939	26·740
·1	7942	22·079	·8	5394	24·341	·5	2886	26·794
·2	7888	22·125	·9	5340	24·391	·6	2833	26·848
·3	7833	22·172	202·0	5286	24·442	·7	2780	26·903
·4	7779	22·218	·1	5232	24·492	·8	2727	26·957
·5	7724	22·264	·2	5178	24·542	·9	2674	27·012
·6	7669	22·311	·3	5124	24·593	207·0	2622	27·066
·7	7615	22·358	·4	5070	24·644	·1	2569	27·121
·8	7560	22·404	·5	5017	24·694	·2	2516	27·179
·9	7506	22·451	·6	4964	24·745	·3	2464	27·231
198·0	7451	22·498	·7	4910	24·796	·4	2411	27·286
·1	7397	22·545	·8	4856	24·847	·5	2358	27·341
·2	7343	22·592	·9	4802	24·898	·6	2305	27·397
·3	7289	22·639	203·0	4749	24·949	·7	2252	27·452
·4	7234	22·686	·1	4695	25·000	·8	2199	27·507
·5	7180	22·734	·2	4641	25·051	·9	2146	27·563
·6	7125	22·781	·3	4588	25·103	208·0	2094	27·618
·7	7071	22·829	·4	4535	25·154	·1	2041	27·674
·8	7016	22·876	·5	4482	25·206	·2	1989	27·730
·9	6962	22·924	·6	4428	25·257	·3	1936	27·786
199·0	6908	22·971	·7	4375	25·309	·4	1884	27·842
·1	6854	23·019	·8	4322	25·361	·5	1831	27·898
·2	6800	23·067	·9	4268	25·413	·6	1778	27·954
·3	6745	23·115	204·0	4215	25·465	·7	1726	28·011
·4	6691	23·163	·1	4161	25·517	·8	1673	28·067
·5	6637	23·211	·2	4107	25·569	·9	1621	28·123
·6	6583	23·259	·3	4053	25·621	209·0	1568	28·180
·7	6529	23·308	·4	4000	25·674	·1	1516	28·237
·8	6474	23·356	·5	3947	25·726	·2	1463	28·293
·9	6420	23·405	·6	3894	25·779	·3	1411	28·350
200·0	6366	23·453	·7	3841	25·831	·4	1358	28·407
·1	6312	23·502	·8	3788	25·884	·5	1306	28·464
·2	6258	23·550	·9	3735	25·937	·6	1254	28·521
·3	6203	23·599	205·0	3682	25·990	·7	1201	28·579
·4	6149	23·648	·1	3625	26·043	·8	1149	28·636

Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.	Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.	Boiling point Fahr.	Altitude above level at which water boils at 212° (temp. of intermediate air being 32° F.).	Approximate corresponding height of aneroid or barometer.
209·9	1096	28·693	211·6	208	29·684	213·3	− 673	30·705
210·0	1044	28·751	·7	156	29·744	·4	− 724	30·766
·1	992	28·809	·8	104	29·803	·5	− 776	30·827
·2	939	28·866	·9	52	29·862	·6	− 828	30·888
·3	887	28·924	212·0	0	29·922	·7	− 880	30·949
·4	835	28·982	·1	− 52	29·981	·8	− 932	31·010
·5	783	29·040	·2	− 104	30·041	·9	− 983	31·071
·6	730	29·098	·3	− 155	30·101	214·0	− 1035	31·132
·7	678	29·156	·4	− 207	30·161	·1	− 1086	31·194
·8	626	29·215	·5	− 259	30·221	·2	− 1138	31·256
·9	573	29·273	·6	− 311	30·281	·3	− 1189	31·318
211·0	521	29·331	·7	− 363	30·341	·4	− 1241	31·380
·1	469	29·390	·8	− 414	30·401	·5	− 1293	31·442
·2	417	29·449	·9	− 466	30·461	·6	− 1344	31·504
·3	365	29·508	213·0	− 518	30·522	·7	− 1396	31·566
·4	313	29·566	·1	− 570	30·583	·8	− 1447	31·628
·5	261	29·625	·2	− 621	30·644	·9	− 1549	31·690

TABLE II.

CORRECTION FOR TEMPERATURE OF INTERMEDIATE AIR.

Mean temperature of intermediate air.	Multiplier.	Mean temperature of intermediate air.	Multiplier.	Mean temperature of intermediate air.	Multiplier.	Mean temperature of intermediate air.	Multiplier.
°		°		°		°	
20	0·9734	37	1·0111	54	1·0488	70	1·0844
21	0·9756	38	1·0133	55	1·0511	71	1·0866
22	0·9778	39	1·0155	56	1·0533	72	1·0888
23	0·9801	40	1·0177	57	1·0555	73	1·0911
24	0·9823	41	1·0199	58	1·0577	74	1·0933
25	0·9845	42	1·0222	59	1·0599	75	1·0955
26	0·9867	43	1·0244	60	1·0622	76	1·0977
27	0·9889	44	1·0266	61	1·0644	77	1·0999
28	0·9912	45	1·0288	62	1·0666	78	1·1022
29	0·9934	46	1·0311	63	1·0688	79	1·1044
30	0·9956	47	1·0333	64	1·0711	80	1·1066
31	0·9978	48	1·0355	65	1·0733	81	1·1088
32	1·0000	49	1·0377	66	1·0755	82	1·1111
33	1·0022	50	1·0399	67	1·0777	83	1·1133
34	1·0044	51	1·0422	68	1·0799	84	1·1156
35	1·0066	52	1·0444	69	1·0822	85	1·1178
36	1·0088	53	1·0466				

TABLES OF NATURAL SINES, TANGENTS, AND SECANTS, WITH THEIR COMPLEMENTS.

Tables of natural sines, cosines, tangents, etc., represent the numerical values of the lengths of the sines, cosines, tangents, etc., of arcs of a circle whose radius = 1.

The natural sines etc., also the arcs for any given natural sines etc., are found from the table in the same manner as is used for logarithmic ones.

The natural sines etc., are easily found from logarithmetic ones, by subtracting 10 from the indices of the latter: the number corresponding to this logarithm is the natural sine etc., required.

Given the logarithmic sine of 36° 44′, namely 9·7767676, to find the natural sine.

$$\begin{array}{rl} L\ \text{sine}\ 36^\circ\ 44' = & 9\cdot7767676 \\ \text{Subtract} & 10\cdot \\ \hline L\ \text{natural sine} = & \bar{1}\cdot7767676 \\ \text{Hence natural sine} = & 0\cdot5980916 \end{array}$$

Conversely, natural sines etc., can be converted into logarithmic ones, by finding the logarithm corresponding to their numerical value and adding 10 to the index.

Given the natural cotangent of 63° 25′, namely 0·5003989, to find its logarithmic cotangent.

$$\begin{array}{rl} \text{Nat. cot.}\ 63^\circ\ 25' = 0\cdot5003989 & \\ \text{log.}\ 0\cdot5003989 = & 0\bar{1}\cdot6993164 \\ \text{Add} & 10\cdot \\ \hline L\ \text{cotangent}\ 63^\circ\ 25' = & 9\cdot6993164 \end{array}$$

	0 Deg.				1 Deg.				2 Deg.		
	Sine	Cosine			Sine	Cosine			Sine	Cosine	
0	0000000	1.0000000	60	0	0174524	9998477	60	0	0348995	9993908	60
1	0002909	1.0000000	59	1	0177432	9998426	59	1	0351902	9993806	59
2	0005818	9999998	58	2	0180341	9998374	58	2	0354809	9993704	58
3	0008727	9999996	57	3	0183249	9998321	57	3	0357716	9993600	57
4	0011636	9999993	56	4	0186158	9998267	56	4	0360623	9993495	56
5	0014544	9999989	55	5	0189066	9998213	55	5	0363530	9993390	55
6	0017453	9999985	54	6	0191974	9998157	54	6	0366437	9993284	54
7	0020362	9999979	53	7	0194883	9998101	53	7	0369344	9993177	53
8	0023271	9999973	52	8	0197791	9998044	52	8	0372251	9993069	52
9	0026180	9999966	51	9	0200699	9997986	51	9	0375158	9992960	51
10	0029089	9999958	50	10	0203608	9997927	50	10	0378065	9992851	50
11	0031998	9999949	49	11	0206516	9997867	49	11	0380971	9992740	49
12	0034907	9999939	48	12	0209424	9997807	48	12	0383878	9992629	48
13	0037815	9999928	47	13	0212332	9997745	47	13	0386785	9992517	47
14	0040724	9999917	46	14	0215241	9997683	46	14	0389692	9992404	46
15	0043633	9999905	45	15	0218149	9997620	45	15	0392598	9992290	45
16	0046542	9999892	44	16	0221057	9997556	44	16	0395505	9992176	44
17	0049451	9999878	43	17	0223965	9997492	43	17	0398411	9992060	43
18	0052360	9999863	42	18	0226873	9997425	42	18	0401318	9991944	42
19	0055268	9999847	41	19	0229781	9997360	41	19	0404224	9991827	41
20	0058177	9999831	40	20	0232690	9997292	40	20	0407131	9991709	40
21	0061086	9999813	39	21	0235598	9997224	39	21	0410037	9991590	39
22	0063995	9999795	38	22	0238506	9997156	38	22	0412944	9991470	38
23	0066904	9999776	37	23	0241414	9997086	37	23	0415850	9991350	37
24	0069813	9999756	36	24	0244322	9997015	36	24	0418757	9991228	36
25	0072721	9999736	35	25	0247230	0996943	35	25	0421663	9991106	35
26	0075630	9999714	34	26	0250138	9996871	34	26	0424569	9990983	34
27	0078539	9999692	33	27	0253046	9996798	33	27	0427475	9990859	33
28	0081448	9999668	32	28	0255954	9996724	32	28	0430382	9990734	32
29	0084357	9999644	31	29	0258862	9996649	31	29	0433288	9990609	31
30	0087265	9999619	30	30	0261769	9996573	30	30	0436194	9990482	30
31	0090174	9999593	29	31	0264677	9996497	29	31	0439100	9990355	29
32	0093083	9999567	28	32	0267585	9996419	28	32	0442006	9990227	28
33	0095992	9999539	27	33	0270493	9996341	27	33	0444912	9990098	27
34	0098900	9999511	26	34	0273401	9996262	26	34	0447818	9989968	26
35	0101809	9999482	25	35	0276309	9996182	25	35	0450724	9989837	25
36	0104718	9999452	24	36	0279216	9996101	24	36	0453630	9989706	24
37	0107627	9999421	23	37	0282124	9996020	23	37	0456536	9989573	23
38	0110535	9999389	22	38	0285032	9995937	22	38	0459442	9989440	22
39	0113444	9999357	21	39	0287940	9995854	21	39	0462347	9989306	21
40	0116353	9999323	20	40	0290847	9995770	20	40	0465253	9989171	20
41	0119261	9999289	19	41	0293755	9995684	19	41	0468159	9989035	19
42	0122170	9999254	18	42	0296662	9995599	18	42	0471065	9988899	18
43	0125079	9999218	17	43	0299570	9995512	17	43	0473970	9988761	17
44	0127987	9999181	16	44	0302478	9995424	16	44	0476876	9988623	16
45	0130896	9999143	15	45	0305385	9995336	15	45	0479781	9988484	15
46	0133805	9999105	14	46	0308293	9995247	14	46	0482687	9988344	14
47	0136713	9999065	13	47	0311200	9995157	13	47	0485592	9988203	13
48	0139622	9996025	12	48	0314108	9995066	12	48	0488498	9988061	12
49	0142530	9998984	11	49	0317015	9994974	11	49	0491403	9987919	11
50	0145439	9998942	10	50	0319922	9994881	10	50	0494308	9987775	10
51	0148348	9998900	9	51	0322830	9994788	9	51	0497214	9987631	9
52	0151256	9998856	8	52	0325737	9994693	8	52	0500119	9987486	8
53	0154165	9998812	7	53	0328644	9994598	7	53	0503024	9987340	7
54	0157073	9998766	6	54	0331552	9994502	6	54	0505929	9987194	6
55	0159982	9998720	5	55	0334459	9994405	5	55	0508835	9987046	5
56	0162890	9998673	4	56	0337366	9994308	4	56	0511740	9986898	4
57	0165799	9998625	3	57	0340274	9994209	3	57	0514645	9986748	3
58	0168707	9998577	2	58	0343181	9994110	2	58	0517550	9986598	2
59	0171616	9998527	1	59	0346088	9994009	1	59	0520455	9986447	1
60	0174524	9998477	0	60	0348995	9993908	0	60	0523360	9986295	0
	Cosine	Sine			Cosine	Sine			Cosine	Sine	
		Deg. 89.				Deg. 88				Deg. 87.	

3 Deg.				4 Deg.				5 Deg.			
	Sine	Cosine			Sine	Cosine			Sine	Cosine	
0	0523360	9986295	60	0	0697565	9975641	60	0	0871557	9961947	60
1	0526264	9986143	59	1	0700467	9975437	59	1	0874455	9961693	59
2	0529169	9985989	58	2	0703368	9975233	58	2	0877353	9961438	58
3	0532074	9985835	57	3	0706270	9975028	57	3	0880251	9961183	57
4	0534979	9985680	56	4	0709171	9974822	56	4	0883148	9960926	56
5	0537883	9985524	55	5	0712073	9974615	55	5	0886046	9960669	55
6	0540788	9985367	54	6	0714974	9974408	54	6	0888943	9960411	54
7	0543693	9985209	53	7	0717876	9974199	53	7	0891840	9960152	53
8	0546597	9985050	52	8	0720777	9973990	52	8	0894738	9959892	52
9	0549502	9984891	51	9	0723678	9973780	51	9	0897635	9959631	51
10	0552406	9984731	50	10	0726580	9973569	50	10	0900532	9959370	50
11	0555311	9984570	49	11	0729481	9973357	49	11	0903429	9959107	49
12	0558215	9984408	48	12	0732382	9973145	48	12	0906326	9958844	48
13	0561119	9984245	47	13	0735283	9972931	47	13	0909223	9958580	47
14	0564024	9984081	46	14	0738184	9972717	46	14	0912119	6958315	46
16	0566928	9983917	45	15	0741085	9972502	45	15	0915016	9958049	45
16	0569832	9983751	44	16	0743986	9972286	44	16	0917913	9957783	44
17	0572736	9983585	43	17	0746887	9972069	43	17	0920809	9957515	43
18	0575640	9983418	42	18	0749787	9971851	42	18	0923706	9957247	42
19	0578544	9983250	41	19	0752688	9971633	41	19	0926602	9956978	41
20	0581448	9983082	40	20	0755589	9971413	40	20	0929499	9956708	40
21	0584352	9982912	39	21	0758489	9971193	39	21	0932395	9956437	39
22	0587256	9982742	38	22	0761390	9970972	38	22	0935291	9956165	38
23	0590160	9982570	37	23	0764290	9970750	37	23	0938187	9955893	37
24	0593064	9982398	36	24	0767190	9970528	36	24	0941083	9855620	36
25	0595967	9982225	35	25	0770091	9970304	35	25	0943979	9955345	35
26	0598871	9982052	34	26	0772991	9970080	34	26	0946875	9955070	34
27	0601775	9981877	33	27	0775891	9969854	33	27	0949771	9954795	33
28	0604678	9981701	32	28	0778791	9969628	32	28	0952666	9954518	32
29	0607582	9981525	31	29	0781691	9969401	31	29	0955562	9954240	31
30	0610485	9981348	30	30	0784591	9969173	30	30	0958458	9953962	30
31	0613389	9981170	29	31	0787491	9968945	29	31	0961353	9953683	29
32	0616292	9980991	28	32	0790391	9968715	28	32	0964248	9953403	28
33	0619196	9980811	27	33	0793290	9968485	27	33	0967144	9953122	27
34	0622099	9980631	26	34	0796190	9968254	26	34	0970039	9952840	26
35	0625002	9980450	25	35	0799090	9968022	25	35	0972934	9952557	25
36	0627905	9980267	24	36	0801989	9967789	24	36	0975829	9952274	24
37	0630808	9980084	23	37	0804889	9967555	23	37	0978724	9951990	23
38	0633711	9979900	22	38	0807788	9967321	22	38	0981619	9951705	22
39	0636614	9979716	21	39	0810687	9967085	21	39	0984514	9951419	21
40	0639517	9979530	20	40	0813587	9966849	20	40	0987408	9951132	20
41	0642420	9979343	19	41	0816486	9966612	19	41	0990303	9950844	19
42	0645323	9979156	18	42	0819385	9966374	18	42	0993197	9950556	18
43	0648226	9978968	17	43	0822284	9966135	17	43	0996092	9950266	17
44	0651129	9978779	16	44	0825183	9965895	16	44	0998986	9949976	16
45	0654031	9978589	15	45	0828082	9965655	15	45	1001881	9949685	15
46	0656934	9978399	14	46	0830981	9965414	14	46	1004775	9949393	14
47	0659836	9978207	13	47	0833880	9965172	13	47	1007669	9949101	13
48	0662739	9978015	12	48	0836778	9964929	12	48	1010563	9948807	12
49	0665641	9977821	11	49	0839677	9964685	11	49	1013457	9948513	11
50	0668544	9977627	10	50	0842576	9964440	10	50	1016351	9948217	10
51	0671446	9977433	9	51	0845474	9964195	9	51	1019245	9947921	9
52	0674349	9977237	8	52	0848373	9963948	8	52	1022138	9947625	8
53	0677251	9977040	7	53	0851271	9963701	7	53	1025032	9947327	7
54	0680153	9976843	6	54	0854169	9963453	6	54	1027925	9947028	6
55	0683055	9976645	5	55	0857067	9963204	5	55	1030819	9946729	5
56	0685957	9976445	4	56	0859966	9962954	4	56	1033712	9946428	4
57	0688859	9976245	3	57	0862864	9962704	3	57	1036605	9946127	3
58	0691761	9976045	2	58	0865762	9962452	2	58	1039499	9945825	2
59	0694663	9975843	1	59	0868660	9962200	1	59	1042392	9945523	1
60	0697565	9975641	0	60	0871557	9961947	0	60	1045285	9945219	0
	Cosine	Sine			Cosine	Sine			Cosine	Sine	
	Deg. 86.				Deg. 85.				Deg. 84.		

6 Deg.				7 Deg.				8 Deg.			
	Sine	Cosine			Sine	Cosine			Sine	Cosine	
0	1045285	9945219	60	0	1218693	9925462	60	0	1391731	9902681	60
1	1048178	9944914	59	1	1221581	9925107	59	1	1394612	9902275	59
2	1051070	9944609	58	2	1224468	9924751	58	2	1397492	9901869	58
3	1053963	9944303	57	3	1227355	9924394	57	3	1400372	9901462	57
4	1056856	9943996	56	4	1230241	9924037	56	4	1403252	9901055	56
5	1059748	9943688	55	5	1233128	9923679	55	5	1406132	9900646	55
6	1062641	9943379	54	6	1236015	9923319	54	6	1409012	9900237	54
7	1065533	9943070	53	7	1238901	9922959	53	7	1411892	9899826	53
8	1068425	9942760	52	8	1241788	9922599	52	8	1414772	9899415	52
9	1071318	9942448	51	9	1244674	9922237	51	9	1417651	9899003	51
10	1074210	9942136	50	10	1247560	9921874	50	10	1420531	9898590	50
11	1077102	9941823	49	11	1250446	9921511	49	11	1423410	9898177	49
12	1079994	9941510	48	12	1253332	9921147	48	12	1426289	9897762	48
13	1082885	9941195	47	13	1256218	9920782	47	13	1429168	9897347	47
14	1085777	9940880	46	14	1259104	9920416	46	14	1432047	9896931	46
15	1088669	9940563	45	15	1261990	9920049	45	15	1434926	9896514	45
16	1091560	9940246	44	16	1264875	9919682	44	16	1437805	9896096	44
17	1094452	9939929	43	17	1267761	9919314	43	17	1440684	9895677	43
18	1097343	9939610	42	18	1270646	9918944	42	18	1443562	9895258	42
19	1100234	9939290	41	19	1273531	9918574	41	19	1446440	9894838	41
20	1103126	9938969	40	20	1276416	9918204	40	20	1449319	9894416	40
21	1106017	9938648	39	21	1279302	9917832	39	21	1452197	9893994	39
22	1108908	9938326	38	22	1282186	9917459	38	22	1455075	9893572	38
23	1111799	9938003	37	23	1285071	9917086	37	23	1457953	9893148	37
24	1114689	9937679	36	24	1287956	9916712	36	24	1460830	9892723	36
25	1117580	9937355	35	25	1290841	9916337	35	25	1463708	9892298	35
26	1120471	9937029	34	26	1293725	9915961	34	26	1466585	9891872	34
27	1123361	9936703	33	27	1296609	9915584	33	27	1469463	9891445	33
28	1126252	9936375	32	28	1299494	9915206	32	28	1472340	9891017	32
29	1129142	9936047	31	29	1302378	9914828	31	29	1475217	9890588	31
30	1132032	9935719	30	30	1305262	9914449	30	30	1478094	9890159	30
31	1134922	9935389	29	31	1308146	9914069	29	31	1480971	9889728	29
32	1137812	9935058	28	32	1311030	9913688	28	32	1483848	9889297	28
33	1140702	9934727	27	33	1313913	9913306	27	33	1486724	9888865	27
34	1143592	9934395	26	34	1316797	9912923	26	34	1489601	9888432	26
35	1146482	9934062	25	35	1319681	9912540	25	35	1492477	9887998	25
36	1149372	9933728	24	36	1322564	9912155	24	36	1495353	9887564	24
37	1152261	9933393	23	37	1325447	9911770	23	37	1498230	9887128	23
38	1155151	9933057	22	38	1328330	9911384	22	38	1501106	9886692	22
39	1158040	9932721	21	39	1331213	9910997	21	39	1503981	9886255	21
40	1160929	9932384	20	40	1334096	9910610	20	40	1506857	9885817	20
41	1163818	9932045	19	41	1336979	9910221	19	41	1509733	9885378	19
42	1166707	9931706	18	42	1339862	9909832	18	42	1512608	9884939	18
43	1169596	9931367	17	43	1342744	9909442	17	43	1515484	9884498	17
44	1172485	9931026	16	44	1345627	9909051	16	44	1518359	9884057	16
45	1175374	9930685	15	45	1348509	9908659	15	45	1521234	9883615	15
46	1178263	9930342	14	46	1351392	9908266	14	46	1524109	9883172	14
47	1181151	9929999	13	47	1354274	9907873	13	47	1526984	9882728	13
48	1184040	9929655	12	48	1357156	9907478	12	48	1529858	9882284	12
49	1186928	9929310	11	49	1360038	9907083	11	49	1532733	9881838	11
50	1189816	9928965	10	50	1362919	9906687	10	50	1535607	9881392	10
51	1192704	9928618	9	51	1365801	9906290	9	51	1538482	9880945	9
52	1195593	9928271	8	52	1368683	9905893	8	52	1541356	9880497	8
53	1198481	9927922	7	53	1371564	9905494	7	53	1544230	9880048	7
54	1201368	9927573	6	54	1374445	9905095	6	54	1547104	9879599	6
55	1204256	9927224	5	55	1377327	9904694	5	55	1549978	9879148	5
56	1207144	9926873	4	56	1380208	9904293	4	56	1552851	9878697	4
57	1210031	9926521	3	57	1383089	9903891	3	57	1555725	9878245	3
58	1212919	9926169	2	58	1385970	9903489	2	58	1558598	9877792	2
59	1215806	9925816	1	59	1388850	9903085	1	59	1561472	9877338	1
60	1218693	9925462	0	60	1391731	9902681	0	60	1564345	9876883	0
	Cosine	Sine			Cosine	Sine			Cosine	Sine	
		Deg. 83.				Deg. 82.				Deg. 81.	

9 Deg.

	Sine	Cosine	
0	1564345	9876883	60
1	1567218	9876428	59
2	1570091	9875972	58
3	1572963	9875514	57
4	1575836	9875057	56
5	1578708	9874598	55
6	1581581	9874138	54
7	1584453	9873678	53
8	1587325	9873216	52
9	1590197	9872754	51
10	1593069	9872291	50
11	1595940	9871827	49
12	1598812	9871363	48
13	1601683	9870897	47
14	1604555	9870431	46
15	1607426	9869964	45
16	1610297	9869496	44
17	1613167	9869027	43
18	1616038	9868557	42
19	1618909	9868087	41
20	1621779	9867615	40
21	1624650	9867143	39
22	1627520	9866670	38
23	1630390	9866196	37
24	1633260	9865722	36
25	1636129	9865246	35
26	1638999	9864770	34
27	1641868	9864293	33
28	1644738	9863815	32
29	1647607	9863336	31
30	1650476	9862856	30
31	1653345	9862375	29
32	1656214	9861894	28
33	1659082	9861412	27
34	1661951	9860929	26
35	1664819	9860445	25
36	1667687	9859960	24
37	1670556	9859475	23
38	1673423	9858988	22
39	1676291	9858501	21
40	1679159	9858013	20
41	1682026	9857524	19
42	1684894	9857035	18
43	1687761	9856544	17
44	1690628	9856053	16
45	1693495	9855561	15
46	1696362	9855068	14
47	1699228	9854574	13
48	1702095	9854079	12
49	1704961	9853583	11
50	1707828	9853087	10
51	1710694	9852590	9
52	1713560	9852092	8
53	1716425	9851593	7
54	1719291	9851093	6
55	1722156	9850593	5
56	1725022	9850091	4
57	1727887	9849589	3
58	1730752	9849086	2
59	1733617	9848582	1
60	1736482	9848078	0
	Cosine	Sine	

Deg. 80.

10 Deg.

	Sine	Cosine	
0	1736482	9848078	60
1	1739346	9847572	59
2	1742211	9847066	58
3	1745075	9846558	57
4	1747939	9846050	56
5	1750803	9845542	55
6	1753667	9845032	54
7	1756531	9844521	53
8	1759395	9844010	52
9	1762258	9843498	51
10	1765121	9842985	50
11	1767984	9842471	49
12	1770847	9841956	48
13	1773710	9841441	47
14	1776573	9840924	46
15	1779435	9840407	45
16	1782298	9839889	44
17	1785160	9839370	43
18	1788022	9838850	42
19	1790884	9838330	41
20	1793746	9837808	40
21	1796607	9837286	39
22	1799469	9836763	38
23	1802330	9836239	37
24	1805191	9835715	36
25	1808052	9835189	35
26	1810913	9834663	34
27	1813774	9834136	33
28	1816635	9833608	32
29	1819495	9833079	31
30	1822355	9832549	30
31	1825215	9832019	29
32	1828075	9831487	28
33	1830935	9830955	27
34	1833795	9830422	26
35	1836654	9829888	25
36	1839514	9829353	24
37	1842373	9828818	23
38	1845232	9828282	22
39	1848091	9827744	21
40	1850949	9827206	20
41	1853808	9826668	19
42	1856666	9826128	18
43	1859524	9825587	17
44	1862382	9825046	16
45	1865240	9824504	15
46	1868098	9823961	14
47	1870956	9823417	13
48	1873813	9822873	12
49	1876670	9822327	11
50	1879528	9821781	10
51	1882385	9821234	9
52	1885241	9820686	8
53	1888098	9820137	7
54	1890954	9819587	6
55	1893811	9819037	5
56	1896667	9818485	4
57	1899523	9817933	3
58	1902379	9817380	2
59	1905234	9816826	1
60	1908090	9816272	0
	Cosine	Sine	

Deg. 79.

11 Deg.

	Sine	Cosine	
0	1908090	9816272	60
1	1910945	9815716	59
2	1913801	9815160	58
3	1916656	9814603	57
4	1919510	9814045	56
5	1922365	9813486	55
6	1925220	9812927	54
7	1928074	9812366	53
8	1930928	9811805	52
9	1933782	9811243	51
10	1936636	9810680	50
11	1939490	9810116	49
12	1942344	9809552	48
13	1945197	9808986	47
14	1948050	9808420	46
15	1950903	9807853	45
16	1953756	9807285	44
17	1956609	9806716	43
18	1959461	9806147	42
19	1962314	9805576	41
20	1965166	9805005	40
21	1968018	9804433	39
22	1970870	9803860	38
23	1973722	9803286	37
24	1976573	9802712	36
25	1979425	9802136	35
26	1982276	9801560	34
27	1985127	9800983	33
28	1987978	9800405	32
29	1990829	9799827	31
30	1993679	9799247	30
31	1996530	9798667	29
32	1999380	9798086	28
33	2002230	9797504	27
34	2005080	9796921	26
35	2007930	9796337	25
36	2010779	9795752	24
37	2013629	9795167	23
38	2016478	9794581	22
39	2019327	9793994	21
40	2022176	9793406	20
41	2025024	9792818	19
42	2027873	9792228	18
43	2030721	9791638	17
44	2033569	9791047	16
45	2036418	9790455	15
46	2039265	9789862	14
47	2042113	9789268	13
48	2044961	9788674	12
49	2047808	9788079	11
50	2050655	9787483	10
51	2053502	9786886	9
52	2056349	9786288	8
53	2059195	9785689	7
54	2062042	9785090	6
55	2064888	9784490	5
56	2067734	9783889	4
57	2070580	9783287	3
58	2073426	9782684	2
59	2076272	9782080	1
60	2079117	9781476	0
	Cosine	Sine	

Deg. 78.

12 Deg.				13 Deg.				14 Deg.			
	Sine	Cosine			Sine	Cosine			Sine	Cosine	
0	2079117	9781476	60	0	2249511	9743701	60	0	2419219	9702957	60
1	2081962	9780871	59	1	2252345	9743046	59	1	2422041	9702253	59
2	2084807	9780265	58	2	2255179	9742390	58	2	2424863	9701548	58
3	2087652	9779658	57	3	2258013	9741734	57	3	2427685	9700842	57
4	2090497	9779050	56	4	2260846	9741077	56	4	2430507	9700136	56
5	2093341	9778442	55	5	2263680	9740419	55	5	2433329	9699428	55
6	2096186	9777832	54	6	2266513	9739760	54	6	2436150	9698720	54
7	2099030	9777222	53	7	2269346	9739100	53	7	2438971	9698011	53
8	2101874	9776611	52	8	2272179	9738439	52	8	2441792	9697301	52
9	2104718	9775999	51	9	2275012	9737778	51	9	2444613	9696591	51
10	2107561	9775387	50	10	2277844	9737116	50	10	2447433	9695879	50
11	2110405	9774773	49	11	2280677	9736453	49	11	2450254	9695167	49
12	2113248	9774159	48	12	2283509	9735789	48	12	2453074	9694453	48
13	2116091	9773544	47	13	2286341	9735124	47	13	2455894	9693740	47
14	2118934	9772928	46	14	2289172	9734459	46	14	2458713	9693025	46
15	2121777	9772311	45	15	2292004	9733793	45	15	2461533	9692309	45
16	2124619	9771693	44	16	2294835	9733125	44	16	2464352	9691593	44
17	2127462	9771075	43	17	2297666	9732458	43	17	2467171	9690875	43
18	2130304	9770456	42	18	2300497	9731789	42	18	2469990	9690157	42
19	2133146	9769836	41	19	2303328	9731119	41	19	2472809	9689438	41
20	2135988	9769215	40	20	2306159	9730449	40	20	2475627	9688719	40
21	2138829	9768593	39	21	2308989	9729777	39	21	2478445	9687998	39
22	2141671	9767970	38	22	2311819	9729105	38	22	2481263	9687277	38
23	2144512	9767347	37	23	2314649	9728432	37	23	2484081	9686555	37
24	2147353	9766723	36	24	2317479	9727759	36	24	2486899	9685832	36
25	2150194	9766098	35	25	2320309	9727084	35	25	2489716	9685108	35
26	2153035	9765472	34	26	2323138	9726409	34	26	2492533	9684383	34
27	2155876	9764845	33	27	2325967	9725733	33	27	2495350	9683658	33
28	2158716	9764218	32	28	2328796	9725056	32	28	2498167	9682931	32
29	2161556	9763589	31	29	2331625	9724378	31	29	2500984	9682204	31
30	2164396	9762960	30	30	2334454	9723699	30	30	2503800	9681476	30
31	2167236	9762330	29	31	2337282	9723020	29	31	2506616	9680748	29
32	2170076	9761699	28	32	2340110	9722339	28	32	2509432	9680018	28
33	2172915	9761068	27	33	2342938	9721658	27	33	2512248	9679288	27
34	2175754	9760435	26	34	2345766	9720976	26	34	2515063	9678557	26
35	2178593	9759802	25	35	2348594	9720294	25	35	2517879	9677825	25
36	2181432	9759168	24	36	2351421	9719610	24	36	2520694	9677092	24
37	2184271	9758533	23	37	2354248	9718926	23	37	2523508	9676358	23
38	2187110	9757897	22	38	2357075	9718240	22	38	2526323	9675624	22
39	2189948	9757260	21	39	2359902	9717554	21	39	2529137	9674888	21
40	2192786	9756623	20	40	2362729	9716867	20	40	2531952	9674152	20
41	2195624	9755985	19	41	2365555	9716180	19	41	2534766	9673415	19
42	2198462	9755345	18	42	2368381	9715491	18	42	2537579	9672678	18
43	2201300	9754706	17	43	2371207	9714802	17	43	2540393	9671939	17
44	2204137	9754065	16	44	2374033	9714112	16	44	2543206	9671200	16
45	2206974	9753423	15	45	2378659	9713421	15	45	2546019	9670459	15
46	2209811	9752781	14	46	2379684	9712729	14	46	2548832	9669718	14
47	2212648	9752138	13	47	2382510	9712036	13	47	2551645	9668977	13
48	2215485	9751494	12	48	2385335	9711343	12	48	2554458	9668234	12
49	2218321	9750849	11	49	2388159	9710649	11	49	2557270	9667490	11
50	2221158	9750203	10	50	2390984	9709953	10	50	2560082	9666746	10
51	2223994	9749556	9	51	2393808	9709258	9	51	2562894	9666001	9
52	2226830	9748909	8	52	2396633	9708561	8	52	2565705	9665255	8
53	2229666	9748261	7	53	2399457	9707863	7	53	2568517	9664508	7
54	2232501	9747612	6	54	2402280	9707165	6	54	2571328	9663761	6
55	2235337	9746962	5	55	2405104	9706466	5	55	2574139	9663012	5
56	2238172	9746311	4	56	2407927	9705766	4	56	2576950	9662263	4
57	2241007	9745660	3	57	2410751	9705065	3	57	2579760	9661513	3
58	2243842	9745008	2	58	2413574	9704363	2	58	2582570	9660762	2
59	2246676	9744355	1	59	2416396	9703661	1	59	2585381	9660011	1
60	2249511	9743701	0	60	2419219	9702957	0	60	2588190	9659258	0
	Cosine	Sine			Cosine	Sine			Cosine	Sine	
		Deg. 77.				Deg. 76.				Deg. 75.	

	15 Deg.				16 Deg.				17 Deg.		
	Sine	Cosine			Sine	Cosine			Sine	Cosine	
0	2588190	9659258	60	0	2756374	9612617	60	0	2923717	9563048	60
1	2591000	9658505	59	1	2759170	9611815	59	1	2926499	9562197	59
2	2593810	9657751	58	2	2761965	9611012	58	2	2929280	9561345	58
3	2596619	9656996	57	3	2764761	9610208	57	3	2932061	9560492	57
4	2599428	9656240	56	4	2767556	9609403	56	4	2934842	9559639	56
5	2602237	9655484	55	5	2770352	9608598	55	5	2937623	9558785	55
6	2605045	9654726	54	6	2773147	9607792	54	6	2940403	9557930	54
7	2607853	9653968	53	7	2775941	9606984	53	7	2943183	9557074	53
8	2610662	9653209	52	8	2778736	9606177	52	8	2945963	9556218	52
9	2613469	9652449	51	9	2781530	9605368	51	9	2948743	9555361	51
10	2616277	9651689	50	10	2784324	9604558	50	10	2951522	9554502	50
11	2619085	9650927	49	11	2787118	9603748	49	11	2954302	9553643	49
12	2621892	9650165	48	12	2789911	9602937	48	12	2957081	9552784	48
13	2624699	9649402	47	13	2792704	9602125	47	13	2959859	9551923	47
14	2627506	9648638	46	14	2795497	9601312	46	14	2962638	9551062	46
15	2630312	9647873	45	15	2798290	9600499	45	15	2965416	9550199	45
16	2633118	9647108	44	16	2801083	9599684	44	16	2968194	9549336	44
17	2635925	9646341	43	17	2803875	9598869	43	17	2970971	9548473	43
18	2638730	9645574	42	18	2806667	9598053	42	18	2973749	9547608	42
19	2641536	9644806	41	19	2809459	9597236	41	19	2976526	9546743	41
20	2644342	9644037	40	20	2812251	9596418	40	20	2979303	9545876	40
21	2647147	9643268	39	21	2815042	9595600	39	21	2982079	9545009	39
22	2649952	9642497	38	22	2817833	9594781	38	22	2984856	9544141	38
23	2652757	9641726	37	23	2820624	9593961	37	23	2987632	9543273	37
24	2655561	9640954	36	24	2823415	9593140	36	24	2950408	9542403	36
25	2658366	9640181	35	25	2826205	9592318	35	25	2993184	9541533	35
26	2661170	9639407	34	26	2828995	9591496	34	26	2995959	9540662	34
27	2663973	9638633	33	27	2831785	9590672	33	27	2998734	9539790	33
28	2666777	9637858	32	28	2834575	9589848	32	28	3001509	9538917	32
29	2669581	9637081	31	29	2837364	9589023	31	29	3004284	9538044	31
30	2672384	9636305	30	30	2840153	9588197	30	30	3007058	9537170	30
31	2675187	9635527	29	31	2842942	9587371	29	31	3009832	9536294	29
32	2677989	9634748	28	32	2845731	9586543	28	32	3012606	9535418	28
33	2680792	9633969	27	33	2848520	9585715	27	33	3015380	9534542	27
34	2683594	9633189	26	34	2851308	9584886	26	34	3018153	9533664	26
35	2686396	9632408	25	35	2854096	9584056	25	35	3020926	9532786	25
36	2689198	9631626	24	36	2856884	9583226	24	36	3023699	9531907	24
37	2692000	9630843	23	37	2859671	9582394	23	37	3026471	9531027	23
38	2694801	9630060	22	38	2862458	9581562	22	38	3029244	9530146	22
39	2697602	9629275	21	39	2865246	9580729	21	39	3032016	9529264	21
40	2700403	9628490	20	40	2868032	9579895	20	40	3034788	9528382	20
41	2703204	9627074	19	41	2870819	9579060	19	41	3037559	9527499	19
42	2706004	9626917	18	42	2873605	9578225	18	42	3040331	9526615	18
43	2708805	9626130	17	43	2876391	9577389	17	43	3043102	9525730	17
44	2711605	9625342	16	44	2879177	9576552	16	44	3045872	9524844	16
45	2714404	9624552	15	45	2881963	9575714	15	45	3048643	9523958	15
46	2717204	9623762	14	46	2884748	9574875	14	46	3051413	9523071	14
47	2720003	9622972	13	47	2887533	9574035	13	47	3054183	9522183	13
48	2722802	9622180	12	48	2890318	9573195	12	48	3056953	9521294	12
49	2725601	9621387	11	49	2893103	9572354	11	49	3059723	9520404	11
50	2728400	9620594	10	50	2895887	9571512	10	50	3062492	9519514	10
51	2731198	9619800	9	51	2898671	9570669	9	51	3065261	9518623	9
52	2733997	9619005	8	52	2901455	9569825	8	52	3068030	9517731	8
53	2736794	9618210	7	53	2904239	9568981	7	53	3070798	9516838	7
54	2739592	9617413	6	54	2907022	9568136	6	54	3073566	9515944	6
55	2742390	9616616	5	55	2909805	9567290	5	55	3076334	9515050	5
56	2745187	9615818	4	56	2912588	9566443	4	56	3079102	9514154	4
57	2747984	9615019	3	57	2915371	9565595	3	57	3081869	9513258	3
58	2750781	9614219	2	58	2918153	9564747	2	58	3084636	9512361	2
59	2753577	9613418	1	59	2920935	9563898	1	59	3087403	9511464	1
60	2756374	9612617	0	60	2923717	9563048	0	60	3090170	9510565	0
	Cosine	Sine			Cosine	Sine			Cosine	Sine	
	Deg. 74.				Deg. 73				Deg. 72.		

18 Deg.				19 Deg.				20 Deg.			
	Sine	**Cosine**			**Sine**	**Cosine**			**Sine**	**Cosine**	
0	3090170	9510565	60	0	3255682	9455186	60	0	3420201	9396926	60
1	3092936	9509666	59	1	3258432	9454238	59	1	3422935	9395931	59
2	3095702	9508766	58	2	3261182	9453290	58	2	3425668	9394935	58
3	3098468	9507865	57	3	3263932	9452341	57	3	3428400	9393938	57
4	3101234	9506963	56	4	3266681	9451391	56	4	3431133	9392940	56
5	3103999	9506061	55	5	3269430	9450441	55	5	3433865	9391942	55
6	3106764	9505157	54	6	3272179	9449489	54	6	3436597	9390943	54
7	3109529	9504253	53	7	3274928	9448537	53	7	3439329	9389942	53
8	3112294	9503348	52	8	3277676	9447584	52	8	3442060	9388942	52
9	3115058	9502443	51	9	3280424	9446630	51	9	3444791	9387940	51
10	3117822	9501536	50	10	3283172	9445675	50	10	3447521	9386938	50
11	3120586	9500629	49	11	3285919	9444720	49	11	3450252	9385934	49
12	3123349	9499721	48	12	3288666	9443764	48	12	3452982	9384930	48
13	3126112	9498812	47	13	3291413	9442807	47	13	3455712	9383925	47
14	3128875	9497902	46	14	3294160	9441849	46	14	3458441	9382920	46
15	3131638	9496991	45	15	3296906	9440890	45	15	3461171	9381913	45
16	3134400	9496080	44	16	3299653	9439931	44	16	3463900	9380906	44
17	3137163	9495168	43	17	3302398	9438971	43	17	3466628	9379898	43
18	3139925	9494255	42	18	3305144	9438010	42	18	3469357	9378889	42
19	3142686	9493341	41	19	3307889	9437048	41	19	3472085	9377880	41
20	3145448	9492426	40	20	3310634	9436085	40	20	3474812	9376869	40
21	3148209	9491511	39	21	3313379	9435122	39	21	3477540	9375858	39
22	3150969	9490595	38	22	3316123	9434157	38	22	3480267	9374846	38
23	3153730	9489678	37	23	3318867	9433192	37	23	3482994	9373833	37
24	3156490	9488760	36	24	3321611	9432227	36	24	3485720	9372820	36
25	3159250	9487842	35	25	3324355	9431260	35	25	3488447	9371806	35
26	3162010	9486922	34	26	3327098	9430293	34	26	3491173	9370790	34
27	3164770	9486002	33	27	3329841	9429324	33	27	3493898	9369774	33
28	3167529	9485081	32	28	3332584	9428355	32	28	3496624	9368758	32
29	3170288	9484159	31	29	3335326	9427386	31	29	3499349	9367740	31
30	3173047	9483237	30	30	3338069	9426415	30	30	3502074	9366722	30
31	3175805	9482313	29	31	3340810	9425444	29	31	3504798	9365703	29
32	3178563	9481389	28	32	3343552	9424471	28	32	3507523	9364683	28
33	3181321	9480464	27	33	3346293	9423498	27	33	3510247	9363662	27
34	3184079	9479538	26	34	3349034	9422525	26	34	3512970	9362641	26
35	3186836	9478612	25	35	3351775	9421550	25	35	3515693	9361618	25
36	3189593	9477684	24	36	3354516	9420575	24	36	3518416	9360595	24
37	3192350	9476756	23	37	3357256	9419598	23	37	3521139	9359571	23
38	3195106	9475827	22	38	3359996	9418621	22	38	3523862	9358547	22
39	3197863	9474897	21	39	3362735	9417644	21	39	3526584	9357521	21
40	3200619	9473966	20	40	3365475	9416665	20	40	3529306	9356495	20
41	3203374	9473035	19	41	3368214	9415686	19	41	3532027	9355468	19
42	3206130	9472103	18	42	3370953	9414705	18	42	3534748	9354440	18
43	3208885	9471170	17	43	3373691	9413724	17	43	3537469	9353412	17
44	3211640	9470236	16	44	3376429	9412743	16	44	3540190	9352382	16
45	3214395	9469301	15	45	3379167	9411760	15	45	3542910	9351352	15
46	3217149	9468366	14	46	3381905	9410777	14	46	3545630	9350321	14
47	3219903	9467430	13	47	3384642	9409793	13	47	3548350	9349289	13
48	3222657	9466493	12	48	3387379	9408808	12	48	3551070	9348257	12
49	3225411	9465555	11	49	3390116	9407822	11	49	3553789	9347223	11
50	3228164	9464616	10	50	3392852	9406835	10	50	3556508	9346189	10
51	3230917	9463677	9	51	3395589	9405848	9	51	3559226	9345154	9
52	3233670	9462736	8	52	3398325	9404860	8	52	3561944	9344119	8
53	3236422	9461795	7	53	3401060	9403871	7	53	3564662	9343082	7
54	3239174	9460854	6	54	3403796	9402881	6	54	3567380	9342045	6
55	3241926	9459911	5	55	3406531	9401891	5	55	3570097	9341007	5
56	3244678	9458968	4	56	3409265	9400899	4	56	3572814	9339968	4
57	3247429	9458023	3	57	3412000	9399907	3	57	3575531	9338928	3
58	3250180	9457078	2	58	3414734	9398914	2	58	3578248	9337888	2
59	3252931	9456132	1	59	3417468	9397921	1	59	3580964	9336846	1
60	3255682	9455186	0	60	3420201	9396926	0	60	3583679	9335804	0
	Cosine	**Sine**			**Cosine**	**Sine**			**Cosine**	**Sine**	
		Deg. 71.				Deg. 70.				Deg. 69.	

21 Deg.

	Sine	Cosine	
0	3583679	9335804	60
1	3586395	9334761	59
2	3589110	9333718	58
3	3591825	9332673	57
4	3594540	9331628	56
5	3597254	9330582	55
6	3599968	9329535	54
7	3602682	9328488	53
8	3605395	9327439	52
9	3608108	9326390	51
10	3610821	9325340	50
11	3613534	9324290	49
12	3616246	9323238	48
13	3618958	9322186	47
14	3621669	9321133	46
15	3624380	9320079	45
16	3627091	9319024	44
17	3629802	9317969	43
18	3632512	9316912	42
19	3635222	9315855	41
20	3637932	9314797	40
21	3640641	9313739	39
22	3643351	9312679	38
23	3646059	9311619	37
24	3648768	9310558	36
25	3651476	9309496	35
26	3654184	9308434	34
27	3656891	9307370	33
28	3659599	9306306	32
29	3662306	9305241	31
30	3665012	9304176	30
31	3667719	9303109	29
32	3670425	9302042	28
33	3673130	9300974	27
34	3675836	9299905	26
35	3678541	9298835	25
36	3581246	9297765	24
37	3683950	9296694	23
38	3686654	9295622	22
39	3689358	9294549	21
40	3692061	9293475	20
41	3694765	9292401	19
42	3697468	9291326	18
43	3700170	9290250	17
44	3702872	9289173	16
45	3705574	9288096	15
46	3708276	9287017	14
47	3710977	9285938	13
48	3713678	9284858	12
49	3716379	9283778	11
50	3719079	9282696	10
51	3721780	9281614	9
52	3724479	9280531	8
53	3727179	9279447	7
54	3729878	9278363	6
55	3732577	9277277	5
56	3735275	9276191	4
57	3737973	9275104	3
58	3740671	9274016	2
59	3743369	9272928	1
60	3746066	9271839	0
	Cosine	Sine	

Deg. 68.

22 Deg.

	Sine	Cosine	
0	3746066	9271839	60
1	3748763	9270748	59
2	3751459	9269658	58
3	3754156	9268566	57
4	3756852	9267474	56
5	3759547	9266380	55
6	3762243	9265286	54
7	3764938	9264192	53
8	3767632	9263096	52
9	3770327	9262000	51
10	3773021	9260902	50
11	3775714	9259805	49
12	3778408	9258706	48
13	3781101	9257606	47
14	3783794	9256506	46
15	3786486	9255405	45
16	3789178	9254303	44
17	3791870	9253201	43
18	3794562	9252097	42
19	3797253	9250993	41
20	3799944	9249888	40
21	3802634	9248782	39
22	3805324	9247676	38
23	3808014	9246568	37
24	3810704	9245460	36
25	3813393	9244351	35
26	3816082	9243242	34
27	3818770	9242131	33
28	3821459	9241020	32
29	3824147	9239908	31
30	3826834	9238795	30
31	3829522	9237682	29
32	3832209	9236567	28
33	3834895	9235452	27
34	3837582	9234336	26
35	3840268	9233220	25
36	3842953	9232102	24
37	3845639	9230984	23
38	3848324	9229865	22
39	3851008	9228745	21
40	3853693	9227624	20
41	3856377	9226503	19
42	3859060	9225381	18
43	3861744	9224258	17
44	3864427	9223134	16
45	3867110	9222010	15
46	3869792	9220884	14
47	3872474	9219758	13
48	3875156	9218632	12
49	3877837	9217504	11
50	3880518	9216375	10
51	3883199	9215246	9
52	3885880	9214116	8
53	3888560	9212986	7
54	3891240	9211854	6
55	3893919	9210722	5
56	3896598	9209589	4
57	3899277	9208455	3
58	3901955	9207320	2
59	3904633	9206185	1
60	3907311	9205049	0
	Cosine	Sine	

Deg. 67.

23 Deg.

	Sine	Cosine	
0	3907311	9205049	60
1	3909989	9203912	59
2	3912666	9202774	58
3	3915343	9201635	57
4	3918019	9200496	56
5	3920695	9199356	55
6	3923371	9198215	54
7	3926047	9197073	53
8	3928722	9195931	52
9	3931397	9194788	51
10	3934071	9193644	50
11	3936745	9192499	49
12	3939419	9191353	48
13	3942093	9190207	47
14	3944766	9189060	46
15	3947439	9187912	45
16	3950111	9186763	44
17	3952783	9185614	43
18	3955455	9184464	42
19	3958127	9183313	41
20	3960798	9182161	40
21	3963468	9181009	39
22	3966139	9179855	38
23	3968809	9178701	37
24	3971479	9177546	36
25	3974148	9176391	35
26	3976818	9175234	34
27	3979486	9174077	33
28	3982155	9172919	32
29	3984823	9171760	31
30	3987491	9170601	30
31	3990158	9169440	29
32	3992825	9168279	28
33	3995492	9167118	27
34	3998158	9165955	26
35	4000825	9164791	25
36	4003490	9163627	24
37	4006156	9162462	23
38	4008821	9161297	22
39	4011486	9160130	21
40	4014150	9158963	20
41	4016814	9157795	19
42	4019478	9156626	18
43	4022141	9155456	17
44	4024804	9154286	16
45	4027467	9153115	15
46	4030129	9151943	14
47	4032791	9150770	13
48	4035453	9149597	12
49	4038114	9148422	11
50	4040775	9147247	10
51	4043436	9146072	9
52	4046096	9144895	8
53	4048756	9143718	7
54	4051416	9142540	6
55	4054075	9141361	5
56	4056734	9140181	4
57	4059393	9139001	3
58	4062051	9137819	2
59	4064709	9136637	1
60	4067366	9135455	0
	Cosine	Sine	

Deg. 66.

24 Deg.				25 Deg.				26 Deg.			
′	Sine	Cosine	′	′	Sine	Cosine	′	′	Sine	Cosine	′
0	4067366	9135455	60	0	4226183	9063078	60	0	4383711	8987940	60
1	4070024	9134271	59	1	4228819	9061848	59	1	4386326	8986665	59
2	4072681	9133087	58	2	4231455	9060618	58	2	4388940	8985389	58
3	4075337	9131902	57	3	4234090	9059386	57	3	4391553	8984112	57
4	4077993	9130716	56	4	4236725	9058154	56	4	4394166	8982834	56
5	4080649	9129529	55	5	4239360	9056922	55	5	4396779	8981555	55
6	4083305	9128342	54	6	4241994	9055688	54	6	4399392	8980276	54
7	4085960	9127154	53	7	4244628	9054454	53	7	4402004	8978996	53
8	4088615	9125965	52	8	4247262	9053219	52	8	4404615	8977715	52
9	4091269	9124775	51	9	4249895	9051983	51	9	4407227	8976433	51
10	4093923	9123584	50	10	4252528	9050746	50	10	4409838	8975151	50
11	4096577	9122393	49	11	4255161	9049509	49	11	4412448	8973868	49
12	4099230	9121201	48	12	4257793	9048271	48	12	4415059	8972584	48
13	4101883	9120008	47	13	4260425	9047032	47	13	4417668	8971299	47
14	4104536	9118815	46	14	4263056	9045792	46	14	4420278	8970014	46
15	4107189	9117620	45	15	4265687	9044551	45	15	4422887	8968727	45
16	4109841	9116425	44	16	4268318	9043310	44	16	4425496	8967440	44
17	4112492	9115229	43	17	4270949	9042068	43	17	4428104	8966153	43
18	4115144	9114033	42	18	4273579	9040825	42	18	4430712	8964864	42
19	4117795	9112835	41	19	4276208	9039582	41	19	4433319	8963575	41
20	4120445	9111637	40	20	4278838	9038338	40	20	4435927	8962285	40
21	4123096	9110438	39	21	4281467	9037093	39	21	4438534	8960994	39
22	4125745	9109238	38	22	4284095	9035847	38	22	4441140	8959703	38
23	4128395	9108038	37	23	4286723	9034600	37	23	4443746	8958411	37
24	4131044	9105837	36	24	4289351	9033353	36	24	4446352	8957118	36
25	4133693	9105635	35	25	4291979	9032105	35	25	4448957	8955824	35
26	4136342	9104432	34	26	4294606	9030856	34	26	4451562	8954529	34
27	4138990	9103228	33	27	4297233	9029606	33	27	4454167	8953234	33
28	4141638	9102024	32	28	4299859	9028356	32	28	4456771	8951938	32
29	4144285	9100819	31	29	4302485	9027105	31	29	4459375	8950641	31
30	4146932	9099613	30	30	4305111	9025853	30	30	4461978	8949344	30
31	4149579	9098406	29	31	4307736	9024600	29	31	4464581	8948045	29
32	4152226	9097199	28	32	4310361	9023347	28	32	4467184	8946746	28
33	4154872	9095990	27	33	4312986	9022092	27	33	4469786	8945446	27
34	4157517	9094781	26	34	4315610	9020838	26	34	4472388	8944146	26
35	4160163	9093572	25	35	4318234	9019582	25	35	4474990	8942844	25
36	4162808	9092361	24	36	4320857	9018325	24	36	4477591	8941542	24
37	4165453	9091150	23	37	4323481	9017068	23	37	4480192	8940240	23
38	4168097	9089938	22	38	4326103	9015810	22	38	4482792	8938936	22
39	4170741	9088725	21	39	4328726	9014551	21	39	4485392	8937632	21
40	4173385	9087511	20	40	4331348	9013292	20	40	4487992	8936326	20
41	4176028	9086297	19	41	4333970	9012031	19	41	4490591	8935021	19
42	4178671	9085082	18	42	4336591	9010770	18	42	4493190	8933714	18
43	4181313	9083866	17	43	4339212	9009508	17	43	4495789	8932406	17
44	4183956	9082649	16	44	4341832	9008246	16	44	4498387	8931098	16
45	4186597	9081432	15	45	4344453	9006982	15	45	4500984	8929789	15
46	4189239	9080214	14	46	4347072	9005718	14	46	4503582	8928480	14
47	4191880	9078995	13	47	4349692	9004453	13	47	4506179	8927169	13
48	4194521	9077775	12	48	4352311	9003188	12	48	4508775	8925858	12
49	4197161	9076554	11	49	4354930	9001921	11	49	4511372	8924546	11
50	4199801	9075333	10	50	4357548	9000654	10	50	4513967	8923234	10
51	4202441	9074111	9	51	4360166	8999386	9	51	4516563	8921920	9
52	4205080	9072888	8	52	4362784	8998117	8	52	4519158	8920606	8
53	4207719	9071665	7	53	4365401	8996848	7	53	4521753	8919291	7
54	4210358	9070440	6	54	4368018	8995578	6	54	4524347	8917975	6
55	4212996	9069215	5	55	4370634	8994307	5	55	4526941	8916659	5
56	4215634	9067989	4	56	4373251	8993035	4	56	4529535	8915342	4
57	4218272	9066762	3	57	4375866	8991763	3	57	4532128	8914024	3
58	4220909	9065535	2	58	4378482	8990489	2	58	4534721	8912705	2
59	4223546	9064307	1	59	4381097	8989215	1	59	4537313	8911385	1
60	4226183	9063078	0	60	4383711	8987940	0	60	4539905	8910065	0
′	Cosine	Sine	′	′	Cosine	Sine	′	′	Cosine	Sine	′
		Deg. 65.				Deg. 64.				Deg. 63.	

27 Deg.

'	Sine	Cosine	'
0	4539905	8910065	60
1	4542497	8908744	59
2	4545088	8907423	58
3	4547679	8906100	57
4	4550269	8904777	56
5	4552859	8903453	55
6	4555449	8902128	54
7	4558038	8900803	53
8	4560627	8899476	52
9	4563216	8898149	51
10	4565804	8896822	50
11	4568392	8895493	49
12	4570979	8894164	48
13	4573566	8892834	47
14	4576153	8891503	46
15	4578739	8890171	45
16	4581325	8888839	44
17	4583910	8887506	43
18	4586496	8886172	42
19	4589080	8884838	41
20	4591665	8883503	40
21	4594248	8882166	39
22	4596832	8880830	38
23	4599415	8879492	37
24	4601998	8878154	36
25	4604580	8876815	35
26	4607162	8875475	34
27	4609744	8874134	33
28	4612325	8872793	32
29	4614906	8871451	31
30	4617486	8870108	30
31	4620066	8868765	29
32	4622646	8867420	28
33	4625225	8866075	27
34	4627804	8864730	26
35	4630382	8863383	25
36	4632960	8862036	24
37	4635538	8860688	23
38	4638115	8859339	22
39	4640692	8857989	21
40	4643269	8856639	20
41	4645845	8855288	19
42	4648420	8853936	18
43	4650996	8852584	17
44	4653571	8851230	16
45	4656145	8849876	15
46	4658719	8848522	14
47	4661293	8847166	13
48	4663866	8845810	12
49	4666439	8844453	11
50	4669012	8843095	10
51	4671584	8841736	9
52	4674156	8840377	8
53	4676727	8839017	7
54	4679298	8837656	6
55	4681869	8836295	5
56	4684439	8834933	4
57	4687009	8833569	3
58	4689578	8832206	2
59	4692147	8830841	1
60	4694716	8829476	0
'	Cosine	Sine	'

Deg. 62.

28 Deg.

'	Sine	Cosine	'
0	4694716	8829476	60
1	4697284	8828110	59
2	4699852	8826743	58
3	4702419	8825376	57
4	4704986	8824007	56
5	4707553	8822638	55
6	4710119	8821269	54
7	4712685	8819898	53
8	4715250	8818527	52
9	4717815	8817155	51
10	4720380	8815782	50
11	4722944	8814409	49
12	4725508	8813035	48
13	4728071	8811660	47
14	4730634	8810284	46
15	4733197	8808907	45
16	4735759	8807530	44
17	4738321	8806152	43
18	4740882	8804774	42
19	4743443	8803394	41
20	4746004	8802014	40
21	4748564	8800633	39
22	4751124	8799251	38
23	4753683	8797869	37
24	4756242	8796486	36
25	4758801	8795102	35
26	4761359	8793717	34
27	4763917	8792332	33
28	4766474	8790946	32
29	4769031	8789559	31
30	4771588	8788171	30
31	4774144	8786783	29
32	4776700	8785394	28
33	4779255	8784004	27
34	4781810	8782613	26
35	4784364	8781222	25
36	4786919	8779830	24
37	4789472	8778437	23
38	4792026	8777043	22
39	4794579	8775649	21
40	4797131	8774254	20
41	4799683	8772858	19
42	4802235	8771462	18
43	4804786	8770064	17
44	4807337	8768666	16
45	4809888	8767268	15
46	4812438	8765868	14
47	4814987	8764468	13
48	4817537	8763067	12
49	4820086	8761665	11
50	4822634	8760263	10
51	4825182	8758859	9
52	4827730	8757455	8
53	4830277	8756051	7
54	4832824	8754645	6
55	4835370	8753239	5
56	4837916	8751832	4
57	4840462	8750425	3
58	4843007	8749016	2
59	4845552	8747607	1
60	4848096	8746197	0
'	Cosine	Sine	'

Deg. 61.

29 Deg.

'	Sine	Cosine	'
0	4848096	8746197	60
1	4850640	8744786	59
2	4853184	8743375	58
3	4855727	8741963	57
4	4858270	8740550	56
5	4860812	8739137	55
6	4863354	8737722	54
7	4865895	8736307	53
8	4868436	8734891	52
9	4870977	8733475	51
10	4873517	8732058	50
11	4876057	8730640	49
12	4878597	8729221	48
13	4881136	8727801	47
14	4883674	8726381	46
15	4886212	8724960	45
16	4888750	8723538	44
17	4891288	8722116	43
18	4893825	8720693	42
19	4896361	8719269	41
20	4898897	8717844	40
21	4901433	8716419	39
22	4903968	8714993	38
23	4906503	8713566	37
24	4909038	8712138	36
25	4911572	8710710	35
26	4914105	8709281	34
27	4916638	8707851	33
28	4919171	8706420	32
29	4921704	8704989	31
30	4924236	8703557	30
31	4926767	8702124	29
32	4929298	8700691	28
33	4931829	8699256	27
34	4934359	8697821	26
35	4936889	8696386	25
36	4939419	8694949	24
37	4941948	8693512	23
38	4944476	8692074	22
39	4947005	8690636	21
40	4949532	8689196	20
41	4952060	8687756	19
42	4954587	8686315	18
43	4957113	8684874	17
44	4959639	8683431	16
45	4962165	8681988	15
46	4964690	8680544	14
47	4967215	8679100	13
48	4969740	8677655	12
49	4972264	8676209	11
50	4974787	8674762	10
51	4977310	8673314	9
52	4979833	8671866	8
53	4982355	8670417	7
54	4984877	8668967	6
55	4987399	8667517	5
56	4989920	8666066	4
57	4992441	8664614	3
58	4994961	8663161	2
59	4997481	8661708	1
60	5000000	8660254	0
'	Cosine	Sine	'

Deg. 60.

30 Deg.

'	Sine	Cosine	'
0	5000000	8660254	60
1	5002519	8658799	59
2	5005037	8657344	58
3	5007556	8655887	57
4	5010073	8654430	56
5	5012591	8652973	55
6	5015107	8651514	54
7	5017624	8650055	53
8	5020140	8648595	52
9	5022655	8647134	51
10	5025170	8645673	50
11	5027685	8644211	49
12	5030199	8642748	48
13	5032713	8641284	47
14	5035227	8639820	46
15	5037740	8638355	45
16	5040252	8636889	44
17	5042765	8635423	43
18	5045276	8633956	42
19	5047788	8632488	41
20	5050298	8631019	40
21	5052809	8629549	39
22	5055319	8628079	38
23	5057828	8626608	37
24	5060338	8625137	36
25	5062846	8623664	35
26	5065355	8622191	34
27	5067863	8620717	33
28	5070370	8619243	32
29	5072877	8617768	31
30	5075384	8616292	30
31	5077890	8614815	29
32	5080396	8613337	28
33	5082901	8611859	27
34	5085406	8610380	26
35	5087910	8608901	25
36	5090414	8607420	24
37	5092918	8605939	23
38	5095421	8604457	22
39	5097924	8602975	21
40	5100426	8601491	20
41	5102928	8600007	19
42	5105429	8598523	18
43	5107930	8597037	17
44	5110431	8595551	16
45	5112931	8594064	15
46	5115431	8592576	14
47	5117930	8591088	13
48	5120429	8589599	12
49	5122927	8588109	11
50	5125425	8586619	10
51	5127923	8585127	9
52	5130420	8583635	8
53	5132916	8582143	7
54	5135413	8580649	6
55	5137908	8579155	5
56	5140404	8577660	4
57	5142899	8576164	3
58	5145393	8574668	2
59	5147887	8573171	1
60	5150381	8571673	0
'	Cosine	Sine	'

Deg. 59.

31 Deg.

'	Sine	Cosine	'
0	5150381	8571673	60
1	5152874	8570174	59
2	5155367	8568675	58
3	5157859	8567175	57
4	5160351	8565674	56
5	5162842	8564173	55
6	5165333	8562671	54
7	5167824	8561168	53
8	5170314	8559664	52
9	5172804	8558160	51
10	5175293	8556655	50
11	5177782	8555149	49
12	5180270	8553643	48
13	5182758	8552135	47
14	5185246	8550627	46
15	5187733	8549119	45
16	5190219	8547609	44
17	5192705	8546099	43
18	5195191	8544588	42
19	5197676	8543077	41
20	5200161	8541564	40
21	5202646	8540051	39
22	5205130	8538538	38
23	5207613	8537023	37
24	5210096	8535508	36
25	5212579	8533992	35
26	5215061	8532475	34
27	5217543	8530958	33
28	5220024	8529440	32
29	5222505	8527921	31
30	5224986	8526402	30
31	5227466	8524881	29
32	5229945	8523360	28
33	5232424	8521839	27
34	5234903	8520316	26
35	5237381	8518793	25
36	5239859	8517269	24
37	5242336	8515745	23
38	5244813	8514219	22
39	5247290	8512693	21
40	5249766	8511167	20
41	5252241	8509639	19
42	5254717	8508111	18
43	5257191	8506582	17
44	5259665	8505053	16
45	5262139	8503522	15
46	5264613	8501991	14
47	5267085	8500459	13
48	5269558	8498927	12
49	5272030	8497394	11
50	5274502	8495860	10
51	5276973	8494325	9
52	5279443	8492790	8
53	5281914	8491254	7
54	5284383	8489717	6
55	5286853	8488179	5
56	5289322	8486641	4
57	5291790	8485102	3
58	5294258	8483562	2
59	5296726	8482022	1
60	5299193	8480481	0
'	Cosine	Sine	'

Deg. 58.

32 Deg.

'	Sine	Cosine	'
0	5299193	8480481	60
1	5301659	8478939	59
2	5304125	8477397	58
3	5306591	8475853	57
4	5309057	8474309	56
5	5311521	8472765	55
6	5313986	8471219	54
7	5316450	8469673	53
8	5318913	8468126	52
9	5321376	8466579	51
10	5323839	8465030	50
11	5326301	8463481	49
12	5328763	8461932	48
13	5331224	8460381	47
14	5333685	8458830	46
15	5336145	8457278	45
16	5338605	8455726	44
17	5341065	8454172	43
18	5343523	8452618	42
19	5345982	8451064	41
20	5348440	8449508	40
21	5350898	8447952	39
22	5353355	8446395	38
23	5355812	8444838	37
24	5358268	8443279	36
25	5360724	8441720	35
26	5363179	8440161	34
27	5365634	8438600	33
28	5368089	8437039	32
29	5370543	8435477	31
30	5372996	8433914	30
31	5375449	8432351	29
32	5377902	8430787	28
33	5380354	8429222	27
34	5382806	8427657	26
35	5385257	8426091	25
36	5387708	8424524	24
37	5390158	8422956	23
38	5392608	8421388	22
39	5395058	8419819	21
40	5397507	8418249	20
41	5399955	8416679	19
42	5402403	8415108	18
43	5404851	8413536	17
44	5407298	8411963	16
45	5409745	8410390	15
46	5412191	8408816	14
47	5414637	8407241	13
48	5417082	8405666	12
49	5419527	8404090	11
50	5421971	8402513	10
51	5424415	8400936	9
52	5426859	8399357	8
53	5429302	8397778	7
54	5431744	8396199	6
55	5434187	8394618	5
56	5436628	8393037	4
57	5439069	8391455	3
58	5441510	8389873	2
59	5443951	8388290	1
60	5446390	8386706	0
'	Cosine	Sine	'

Deg. 57.

33 Deg.

′	Sine	Cosine	′
0	5446390	8386706	60
1	5448830	8385121	59
2	5451269	8383536	58
3	5453707	8381950	57
4	5456145	8380363	56
5	5458583	8378775	55
6	5461020	8377187	54
7	5463456	8375598	53
8	5465892	8374009	52
9	5468328	8372418	51
10	5470763	8370827	50
11	5473198	8369236	49
12	5475632	8367643	48
13	5478066	8366050	47
14	5480499	8364456	46
15	5482932	8362862	45
16	5485365	8361266	44
17	5487797	8359670	43
18	5490228	8358074	42
19	5492659	8356476	41
20	5495090	8354878	40
21	5497520	8353279	39
22	5499950	8351680	38
23	5502379	8350080	37
24	5504807	8348479	36
25	5507236	8346877	35
26	5509663	8345275	34
27	5512091	8343672	33
28	5514518	8342068	32
29	5516944	8340463	31
30	5519370	8338858	30
31	5521795	8337252	29
32	5524220	8335646	28
33	5526645	8334038	27
34	5529069	8332430	26
35	5531492	8330822	25
36	5533915	8329212	24
37	5536338	8327602	23
38	5538760	8325991	22
39	5541182	8324380	21
40	5543603	8322768	20
41	5546024	8321155	19
42	5548444	8319541	18
43	5550864	8317927	17
44	5553283	8316312	16
45	5555702	8314696	15
46	5558121	8313080	14
47	5560539	8311463	13
48	5562956	8309845	12
49	5565373	8308226	11
50	5567790	8306607	10
51	5570206	8304987	9
52	5572621	8303366	8
53	5575036	8301745	7
54	5577451	8300123	6
55	5579865	8298500	5
56	5582279	8296877	4
57	5584692	8295252	3
58	5587105	8293628	2
59	5589517	8292002	1
60	5591929	8290376	0
′	Cosine	Sine	′

Deg. 56.

34 Deg.

′	Sine	Cosine	′
0	5591929	8290376	60
1	5594340	8288749	59
2	5596751	8287121	58
3	5599162	8285493	57
4	5601572	8283864	56
5	5603981	8282234	55
6	5606390	8280603	54
7	5608798	8278972	53
8	5611206	8277340	52
9	5613614	8275708	51
10	5616021	8274074	50
11	5618428	8272440	49
12	5620834	8270806	48
13	5623239	8269170	47
14	5625645	8267534	46
15	5628049	8265897	45
16	5630453	8264260	44
17	5632857	8262622	43
18	5635260	8260983	42
19	5637663	8259343	41
20	5640066	8257703	40
21	5642467	8256062	39
22	5644869	8254420	38
23	5647270	8252778	37
24	5649670	8251135	36
25	5652070	8249491	35
26	5654469	8247847	34
27	5656868	8246202	33
28	5659267	8244556	32
29	5661665	8242909	31
30	5664062	8241262	30
31	5666459	8239614	29
32	5668856	8237965	28
33	5671252	8236316	27
34	5673648	8234666	26
35	5676043	8233015	25
36	5678437	8231364	24
37	5680832	8229712	23
38	5683225	8228059	22
39	5685619	8226405	21
40	5688011	8224751	20
41	5690403	8223096	19
42	5692795	8221440	18
43	5695187	8219784	17
44	5697577	8218127	16
45	5699968	8216469	15
46	5702357	8214811	14
47	5704747	8213152	13
48	5707136	8211492	12
49	5709524	8209832	11
50	5711912	8208170	10
51	5714299	8206509	9
52	5716686	8204846	8
53	5719073	8203183	7
54	5721459	8201519	6
55	5723844	8199854	5
56	5726229	8198189	4
57	5728614	8196523	3
58	5730998	8194856	2
59	5733381	8193189	1
60	5735764	8191520	0
′	Cosine	Sine	′

Deg. 55.

35 Deg.

′	Sine	Cosine	′
0	5735764	8191520	60
1	5738147	8189852	59
2	5740529	8188182	58
3	5742911	8186512	57
4	5745292	8184841	56
5	5747672	8183169	55
6	5750053	8181497	54
7	5752432	8179824	53
8	5754811	8178151	52
9	5757190	8176476	51
10	5759568	8174801	50
11	5761946	8173125	49
12	5764323	8171449	48
13	5766700	8169772	47
14	5769076	8168094	46
15	5771452	8166416	45
16	5773827	8164736	44
17	5776202	8163056	43
18	5778576	8161376	42
19	5780950	8159695	41
20	5783323	8158013	40
21	5785696	8156330	39
22	5788069	8154647	38
23	5790440	8152963	37
24	5792812	8151278	36
25	5795183	8149593	35
26	5797553	8147906	34
27	5799923	8146220	33
28	5802292	8144532	32
29	5804661	8142844	31
30	5807030	8141155	30
31	5809397	8139466	29
32	5811765	8137775	28
33	5814132	8136084	27
34	5816498	8134393	26
35	5818864	8132701	25
36	5821230	8131008	24
37	5823595	8129314	23
38	5825959	8127620	22
39	5828323	8125925	21
40	5830687	8124229	20
41	5833050	8122532	19
42	5835412	8120835	18
43	5837774	8119137	17
44	5840136	8117439	16
45	5842497	8115740	15
46	5844857	8114040	14
47	5847217	8112339	13
48	5849577	8110638	12
49	5851936	8108936	11
50	5854294	8107234	10
51	5856652	8105530	9
52	5859010	8103826	8
53	5861367	8102122	7
54	5863724	8100416	6
55	5866080	8098710	5
56	5868435	8097004	4
57	5870790	8095296	3
58	5873145	8093588	2
59	5875499	8091879	1
60	5877853	8090170	0
′	Cosine	Sine	′

Deg. 54.

36 Deg.				37 Deg.				38 Deg.			
′	Sine	Cosine	′	′	Sine	Cosine	′	′	Sine	Cosine	′
0	5877853	8090170	60	0	6018150	7986355	60	0	6156615	7880108	60
1	5880206	8088460	59	1	6020473	7984604	59	1	6158907	7878316	59
2	5882558	8086749	58	2	6022795	7982853	58	2	6161198	7876524	58
3	5884910	8085037	57	3	6025117	7981100	57	3	6163489	7874732	57
4	5887262	8083325	56	4	6027439	7979347	56	4	6165780	7872939	56
5	5889613	8081612	55	5	6029760	7977594	55	5	6168069	7871145	55
6	5891964	8079899	54	6	6032080	7975839	54	6	6170359	7869350	54
7	5894314	8078185	53	7	6034400	7974084	53	7	6172648	7867555	53
8	5896663	8076470	52	8	6036719	7972329	52	8	6174936	7865759	52
9	5899012	8074754	51	9	6039038	7970572	51	9	6177224	7863963	51
10	5901361	8073038	50	10	6041356	7968815	50	10	6179511	7862165	50
11	5903709	8071321	49	11	6043674	7967058	49	11	6181798	7860367	49
12	5906057	8069603	48	12	6045991	7965299	48	12	6184084	7858569	48
13	5908404	8067885	47	13	6048308	7963540	47	13	6186370	7856770	47
14	5910750	8066166	46	14	6050624	7961780	46	14	6188655	7854970	46
15	5913096	8064446	45	15	6052940	7960020	45	15	6190939	7853169	45
16	5915442	8062726	44	16	6055255	7958259	44	16	6193224	7851368	44
17	5917787	8061005	43	17	6057570	7956497	43	17	6195507	7849566	43
18	5920132	8059283	42	18	6059884	7954735	42	18	6197790	7847764	42
19	5922476	8057560	41	19	6062198	7952972	41	19	6200073	7845961	41
20	5924819	8055837	40	20	6064511	7951208	40	20	6202355	7844157	40
21	5927163	8054113	39	21	6066824	7949444	39	21	6204636	7842352	39
22	5929505	8052389	38	22	6069136	7947678	38	22	6206917	7840547	38
23	5931847	8050664	37	23	6071447	7945913	37	23	6209198	7838741	37
24	5934189	8048938	36	24	6073758	7944146	36	24	6211478	7836935	36
25	5936530	8047211	35	25	6076069	7942379	35	25	6213757	7835127	35
26	5938871	8045484	34	26	6078379	7940611	34	26	6216036	7833320	34
27	5941211	8043756	33	27	6080689	7938843	33	27	6218314	7831511	33
28	5943550	8042028	32	28	6082998	7937074	32	28	6220592	7829702	32
29	5945889	8040299	31	29	6085306	7935304	31	29	6222870	7827892	31
30	5948228	8038569	30	30	6087614	7933533	30	30	6225146	7826082	30
31	5950566	8036838	29	31	6089922	7931762	29	31	6227423	7824270	29
32	5952904	8035107	28	32	6092229	7929990	28	32	6229698	7822459	28
33	5955241	8033375	27	33	6094535	7928218	27	33	6231974	7820646	27
34	5957577	8031642	26	34	6096841	7926445	26	34	6234248	7818833	26
35	5959913	8029909	25	35	6099147	7924671	25	35	6236522	7817019	25
36	5962249	8028175	24	36	6101452	7922896	24	36	6238796	7815205	24
37	5964584	8026440	23	37	6103756	7921121	23	37	6241069	7813390	23
38	5966918	8024705	22	38	6106060	7919345	22	38	6243342	7811574	22
39	5969252	8022969	21	39	6108363	7917569	21	39	6245614	7809757	21
40	5971586	8021232	20	40	6110666	7915792	20	40	6247885	7807940	20
41	5973919	8019495	19	41	6112969	7914014	19	41	6250156	7806123	19
42	5976251	8017756	18	42	6115270	7912235	18	42	6252427	7804304	18
43	5978583	8016018	17	43	6117572	7910456	17	43	6254696	7802485	17
44	5980915	8014278	16	44	6119873	7908676	16	44	6256966	7800665	16
45	5983246	8012538	15	45	6122173	7906896	15	45	6259235	7798845	15
46	5985577	8010797	14	46	6124473	7905115	14	46	6261503	7797024	14
47	5987906	8009056	13	47	6126772	7903333	13	47	6263771	7795202	13
48	5990236	8007314	12	48	6129071	7901550	12	48	6266038	7793380	12
49	5992565	8005571	11	49	6131369	7899767	11	49	6268305	7791557	11
50	5994893	8003827	10	50	6133666	7897983	10	50	6270571	7789733	10
51	5997221	8002083	9	51	6135964	7896198	9	51	6272837	7787909	9
52	5999549	8000338	8	52	6138260	7894413	8	52	6275102	7786084	8
53	6001876	7998593	7	53	6140556	7892627	7	53	6277366	7784258	7
54	6004202	7996847	6	54	6142852	7890841	6	54	6279631	7782431	6
55	6006528	7995100	5	55	6145147	7889054	5	55	6281894	7780604	5
56	6008854	7993352	4	56	6147442	7887266	4	56	6284157	7778777	4
57	6011179	7991604	3	57	6149736	7885477	3	57	6286420	7776949	3
58	6013503	7989855	2	58	6152029	7883688	2	58	6288682	7775120	2
59	6015827	7988105	1	59	6154322	7881898	1	59	6290943	7773290	1
60	6018150	7986355	0	60	6156615	7880108	0	60	6293204	7771460	0
′	Cosine	Sine	′	′	Cosine	Sine	′	′	Cosine	Sine	′
		Deg. 53.				Deg. 52.				Deg. 51.	

39 Deg.				40 Deg.				41 Deg.			
′	**Sine**	**Cosine**	′	′	**Sine**	**Cosine**	′	′	**Sine**	**Cosine**	′
0	6293204	7771460	60	0	6427876	7660444	60	0	6560590	7547096	60
1	6295464	7769629	59	1	6430104	7658574	59	1	6562785	7545187	59
2	6297724	7767797	58	2	6432332	7656704	58	2	6564980	7543278	58
3	6299983	7765965	57	3	6434559	7654832	57	3	6567174	7541368	57
4	6302242	7764132	56	4	6436785	7652960	56	4	6569367	7539457	56
5	6304500	7762298	55	5	6439011	7651087	55	5	6571560	7537546	55
6	6306758	7760464	54	6	6441236	7649214	54	6	6573752	7535634	54
7	6309015	7758629	53	7	6443461	7647340	53	7	6575944	7533721	53
8	6311272	7756794	52	8	6445685	7645465	52	8	6578135	7531808	52
9	6313528	7754957	51	9	6447909	7643590	51	9	6580326	7529894	51
10	6315784	7753121	50	10	6450132	7641714	50	10	6582516	7527980	50
11	6318039	7751283	49	11	6452355	7639838	49	11	6584706	7526065	49
12	6320293	7749445	48	12	6454577	7637960	48	12	6586895	7524149	48
13	6322547	7747606	47	13	6456798	7636082	47	13	6589083	7522233	47
14	6324800	7745767	46	14	6459019	7634204	46	14	6591271	7520316	46
15	6327053	7743926	45	15	6461240	7632325	45	15	6593458	7518398	45
16	6329306	7742086	44	16	6463460	7630445	44	16	6595645	7516480	44
17	6331557	7740244	43	17	6465679	7628564	43	17	6597831	7514561	43
18	6333809	7738402	42	18	6467898	7626683	42	18	6600017	7512641	42
19	6336059	7736559	41	19	6470116	7624802	41	19	6602202	7510721	41
20	6338310	7734716	40	20	6472334	7622919	40	20	6604386	7508800	40
21	6340559	7732872	39	21	6474551	7621036	39	21	6606570	7506879	39
22	6342808	7731027	38	22	6476767	7619152	38	22	6608754	7504957	38
23	6345057	7729182	37	23	6478984	7617268	37	23	6610936	7503034	37
24	6347305	7727336	36	24	6481199	7615383	36	24	6613119	7501111	36
25	6349553	7725489	35	25	6483414	7613497	35	25	6615300	7499187	35
26	6351800	7723642	34	26	6485628	7611611	34	26	6617482	7497262	34
27	6354046	7721794	33	27	6487842	7609724	33	27	6619662	7495337	33
28	6356292	7719945	32	28	6490056	7607837	32	28	6621842	7493411	32
29	6358537	7718096	31	29	6492268	7605949	31	29	6624022	7491484	31
30	6360782	7716246	30	30	6494480	7604060	30	30	6626200	7489557	30
31	6363026	7714395	29	31	6496692	7602170	29	31	6628379	7487629	29
32	6365270	7712544	28	32	6498903	7600280	28	32	6630557	7485701	28
33	6367513	7710692	27	33	6501114	7598389	27	33	6632734	7483772	27
34	6369756	7708840	26	34	6503324	7596498	26	34	6634910	7481842	26
35	6371998	7706986	25	35	6505533	7594606	25	35	6637087	7479912	25
36	6374240	7705132	24	36	6507742	7592713	24	36	6639262	7477981	24
37	6376481	7703278	23	37	6509951	7590820	23	37	6641437	7476049	23
38	6378721	7701423	22	38	6512158	7588926	22	38	6643612	7474117	22
39	6380961	7699567	21	39	6514366	7587031	21	39	6645785	7472184	21
40	6383201	7697710	20	40	6516572	7585136	20	40	6647959	7470251	20
41	6385440	7695853	19	41	6518778	7583240	19	41	6650131	7468317	19
42	6387678	7693996	18	42	6520984	7581343	18	42	6652304	7466382	18
43	6389916	7692137	17	43	6523189	7579446	17	43	6654475	7464446	17
44	6392153	7690278	16	44	6525394	7577548	16	44	6656646	7462510	16
45	6394390	7688418	15	45	6527598	7575650	15	45	6658817	7460574	15
46	6396626	7686558	14	46	6529801	7573751	14	46	6660987	7458636	14
47	6398862	7684697	13	47	6532004	7571851	13	47	6663156	7456699	13
48	6401097	7682835	12	48	6534206	7569951	12	48	6665325	7454760	12
49	6403332	7680973	11	49	6536408	7568050	11	49	6667493	7452821	11
50	6405566	7679110	10	50	6538609	7566148	10	50	6669661	7450881	10
51	6407799	7677246	9	51	6540810	7564246	9	51	6671828	7448941	9
52	6410032	7675382	8	52	6543010	7562343	8	52	6673994	7446999	8
53	6412264	7673517	7	53	6545209	7560439	7	53	6676160	7445058	7
54	6414496	7671652	6	54	6547408	7558535	6	54	6678326	7443115	6
55	6416728	7669785	5	55	6549607	7556630	5	55	6680490	7441173	5
56	6418958	7667918	4	56	6551804	7554724	4	56	6682655	7439229	4
57	6421189	7666051	3	57	6554002	7552818	3	57	6684818	7437285	3
58	6423418	7664183	2	58	6556198	7550911	2	58	6686981	7435340	2
59	6425647	7662314	1	59	6558395	7549004	1	59	6689144	7433394	1
60	6427876	7660444	0	60	6560590	7547096	0	60	6691306	7431448	0
′	**Cosine**	**Sine**	′	′	**Cosine**	**Sine**	′	′	**Cosine**	**Sine**	′
		Deg. 50.				Deg. 49.				Deg. 48.	

42 Deg.				**43 Deg.**				**44 Deg.**			
′	**Sine**	**Cosine**	′	′	**Sine**	**Cosine**	′	′	**Sine**	**Cosine**	′
0	6691306	7431448	60	0	6819984	7313537	60	0	6946584	7193398	60
1	6693468	7429502	59	1	6822111	7311553	59	1	6948676	7191377	59
2	6695628	7427554	58	2	6824237	7309568	58	2	6950767	7189355	58
3	6697789	7425606	57	3	6826363	7307583	57	3	6952858	7187333	57
4	6699948	7423658	56	4	6828489	7305597	56	4	6954949	7185310	56
5	6702108	7421708	55	5	6830613	7303610	55	5	6957039	7183287	55
6	6704266	7419758	54	6	6832738	7301623	54	6	6959128	7181263	54
7	6706424	7417808	53	7	6834861	7299635	53	7	6961217	7179238	53
8	6708582	7415857	52	8	6836984	7297646	52	8	6963305	7177213	52
9	6710739	7413905	51	9	6839107	7295657	51	9	6965392	7175187	51
10	6712895	7411953	50	10	6841229	7293668	50	10	6967479	7173161	50
11	6715051	7410000	49	11	6843350	7291677	49	11	6969565	7171134	49
12	6717205	7408046	48	12	6845471	7289686	48	12	6971651	7169106	48
13	6719361	7406092	47	13	6847591	7287695	47	13	6973736	7167078	47
14	6721515	7404137	46	14	6849711	7285703	46	14	6975821	7165049	46
15	6723668	7402181	45	15	6851830	7283710	45	15	6977905	7163019	45
16	6725821	7400225	44	16	6853948	7281716	44	16	6979988	7160989	44
17	6727973	7398268	43	17	6856066	7279722	43	17	6982071	7158959	43
18	6730125	7396311	42	18	6858184	7277728	42	18	6984153	7156927	42
19	6732276	7394353	41	19	6860300	7275732	41	19	6986234	7154895	41
20	6734427	7392394	40	20	6862416	7273736	40	20	6988315	7152863	40
21	6736577	7390435	39	21	6864532	7271740	39	21	6990396	7150830	39
22	6738727	7388475	38	22	6866647	7269743	38	22	6992476	7148796	38
23	6740876	7386515	37	23	6868761	7267745	37	23	6994555	7146762	37
24	6743024	7384553	36	24	6870875	7265747	36	24	6996633	7144727	36
25	6745172	7382592	35	25	6872988	7263748	35	25	6998711	7142691	35
26	6747319	7380629	34	26	6875101	7261748	34	26	7000789	7140655	34
27	6749466	7378666	33	27	6877213	7259748	33	27	7002866	7138618	33
28	6751612	7376703	32	28	6879325	7257747	32	28	7004942	7136581	32
29	6753757	7374738	31	29	6881435	7255746	31	29	7007018	7134543	31
30	6755902	7372773	30	30	6883546	7253744	30	30	7009093	7132504	30
31	6758046	7370808	29	31	6885655	7251741	29	31	7011167	7130465	29
32	6760190	7368842	28	32	6887765	7249738	28	32	7013241	7128426	28
33	6762331	7366875	27	33	6889873	7247734	27	33	7015314	7126385	27
34	6764476	7364908	26	34	6891981	7245729	26	34	7017387	7124344	26
35	6766618	7362940	25	35	6894089	7243724	25	35	7019459	7122303	25
36	6768760	7360971	24	36	6896195	7241719	24	36	7021531	7120260	24
37	6770901	7359002	23	37	6898302	7239712	23	37	7023601	7118218	23
38	6773041	7357032	22	38	6900407	7237705	22	38	7025672	7116174	22
39	6775181	7355061	21	39	6902512	7235698	21	39	7027741	7114130	21
40	6777320	7353090	20	40	6904617	7233690	20	40	7029811	7112086	20
41	6779459	7351118	19	41	6906721	7231681	19	41	7031879	7110041	19
42	6781597	7349146	18	42	6908824	7229671	18	42	7033947	7107995	18
43	6783734	7347173	17	43	6910927	7227661	17	43	7036014	7105948	17
44	6785871	7345199	16	44	6913029	7225651	16	44	7038081	7103901	16
45	6788007	7343225	15	45	6915131	7223640	15	45	7040147	7101854	15
46	6790143	7341250	14	46	6917232	7221628	14	46	7042213	7099806	14
47	6792278	7339275	13	47	6919332	7219615	13	47	7044278	7097757	13
48	6794413	7337299	12	48	6921432	7217602	12	48	7046342	7095707	12
49	6796547	7335322	11	49	6923531	7215589	11	49	7048406	7093657	11
50	6798681	7333345	10	50	6925630	7213574	10	50	7050469	7091607	10
51	6800813	7331367	9	51	6927728	7211559	9	51	7052532	7089556	9
52	6802946	7329388	8	52	6929825	7209544	8	52	7054594	7087504	8
53	6805078	7327409	7	53	6931922	7207528	7	53	7056655	7085451	7
54	6807209	7325429	6	54	6934018	7205511	6	54	7058716	7083398	6
55	6809339	7323449	5	55	6936114	7203494	5	55	7060776	7081345	5
56	6811469	7321467	4	56	6938209	7201476	4	56	7062835	7079291	4
57	6813599	7319486	3	57	6940304	7199457	3	57	7064894	7077236	3
58	6815728	7317503	2	58	6942398	7197438	2	58	7066953	7075180	2
59	6817856	7315521	1	59	6944491	7195418	1	59	7069011	7073124	1
60	6819984	7313537	0	60	6946584	7193398	0	60	7071068	7071068	0
′	**Cosine**	**Sine**	′	′	**Cosine**	**Sine**	′	′	**Cosine**	**Sine**	′
		Deg. 47.				**Deg. 46.**				**Deg. 45.**	

TANGENTS.

′	0°	1°	2°	3°	4°	5°	6°	′
0	0·0000000	0·0174551	0·0349208	0·0524078	0·0699268	0·0874887	0·1051042	60
1	·0002909	·0177460	·0352120	·0526995	·0702191	·0877818	·1053983	59
2	·0005818	·0180370	·0355033	·0529912	·0705115	·0880749	·1056925	58
3	·0008727	·0183280	·0357945	·0532829	·0708038	·0883681	·1059866	57
4	·0011636	·0186190	·0360858	·0535746	·0710961	·0886612	·1062808	56
5	·0014544	·0189100	·0363771	·0538663	·0713885	·0889544	·1065750	55
6	·0017453	·0192010	·0366683	·0541581	·0716809	·0892476	·1068692	54
7	·0020362	·0194920	·0369596	·0544498	·0719733	·0895408	·1071634	53
8	·0023271	·0197830	·0372509	·0547416	·0722657	·0898341	·1074576	52
9	·0026180	·0200740	·0375422	·0550333	·0725581	·0901273	·1077519	51
10	0·0029089	0·0203650	0·0378335	0·0553251	0·0728505	0·0904206	0·1080462	50
11	·0031998	·0206560	·0381248	·0556169	·0731430	·0907138	·1083405	49
12	·0034907	·0209470	·0384161	·0559087	·0734354	·0910071	·1086348	48
13	·0037816	·0212380	·0387074	·0562005	·0737279	·0913004	·1089291	47
14	·0040725	·0215291	·0389988	·0564923	·0740203	·0915938	·1092234	46
15	·0043634	·0218201	·0392901	·0567841	·0743128	·0918871	·1095178	45
16	·0046542	·0221111	·0395814	·0570759	·0746053	·0921804	·1098122	44
17	·0049451	·0224021	·0398728	·0573678	·0748979	·0924738	·1101066	43
18	·0052360	·0226932	·0401641	·0576596	·0751904	·0927672	·1104010	42
19	·0055269	·0229842	·0404555	·0579515	·0754829	·0930606	·1106955	41
20	0·0058178	0·0232753	0·0407469	0·0582434	0·0757755	0·0933540	0·1109899	40
21	·0061087	·0235663	·0410383	·0585352	·0760680	·0936474	·1112844	39
22	·0063996	·0238574	·0413296	·0588271	·0763606	·0939409	·1115789	38
23	·0066905	·0241484	·0416210	·0591190	·0766532	·0942344	·1118734	37
24	·0069814	·0244395	·0419124	·0594109	·0769458	·0945278	·1121680	36
25	·0072723	·0247305	·0422038	·0597029	·0772384	·0948213	·1124625	35
26	·0075632	·0250216	·0424952	·0599948	·0775311	·0951148	·1127571	34
27	·0078541	·0253127	·0427866	·0602867	·0778237	·0954084	·1130517	33
28	·0081450	·0256038	·0430781	·0605787	·0781164	·0957019	·1133463	32
29	·0084360	·0258948	·0433695	·0608706	·0784090	·0959955	·1136410	31
30	0·0087269	0·0261859	0·0436609	0·0611626	0·0787017	0·0962890	0·1139356	30
31	·0090178	·0264770	·0439524	·0614546	·0789944	·0965826	·1142303	29
32	·0093087	·0267681	·0442438	·0617466	·0792871	·0968763	·1145250	28
33	·0095996	·0270592	·0445353	·0620386	·0795798	·0971699	·1148197	27
34	·0098905	·0273503	·0448268	·0623306	·0798726	·0974635	·1151144	26
35	·0101814	·0276414	·0451183	·0626226	·0801653	·0977572	·1154092	25
36	·0104724	·0279325	·0454097	·0629147	·0804581	·0980509	·1157039	24
37	·0107633	·0282236	·0457012	·0632067	·0807509	·0983446	·1159987	23
38	·0110542	·0285148	·0459927	·0634988	·0810437	·0986383	·1162936	22
39	·0113451	·0288059	·0462842	·0637908	·0813365	·0989320	·1165884	21
40	0·0116361	0·0290970	0·0465757	0·0640829	0·0816293	0·0992257	0·1168832	20
41	·0119270	·0293882	·0468673	·0643750	·0819221	·0995194	·1171781	19
42	·0122179	·0296793	·0471588	·0646671	·0822150	·0998133	·1174730	18
43	·0125088	·0299705	·0474503	·0649592	·0825078	·1001071	·1177679	17
44	·0127998	·0302616	·0477419	·0652513	·0828007	·1004009	·1180628	16
45	·0130907	·0305528	·0480334	·0655435	·0830936	·1006947	·1183578	15
46	·0133817	·0308439	·0483250	·0658356	·0833865	·1009886	·1186528	14
47	·0136726	·0311351	·0486166	·0661278	·0836794	·1012824	·1189478	13
48	·0139635	·0314263	·0489082	·0664199	·0839723	·1015763	·1192428	12
49	·0142545	·0317174	·0491997	·0667121	·0842653	·1018702	·1195378	11
50	0·0145454	0·0320086	0·0494913	0·0670043	0·0845583	0·1021641	0·1198329	10
51	·0148364	·0322998	·0497829	·0672965	·0848512	·1024580	·1201279	9
52	·0151273	·0325910	·0500746	·0675887	·0851442	·1027520	·1204230	8
53	·0154183	·0328822	·0503662	·0678809	·0854372	·1030460	·1207182	7
54	·0157093	·0331734	·0506578	·0681732	·0857302	·1033399	·1210133	6
55	·0160002	·0334646	·0509495	·0684654	·0860233	·1036340	·1213085	5
56	·0162912	·0337558	·0512411	·0687577	·0863163	·1039280	·1216036	4
57	·0165821	·0340471	·0515328	·0690499	·0866094	·1042220	·1218988	3
58	·0168731	·0343383	·0518244	·0693422	·0869025	·1045161	·1221941	2
59	·0171641	·0346295	·0521161	·0696345	·0871956	·1048101	·1224893	1
60	0·0174551	0·0349208	0·0524078	0·0699268	0·0874887	0·1051042	0·1227846	0
′	89°	88°	87°	88°	85°	84°	83°	′

CO-TANGENTS.

TANGENTS.

′	7°	8°	9°	10°	11°	12°	13°	′
0	0·1227846	0·1405408	0·1583844	0·1763270	0·1943803	0·2125566	0·2308682	60
1	·1230798	·1408375	·1586826	·1766269	·1946822	·2128606	·2311746	59
2	·1233752	·1411342	·1589809	.1769269	·1949841	·2131647	·2314811	58
3	·1236705	·1414308	·1592791	·1772269	·1952861	·2134688	·2317876	57
4	·1239658	·1417276	·1595774	·1775270	·1955881	·2137730	·2320941	56
5	·1242612	·1420243	·1598757	·1778270	·1958901	·2140772	·2324007	55
6	·1245566	·1423211	·1601740	·1781271	·1961922	·2143814	·2327073	54
7	·1248520	·1426179	·1604724	·1784273	·1964943	·2146857	·2330140	53
8	·1251474	·1429147	·1607708	·1787274	·1967964	·2149900	·2333207	52
9	·1254429	·1432115	·1610692	·1790276	·1970986	·2152944	·2336274	51
10	0·1257384	0·1435084	0·1613677	0·1793279	0·1974008	0·2155988	0·2339342	50
11	·1260339	·1438053	·1616662	·1796281	·1977031	·2159032	·2342410	49
12	·1263294	·1441022	·1619647	·1799284	·1980053	·2162077	·2345479	48
13	·1266249	·1443991	·1622632	·1802287	·1983076	·2165122	·2348548	47
14	·1269205	·1446961	·1625618	·1805291	·1986100	·2168167	·2351617	46
15	·1272161	·1449931	·1628603	·1808295	·1989124	·2171213	·2354687	45
16	·1275117	·1452901	·1631590	·1811299	·1992148	·2174259	·2357758	44
17	·1278073	·1455872	·1634576	·1814303	·1995172	·2177306	·2360829	43
18	·1281030	·1458842	·1637563	·1817308	·1998197	·2180353	·2363900	42
19	·1283986	·1461813	·1640550	·1820313	·2001222	·2183400	·2366971	41
20	0·1286943	0·1464784	0·1643537	0·1823319	0·2004248	0·2186448	0·2370044	40
21	·1289900	·1467756	·1646525	·1826324	·2007274	·2189496	·2373116	39
22	·1292858	·1470727	·1649513	·1829330	·2010300	·2192544	·2376189	38
23	·1295815	·1473699	·1652501	·1832337	·2013327	·2195593	·2379262	37
24	·1298773	·1476672	·1655489	·1835343	·2016354	·2198643	·2382336	36
25	·1301731	·1479644	·1658478	·1838350	·2019381	·2201692	·2385410	35
26	·1304690	·1482617	·1661467	·1841358	·2022409	·2204742	·2388485	34
27	·1307648	·1485590	·1664456	·1844365	·2025437	·2207793	·2391560	33
28	·1310607	·1488563	·1667446	·1847373	·2028465	·2210844	·2394635	32
29	·1313566	·1491536	·1670436	·1850382	·2031494	·2213895	·2397711	31
30	0·1316525	0·1494510	0·1673426	0·1853390	0·2034523	0·2216947	0·2400788	30
31	·1319484	·1497484	·1676417	·1856399	·2037552	·2219999	·2403864	29
32	·1322444	·1500458	·1679407	·1859409	·2040582	·2223051	·2406942	28
33	·1325404	·1503433	·1682398	·1862418	·2043612	·2226104	·2410019	27
34	·1328364	·1506408	·1685390	·1865428	·2046643	·2229157	·2413097	26
35	·1331324	·1509383	·1688381	·1868439	·2049674	·2232211	·2416176	25
36	·1334285	·1512358	·1691373	·1871449	·2052705	·2235265	·2419255	24
37	·1337246	·1515333	·1694366	·1874460	·2055737	·2238319	·2422334	23
38	·1340207	·1518309	·1697358	·1877471	·2058769	·2241374	·2425414	22
39	·1343168	·1521285	·1700351	·1880483	·2061801	·2244429	·2428494	21
40	0·1346129	0·1524262	0·1703344	0·1883495	0·2064834	0·2247485	0·2431575	20
41	·1349091	·1527238	·1706338	·1886507	·2067867	·2250541	·2434656	19
42	·1352053	·1530215	·1709331	·1889520	·2070900	·2253597	·2437737	18
43	·1355015	·1533192	·1712325	·1892533	·2073934	·2256654	·2440819	17
44	·1357978	·1536170	·1715320	·1895546	·2076968	·2259711	·2443902	16
45	·1360940	·1539147	·1718314	·1898559	·2080003	·2262769	·2446984	15
46	·1363903	·1542125	·1721309	·1901573	·2083038	·2265827	·2450068	14
47	·1366866	·1545103	·1724304	·1904587	·2086073	·2268885	·2453151	13
48	·1369830	·1548082	·1727300	·1907602	·2089109	·2271944	·2456236	12
49	·1372793	·1551061	·1730296	·1910617	·2092145	·2275003	·2459320	11
50	0·1375757	0·1554040	0·1733292	0·1913632	0·2095181	0·2278063	0·2462405	10
51	·1378721	·1557019	·1736288	·1916648	·2098218	·2281123	·2465491	9
52	·1381685	·1559998	·1739285	·1919664	·2101255	·2284184	·2468577	8
53	·1384650	·1562978	·1742282	·1922680	·2104293	·2287244	·2471663	7
54	·1387615	·1565958	·1745279	·1925696	·2107331	·2290306	·2474750	6
55	·1390580	·1568939	·1748277	·1928713	.2110369	·2293367	·2477837	5
56	·1393545	·1571919	·1751275	·1931731	·2113407	·2296429	·2480925	4
57	·1396510	·1574900	·1754273	·1934748	·2116446	·2299492	·2484013	3
58	·1399476	·1577881	·1757272	·1937766	·2119486	·2302555	·2487102	2
59	·1402442	·1580863	·1760271	·1940784	·2122525	·2305618	·2490191	1
60	0·1405408	0·1583844	0·1763270	0·1943803	0·2125566	0·2308682	0·2493280	0
′	82°	81°	80°	79°	78°	77°	76°	′

CO-TANGENTS.

TANGENTS.

′	14°	15°	16°	17°	18°	19°	20°	′
0	0·2493280	0·2679492	0·2867454	0·3057307	0·3249197	0·3443276	0·3639702	60
1	·2496370	·2682610	·2870602	·3060488	·3252413	·3446530	·3642997	59
2	·2499460	·2685728	·2873751	·3063670	·3255630	·3449785	·3646292	58
3	·2502551	·2688847	·2876900	·3066852	·3258848	·3453040	·3649588	57
4	·2505642	·2691967	·2880050	·3070034	·3262066	·3456296	·3652885	68
5	·2508734	·2695087	·2883201	·3073218	·3265284	·3459553	·3656182	65
6	·2511826	·2698207	·2886352	·3076402	·3268504	·3462810	·3659480	54
7	·2514919	·2701328	·2889503	·3079586	·3271724	·3466068	·3662779	63
8	·2518012	·2704449	·2892655	·3082771	·3274944	·3469327	·3666079	52
9	·2521106	·2707571	·2895808	·3085957	·3278165	·3472586	·3669379	51
10	0·2524200	0·2710694	0·2898961	0·3089143	0·3281387	0·3475846	0·3672680	50
11	·2527294	·2713817	·2902114	·3092330	·3284610	·3479107	·3675981	49
12	·2530389	·2716940	·2905269	·3095517	·3287833	·3482368	·3679284	48
13	·2533484	·2720064	·2908423	·3098705	·3291056	·3485630	·3682587	47
14	·2536580	·2723188	·2911578	·3101893	·3294281	·3488893	·3685890	46
16	·2539676	·2726313	·2914734	·3105083	·3297505	·3492156	·3689195	45
16	·2542773	·2729438	·2917890	·3108272	·3300731	·3495420	·3692500	44
17	·2545870	·2732564	·2921047	·3111462	·3303957	·3498685	·3695806	43
18	·2548968	·2735690	·2924205	·3114653	·3307184	·3501950	·3699112	42
19	·2552066	·2738817	·2927363	·3117845	·3310411	·3505216	·3702420	41
20	0·2555165	0·2741945	0·2930521	0·3121036	0·3313639	0·3508483	0·3705728	40
21	·2558264	·2745072	·2933680	·3124229	·3316868	·3511750	·3709036	89
22	·2561363	·2748201	·2936839	·3127422	·3320097	·3515018	·3712346	38
23	·2564463	·2751330	·2939999	·3130616	·3323327	·3518287	·3715656	37
24	·2567564	·2754459	·2943160	·3133810	·3326557	·3521556	·3718967	36
25	·2570664	·2757589	·2946321	·3137005	·3329788	·3524826	·3722278	35
26	·2573766	·2760719	·2949483	·3140200	·3333020	·3528096	·3725590	34
27	·2576868	·2763850	·2952645	·3143396	·3336252	·3531368	·3728903	33
28	·2579970	·2766981	·2955808	·3146593	·3339485	·3534640	·3732217	32
29	·2583073	·2770113	·2958971	·3149790	·3342719	·3537912	·3735532	31
30	0·2586176	0·2773245	0·2962135	0·3152988	0·3345953	0·3541186	0·3738847	30
31	·2589280	·2776378	·2965299	·3156186	·3349188	·3544460	·3742163	29
32	·2592384	·2779512	·2968464	·3159385	·3352424	·3547734	·3745479	28
33	·2595488	·2782646	·2971630	·3162585	·3355660	·3551010	·3748797	27
34	·2598593	·2785780	·2974796	·3165785	·3358896	·3554286	·3752115	26
35	·2601699	·2788915	·2977962	·3168986	·3362134	·3557562	·3755433	25
36	·2604805	·2792050	·2981129	·3172187	·3365372	·3560840	·3758753	24
37	·2607911	·2795186	·2984297	·3175389	·3368610	·3564118	·3762073	23
38	·2611018	·2798322	·2987465	·3178591	·3371850	·3567397	·3765394	22
39	·2614126	·2801459	·2990634	·3181794	·3375090	·3570676	·3768716	21
40	0·2617234	0·2804597	0·2993803	0·3184998	0·3378330	0·3573956	0·3772038	20
41	·2620342	·2807735	·2996973	·3188202	·3381571	·3577237	·3775361	19
42	·2623451	·2810873	·3000144	·3191407	·3384813	·3580518	·3778685	18
43	·2626560	·2814012	·3003315	·3194613	·3388056	·3583801	·3782010	17
44	·2629670	·2817152	·3006486	·3197819	·3391299	·3587083	·3785335	16
45	·2632780	·2820292	·3009658	·3201025	·3394543	·3590367	·3788661	15
46	·2635891	·2823432	·3012831	·3204232	·3397787	·3593651	·3791988	14
47	·2639002	·2826573	·3016004	·3207440	·3401032	·3596936	·3795315	13
48	·2642114	·2829715	·3019178	·3210649	·3404278	·3600222	·3798644	12
49	·2645226	·2832857	·3022352	·3213858	·3407524	·3603508	·3801973	11
50	0·2648339	0·2835999	0·3025527	0·3217067	0·3410771	0·3606795	0·3805302	10
51	·2651452	·2839143	·3028703	·3220278	·3414019	·3610082	·3808633	9
52	·2654566	·2842286	·3031879	·3223489	·3417267	·3613371	·3811964	8
53	·2657680	·2845430	·3035055	·3226700	·3420516	·3616660	·3815296	7
54	·2660794	·2848575	·3038232	·3229912	·3423765	·3619949	·3818629	6
55	·2663909	·2851720	·3041410	·3233125	·3427015	·3623240	·3821962	5
56	·2667025	·2854866	·3044588	·3236338	·3430266	·3626531	·3825296	4
57	·2670141	·2858012	·3047767	·3239552	·3433518	·3629823	·3828631	3
58	·2673257	·2861159	·3050946	·3242766	·3436770	·3633115	·3831967	2
59	·2676374	·2864306	·3054126	·3245981	·3440023	·3636408	·3835303	1
60	0·2679492	0·2867454	0·3057307	0·3249197	0·3443276	0·3639702	0·3838640	0
′	75°	74°	73°	72°	71°	70°	69°	′

CO-TANGENTS.

TANGENTS.

′	21°	22°	23°	24°	25°	26°	27°	′
0	0·3838640	0·4040262	0.4244748	0·4452287	0·4663077	0·4877326	0·5095254	60
1	·3841978	·4043646	·4248182	·4455773	·4666618	·4880927	·5098919	59
2	·3845317	·4047031	·4251616	·4459260	·4670161	·4884530	·5102585	58
3	·3848656	·4050417	·4255051	·4462747	·4673705	·4888133	·5106252	57
4	·3851996	·4053804	·4258487	·4466236	·4677250	·4891737	·5109919	56
5	·3855337	·4057191	·4261924	·4469726	·4680796	·4895343	·5113588	55
6	·3858679	·4060579	·4265361	·4473216	·4684342	·4898949	·5117259	54
7	·3862021	·4063968	·4268800	·4476708	·4687890	·4902557	·5120930	53
8	·3865364	·4067358	·4272239	·4480200	·4691439	·4906166	·5124602	52
9	·3868708	·4070748	·4275680	·4483693	·4694988	·4909775	·5128275	51
10	0·3872053	0·4074139	0·4279121	0·4487187	0·4698539	0·4913386	0·5131950	50
11	·3875398	·4077531	·4282563	·4490682	·4702090	·4916997	·5135625	49
12	·3878744	·4080924	·4286005	·4494178	·4705643	·4920610	·5139302	48
13	·3882091	·4084318	·4289449	·4497675	·4709196	·4924224	·5142980	47
14	·3885439	·4087713	·4292894	·4501173	·4712751	·4927838	·5146658	46
15	·3888787	·4091108	·4296339	·4504672	·4716306	·4931454	·5150338	45
16	·3892136	·4094504	·4299785	·4508171	·4719863	·4935071	·5154019	44
17	·3895486	·4097901	·4303232	·4511672	·4723420	·4938689	·5157702	43
18	·3898837	·4101299	·4306680	·4515173	·4726978	·4942308	·5161385	42
19	·3902189	·4104697	·4310129	·4518676	·4730538	·4945928	·5165069	41
20	0·3905541	0·4108097	0·4313579	0·4522179	0·4734098	0·4949549	0·5168755	40
21	·3908894	·4111497	·4317030	·4525683	·4737659	·4953171	·5172441	39
22	·3912247	·4114898	·4320481	·4529188	·4741222	·4956795	·5176129	38
23	·3915602	·4118300	·4323933	·4532694	·4744785	·4960418	·5179818	37
24	·3918957	·4121703	·4327386	·4536201	·4748349	·4964043	·5183508	36
25	·3922313	·4125106	·4330840	·4539709	·4751914	·4967669	·5187199	35
26	·3925670	·4128510	·4334295	·4543218	·4755481	·4971297	·5190891	34
27	·3929027	·4131915	·4337751	·4546728	·4759048	·4974925	·5194584	33
28	·3932386	·4135321	·4341208	·4550238	·4762616	·4978554	·5198278	32
29	·3935745	·4138728	·4344665	·4553750	·4766185	·4982185	·5201974	31
30	0·3939105	0·4142136	0·4348124	0·4557263	0·4769755	0·4985816	0·5205671	30
31	·3942465	·4145544	·4351583	·4560776	·4773326	·4989449	·5209368	29
32	·3945827	·4148953	·4355043	·4564290	·4776899	·4993082	·5213067	28
33	·3949189	·4152363	·4358504	·4567806	·4780472	·4996717	·5216767	27
34	·3952552	·4155774	·4361966	·4571322	·4784046	·5000352	·5220468	26
35	·3955916	·4159186	·4365429	·4574839	·4787621	·5003989	·5224170	25
36	·3959280	·4162598	·4368893	·4578357	·4791197	·5007627	·5227874	24
37	·3962645	·4166012	·4372357	·4581877	·4794774	·5011266	·5231578	23
38	·3966011	·4169426	·4375823	·4585397	·4798352	·5014906	·5235284	22
39	·3969378	·4172841	·4379289	·4588918	·4801932	·5018547	·5238990	21
40	0·3972746	0·4176257	0·4382756	0·4592439	0·4805512	0·5022189	0·5242698	20
41	·3976114	·4179673	·4386224	·4595962	·4809093	·5025832	·5246407	19
42	·3979483	·4183091	·4389693	·4599486	·4812675	·5029476	·5250117	18
43	·3982853	·4186509	·4393163	·4603011	·4816258	·5033121	·5253829	17
44	·3986224	·4189928	·4396634	·4606537	·4819842	·5036768	·5257541	16
45	·3989595	·4193348	·4400105	·4610063	·4823427	·5040415	·5261255	15
46	·3992968	·4196769	·4403578	·4613591	·4827014	·5044063	·5264969	14
47	·3996341	·4200190	·4407051	·4617119	·4830601	·5047713	·5268685	13
48	·3999715	·4203613	·4410526	·4620649	·4834189	·5051363	·5272402	12
49	·4003089	·4207036	·4414001	·4624179	·4837778	·5055015	·5276120	11
50	0·4006465	0·4210460	0·4417477	0·4627710	0·4841368	0·5058668	0·5279839	10
51	·4009841	·4213885	·4420954	·4631243	·4844959	·5062322	·5283560	9
52	·4013218	·4217311	·4424432	·4634776	·4848552	·5065977	·5287281	8
53	·4016596	·4220738	·4427910	·4638310	·4852145	·5069633	·5291004	7
54	·4019974	·4224165	·4431390	·4641845	·4855739	·5073290	·5294727	6
55	·4023354	·4227594	·4434871	·4645382	·4859334	·5076948	·5298452	5
56	·4026734	·4231023	·4438352	·4648919	·4862931	·5080607	·5302178	4
57	·4030115	·4234453	·4441834	·4652457	·4866528	·5084267	·5305906	3
58	·4033496	·4237884	·4445318	·4655996	·4870126	·5087929	·5309634	2
59	·4036879	·4241316	·4448802	·4659536	·4873726	·5091591	·5313364	1
60	0·4040262	0·4244748	0·4452287	0·4663077	0·4877326	0·5095254	0·5317094	0
′	68°	67°	66°	65°	64°	63°	62°	′

CO-TANGENTS.

TANGENTS.

′	28°	29°	30°	31°	32°	33°	34°	′
0	0·5317094	0·5543091	0·5773503	0·6008606	0·6248694	0·6494076	0·6745085	60
1	·5320826	·5546894	·5777382	·6012566	·6252739	·6498212	·6749318	59
2	·5324559	·5550698	·5781262	·6016527	·6256786	·6502350	·6753553	58
3	·5328293	·5554504	·5785144	·6020490	·6260834	·6506490	·6757790	57
4	·5332029	·5558311	·5789027	·6024454	·6264884	·6510631	·6762028	56
5	·5335765	·5562119	·5792912	·6028419	·6268935	·6514774	·6766268	55
6	·5339503	·5565929	·5796797	·6032386	·6272988	·6518918	·6770509	54
7	·5343242	·5569739	·5800684	·6036354	·6277042	·6523064	·6774752	53
8	·5346981	·5573551	·5804573	·6040323	·6281098	·6527211	·6778997	52
9	·5350723	·5577364	·5808462	·6044294	·6285155	·6531360	·6783243	51
10	0·5354465	0·5581179	0·5812353	0·6048266	0·6289214	0·6535511	0·6787492	50
11	·5358208	·5584994	·5816245	·6052240	·6293274	·6539663	·6791741	49
12	·5361953	·5588811	·5820139	·6056215	·6297336	·6543817	·6795993	48
13	·5365699	·5592629	·5824034	·6060192	·6301399	·6547972	·6800246	47
14	·5369446	·5596449	·5827930	·6064170	·6305464	·6552129	·6804501	46
15	·5373194	·5600269	·5831828	·6068149	·6309530	·6556287	·6808758	45
16	·5376943	·5604091	·5835726	·6072130	·6313598	·6560447	·6813016	44
17	·5380694	·5607914	·5839627	·6076112	·6317667	·6564609	·6817276	43
18	·5384445	·5611738	·5843528	·6080095	·6321738	·6568772	·6821537	42
19	·5388198	·5615564	·5847431	·6084080	·6325810	·6572937	·6825801	41
20	0·5391952	0·5619391	0·5851335	0·6088067	0·6329883	0·6577103	0·6830066	40
21	·5395707	·5623219	·5855241	·6092054	·6333959	·6581271	·6834333	39
22	·5399464	·5627048	·5859148	·6096043	·6338035	·6585441	·6838601	38
23	·5403221	·5630879	·5863056	·6100034	·6342113	·6589612	·6842871	37
24	·5406980	·5634710	·5866965	·6104026	·6346193	·6593785	·6847143	36
25	·5410740	·5638543	·5870876	·6108019	·6350274	·6597960	·6851416	35
26	·5414501	·5642378	·5874788	·6112014	·6354357	·6602136	·6855692	34
27	·5418263	·5646213	·5878702	·6116011	·6358441	·6606313	·6859969	33
28	·5422027	·5650050	·5882616	·6120008	·6362527	·6610492	·6864247	32
29	·5425791	·5653888	·5886533	·6124007	·6366614	·6614673	·6868528	31
30	0·5429557	0·5657728	0·5890450	0·6128008	0·6370703	0·6618856	0·6872810	30
31	·5433324	·5661568	·5894369	·6132010	·6374793	·6623040	·6877093	29
32	·5437092	·5665410	·5898289	·6136013	·6378885	·6627225	·6881379	28
33	·5440862	·5669254	·5902211	·6140018	·6382978	·6631413	·6885666	27
34	·5444632	·5673098	·5906134	·6144024	·6387073	·6635601	·6889955	26
35	·5448404	·5676944	·5910058	·6148032	·6391169	·6639792	·6894246	25
36	·5452177	·5680791	·5913984	·6152041	·6395267	·6643984	·6898538	24
37	·5455951	·5684639	·5917910	·6156052	·6399366	·6648178	·6902832	23
38	·5459727	·5688488	·5921839	·6160064	·6403467	·6652373	·6907128	22
39	·5463503	·5692339	·5925768	·6164077	·6407569	·6656570	·6911425	21
40	0·5467281	0·5696191	0·5929699	0·6168092	0·6411673	0·6660769	0·6915725	20
41	·5471060	·5700045	·5933632	·6172108	·6415779	·6664969	·6920026	19
42	·5474840	·5703899	·5937565	·6176126	·6419886	·6669171	·6924328	18
43	·5478621	·5707755	·5941501	·6180145	·6423994	·6673374	·6928633	17
44	·5482404	·5711612	·5945437	·6184166	·6428105	·6677580	·6932939	16
45	·5486188	·5715471	·5949375	·6188188	·6432216	·6681786	·6937247	16
46	·5489973	·5719331	·5953314	·6192211	·6436329	·6685995	·6941557	14
47	·5493759	·5723192	·5957255	·6196236	·6440444	·6690205	·6945868	13
48	·5497547	·5727054	·5961196	·6200263	·6444560	·6694417	·6950181	12
49	·5501335	·5730918	·5965140	·6204291	·6448678	·6698630	·6954496	11
50	0·5505125	0·5734783	0·5969084	0·6208320	0·6452797	0·6702845	0·6958813	10
51	·5508916	·5738649	·5973030	·6212351	·6456918	·6707061	·6963131	9
52	·5512708	·5742516	·5976978	·6216383	·6461041	·6711280	·6967451	8
53	·5516502	·5746385	·5980926	·6220417	·6465165	·6715500	·6971773	7
54	·5520297	·5750255	·5984877	·6224452	·6469290	·6719721	·6976097	6
55	·5524093	·5754126	·5988828	·6228488	·6473417	·6723944	·6980422	5
56	·5527890	·5757999	·5992781	·6232527	·6477546	·6728169	·6984749	4
57	·5531688	·5761873	·5996735	·6236566	·6481676	·6732396	·6989078	3
58	·5535488	·5765748	·6000691	·6240607	·6485808	·6736624	·6993409	2
59	·5539288	·5769625	·6004648	·6244650	·6489941	·6740854	·6997741	1
60	0·5543091	0·5773503	0·6008606	0·6248694	0·6494076	0·6745085	0·7002075	0
′	61°	60°	59°	58°	57°	56°	55°	′

CO-TANGENTS.

TANGENTS.

′	35°	36°	37°	38°	39°	40°	41°	′
0	0·7002075	0·7265425	0·7535541	0·7812856	0·8097840	0·8390996	0·8692867	60
1	·7006411	·7269871	·7540102	·7817542	·8102658	·8395955	·8697976	59
2	·7010749	·7274318	·7544666	·7822229	·8107478	·8400915	·8703087	58
3	·7015089	·7278767	·7549232	·7826919	·8112300	·8405878	·8708200	57
4	·7019430	·7283218	·7553799	·7831611	·8117124	·8410844	·8713316	56
5	·7023773	·7287671	·7558369	·7836305	·8121951	·8415812	·8718435	55
6	·7028118	·7292125	·7562941	·7841002	·8126780	·8420782	·8723556	54
7	·7032464	·7296582	·7567514	·7845700	·8131611	·8425755	·8728680	53
8	·7036813	·7301041	·7572090	·7850400	·8136444	·8430730	·8733806	52
9	·7041163	·7305501	·7576668	·7855103	·8141280	·8435708	·8738935	51
10	0·7045515	0·7309963	0·7581248	0·7859808	0·8146118	0·8440688	0·8744067	50
11	·7049869	·7314428	·7585829	·7864515	·8150958	·8445670	·8749201	49
12	·7054224	·7318894	·7590413	·7869224	·8155801	·8450655	·8754338	48
13	·7058581	·7323362	·7594999	·7873935	·8160646	·8455643	·8759478	47
14	·7062940	·7327832	·7599587	·7878649	·8165493	·8460633	·8764620	46
15	·7067301	·7332303	·7604177	·7883364	·8170343	·8465625	·8769765	45
16	·7071664	·7336777	·7608769	·7888082	·8175195	·8470620	·8774912	44
17	·7076028	·7341253	·7613363	·7892802	·8180049	·8475617	·8780062	43
18	·7080395	·7345730	·7617959	·7897524	·8184905	·8480617	·8785215	42
19	·7084763	·7350210	·7622557	·7902248	·8189764	·8485619	·8790370	41
20	0·7089133	0·7354691	0·7627157	0·7906975	0·8194625	0·8490624	0·8795528	40
21	·7093504	·7359174	·7631759	·7911703	·8199488	·8495631	·8800688	39
22	·7097878	·7363660	·7636363	·7916434	·8204354	·8500640	·8805852	38
23	·7102253	·7368147	·7640969	·7921167	·8209222	·8505653	·8811017	37
24	·7106630	·7372636	·7645577	·7925902	·8214093	·8510667	·8816186	36
25	·7111009	·7377127	·7650188	·7930640	·8218965	·8515684	·8821357	35
26	·7115390	·7381620	·7654800	·7935379	·8223840	·8520704	·8826531	34
27	·7119772	·7386115	·7659414	·7940121	·8228718	·8525726	·8831707	33
28	·7124157	·7390611	·7664031	·7944865	·8233597	·8530750	·8836886	32
29	·7128543	·7395110	·7668649	·7949611	·8238479	·8535777	·8842068	31
30	0·7132931	0·7399611	0·7673270	0·7954359	0·8243364	0·8540807	0·8847253	30
31	·7137320	·7404113	·7677893	·7959110	·8248251	·8545839	·8852440	29
32	·7141712	·7408618	·7682517	·7963862	·8253140	·8550873	·8857630	28
33	·7146106	·7413124	·7687144	·7968617	·8258031	·8555910	·8862822	27
34	·7150501	·7417633	·7691773	·7973374	·8262925	·8560950	·8868017	26
35	·7154898	·7422143	·7696404	·7978134	·8267821	·8565992	·8873215	25
36	·7159297	·7426655	·7701037	·7982895	·8272719	·8571037	·8878415	24
37	·7163698	·7431170	·7705672	·7987659	·8277620	·8576084	·8883619	23
38	·7168100	·7435686	·7710309	·7992425	·8282523	·8581133	·8888825	22
39	·7172505	·7440204	·7714948	·7997193	·8287429	·8586185	·8894033	21
40	0·7176911	0·7444724	0·7719589	0·8001963	0·8292337	0·8591240	0·8899244	20
41	·7181319	·7449246	·7724233	·8006736	·8297247	·8596297	·8904458	19
42	·7185729	·7453770	·7728878	·8011511	·8302160	·8601357	·8909675	18
43	·7190141	·7458296	·7733526	·8016288	·8307075	·8606419	·8914894	17
44	·7194554	·7462824	·7738176	·8021067	·8311992	·8611484	·8920116	16
45	·7198970	·7467354	·7742827	·8025849	·8316912	·8616551	·8925341	15
46	·7203387	·7471886	·7747481	·8030632	·8321834	·8621621	·8930569	14
47	·7207806	·7476420	·7752137	·8035418	·8326759	·8626694	·8935799	13
48	·7212227	·7480956	·7756795	·8040206	·8331686	·8631768	·8941032	12
49	·7216650	·7485494	·7761455	·8044997	·8336615	·8636846	·8946268	11
50	0·7221075	0·7490033	0·7766118	0·8049790	0·8341547	0·8641926	0·8951506	10
51	·7225502	·7494575	·7770782	·8054584	·8346481	·8647009	·8956747	9
52	·7229930	·7499119	·7775448	·8059382	·8351418	·8652094	·8961991	8
53	·7234361	·7503665	·7780117	·8064181	·8356357	·8657181	·8967238	7
54	·7238793	·7508212	·7784788	·8068983	·8361298	·8662272	·8972487	6
55	·7243227	·7512762	·7789460	·8073787	·8366242	·8667365	·8977739	5
56	·7247663	·7517314	·7794135	·8078593	·8371188	·8672460	·8982994	4
57	·7252101	·7521867	·7798812	·8083401	·8376136	·8677558	·8988251	3
58	·7256540	·7526423	·7803492	·8088212	·8381087	·8682659	·8993512	2
59	·7260982	·7530981	·7808173	·8093025	·8386041	·8687762	·8998775	1
60	0·7265425	0·7535541	0·7812856	0·8097840	0·8390996	0·8692867	0·9004040	0
′	54°	53°	52°	51°	50°	49°	48°	′

CO-TANGENTS.

TANGENTS.

′	42°	43°	44°	45°	46°	47°	48°	′
0	0·9004040	0·9325151	0·9656888	1·0000000	1·0355303	1·0723687	1·1106125	60
1	·9009309	·9330591	·9662511	1·0005819	1·0361333	1·0729943	1·1112624	59
2	·9014580	·9336034	·9668137	1·0011642	1·0367367	1·0736203	1·1119127	58
3	·9019854	·9341479	·9673767	1·0017469	1·0373404	1·0742467	1·1125635	57
4	·9025131	·9346928	·9679399	1·0023298	1·0379445	1·0748734	1·1132146	56
5	·9030411	·9352380	·9685035	1·0029131	1·0385489	1·0755006	1·1138662	55
6	·9035693	·9357834	·9690674	1·0034968	1·0391538	1·0761282	1·1145182	54
7	·9040979	·9363292	·9696316	1·0040807	1·0397589	1·0767561	1·1151706	53
8	·9046267	·9368753	·9701962	1·0046651	1·0403645	1·0773845	1·1158235	52
9	·9051557	·9374216	·9707610	1·0052497	1·0409704	1·0780132	1·1164768	51
10	0·9056851	0·9379683	0·9713262	1·0058348	1·0415767	1·0786423	1·1171305	50
11	·9062147	·9385153	.9718917	1·0064201	1·0421833	1·0792718	1·1177846	49
12	·9067446	·9390625	·9724575	1·0070058	1·0427904	1·0799018	1·1184391	48
13	·9072748	·9396101	·9730236	1·0075918	1·0433977	1·0805321	1·1190941	47
14	·9078053	·9401579	·9735901	1·0081782	1·0440055	1·0811628	1·1197495	46
15	·9083360	·9407061	·9741569	1·0087649	1·0446136	1·0817939	1·1204053	45
16	·9088671	·9412545	·9747240	1·0093520	1·0452221	1·0824254	1·1210616	44
17	·9093984	·9418033	·9752914	1·0099394	1·0458310	1·0830573	1·1217183	43
18	·9099300	·9423523	·9758591	1·0105272	1·0464402	1·0836896	1·1223754	42
19	·9104619	·9429017	·9764272	1·0111153	1·0470498	1·0843223	1·1230329	41
20	0·9109940	0·9434513	0·9769956	1·0117038	1·0476598	1·0849554	1·1236909	40
21	·9115265	·9440013	·9775643	1·0122925	1·0482702	1·0855889	1·1243493	39
22	·9120592	·9445516	·9781333	1·0128817	1·0488809	1·0862228	1·1250081	38
23	·9125922	·9451021	·9787027	1·0134712	1·0494920	1·0868571	1·1256674	37
24	·9131255	·9456530	·9792724	1·0140610	1·0501034	1·0874918	1·1263271	36
25	·9136591	·9462042	·9798424	1·0146512	1·0507153	1·0881269	1·1269872	35
26	·9141929	·9467556	·9804127	1·0152418	1·0513275	1·0887624	1·1276478	34
27	·9147270	·9473074	·9809833	1·0158326	1·0519401	1·0893984	1·1283088	33
28	·9152615	·9478595	·9815543	1·0164239	1·0525531	1·0900347	1·1289702	32
29	·9157962	·9484119	·9821256	1·0170155	1·0531664	1·0906714	1·1296321	31
30	0·9163312	0·9489646	0·9826973	1·0176074	1·0537801	1·0913085	1·1302944	30
31	·9168665	·9495176	·9832692	1·0181997	1·0543942	1·0919460	1·1309571	29
32	·9174020	·9500709	·9838415	1·0187923	1·0550087	1·0925840	1·1316203	28
33	·9179379	·9506245	·9844141	1·0193853	1·0556235	1·0932223	1·1322839	27
34	·9184740	·9511784	·9849871	1·0199786	1·0562388	1·0938610	1·1329479	26
35	·9190104	·9517326	·9855603	1·0205723	1·0568544	1·0945002	1·1336124	25
36	·9195471	·9522871	·9861339	1·0211664	1·0574704	1·0951397	1·1342773	24
37	·9200841	·9528420	·9867079	1·0217608	1·0580867	1·0957797	1·1349427	23
38	·9206214	·9533971	·9872821	1·0223555	1·0587035	1·0964201	1·1356085	22
39	·9211590	·9539526	·9878567	1·0229506	1·0593206	1·0970609	1·1362747	21
40	0·9216969	0·9545083	0·9884316	1·0235461	1·0599381	1·0977020	1·1369414	20
41	·9222350	·9550644	·9890069	1·0241419	1·0605560	1·0983436	1·1376086	19
42	·9227734	·9556208	·9895825	1·0247381	1.0611742	1·0989857	1·1382761	18
43	·9233122	·9561774	·9901584	1·0253346	1·0617929	1·0996281	1·1389441	17
44	·9238512	·9567344	·9907346	1·0259315	1·0624119	1·1002709	1·1396126	16
45	·9243905	·9572917	·9913112	1·0265287	1·0630313	1·1009141	1·1402815	15
48	·9249301	·9578494	·9918881	1·0271263	1·0636511	1·1015578	1·1409508	14
47	·9254700	·9584073	·9924654	1·0277243	1·0642713	1·1022019	1·1416206	13
48	·9260102	·9589655	·9930429	1·0283226	1·0648918	1·1028463	1·1422908	12
49	·9265506	·9595241	·9936208	1·0289212	1·0655128	1·1034912	1·1429615	11
50	0·9270914	0·9600829	0·9941991	1·0295203	1·0661341	1·1041365	1·1436326	10
51	·9276324	·9606421	·9947777	1·0301196	1·0667558	1·1047823	1·1443041	9
52	·9281738	·9612016	·9953566	1·0307194	1·0673779	1·1054284	1·1449762	8
53	·9287154	·9617614	·9959358	1·0313195	1·0680004	1·1060750	1·1456486	7
54	·9292573	·9623215	·9965154	1·0319199	1·0686233	1·1067219	1·1463215	6
55	·9297996	·9628819	·9970953	1·0325208	1·0692466	1·1073693	1·1469949	5
56	·9303421	·9634427	·9976756	1·0331220	1·0698702	1·1080171	1·1476687	4
57	·9308849	·9640037	·9982562	1·0337235	1·0704943	1·1086653	1·1483429	3
58	·9314280	·9645651	·9988371	1·0343254	1·0711187	1·1093140	1·1490176	2
59	·9319714	·9651268	·9994184	1·0349277	1·0717435	1·1099630	1·1496928	1
60	0·9325151	0·9656888	1·0000000	1·0355303	1·0723687	1·1106125	1·1503684	0
′	47°	46°	45°	44°	43°	42°	41°	′

CO-TANGENTS.

TANGENTS.

′	49°	50°	51°	52°	53°	54°	55°	′
0	1·1503684	1·1917536	1·2348972	1·2799416	1·3270448	1·3763819	1·4281480	80
1	1·1510445	1·1924579	1·2356319	1·2807094	1·3278483	1·3772242	1·4290326	59
2	1·1517210	1·1931626	1·2363672	1·2814776	1·3286524	1·3780672	1·4299178	58
3	1·1523979	1·1938679	1·2371030	1·2822465	1·3294571	1·3789108	1·4308039	57
4	1·1530754	1·1945736	1·2378393	1·2830160	1·3302624	1·3797551	1·4316906	56
5	1·1537532	1·1952799	1·2385762	1·2837860	1·3310684	1·3806001	1·4325781	55
6	1·1544316	1·1959866	1·2393136	1·2845566	1·3318750	1·3814458	1·4334664	54
7	1·1551104	1·1966938	1·2400515	1·2853277	1·3326822	1·3822922	1·4343554	53
8	1·1557896	1·1974015	1·2407900	1·2860995	1·3334900	1·3831392	1·4352451	52
9	1·1564693	1·1981097	1·2415290	1·2868718	1·3342984	1·3839869	1·4361356	51
10	1·1571495	1·1988184	1·2422685	1·2876447	1·3351075	1·3848353	1·4370268	50
11	1·1578301	1·1995276	1·2430086	1·2884182	1·3359172	1·3856844	1·4379187	49
12	1·1585112	1·2002373	1·2437492	1·2891922	1·3367276	1·3865342	1·4388114	48
13	1·1591927	1·2009475	1·2444903	1·2899669	1·3375386	1·3873847	1·4397049	47
14	1·1598747	1·2016581	1·2452320	1·2907421	1·3383502	1·3882358	1·4405991	46
15	1·1605571	1·2023693	1·2459742	1·2915179	1·3391624	1·3890876	1·4414940	45
16	1·1612400	1·2030810	1·2467169	1·2922943	1·3399753	1·3899401	1·4423897	44
17	1·1619234	1·2037932	1·2474602	1·2930713	1·3407888	1·3907934	1·4432862	43
18	1·1626073	1·2045058	1·2482040	1·2938488	1·3416029	1·3916473	1·4441834	42
19	1·1632916	1·2052190	1·2489484	1·2946270	1·3424177	1·3925019	1·4450814	41
20	1·1639763	1·2059327	1·2496933	1·2954057	1·3432331	1·3933571	1·4459801	40
21	1·1646615	1·2066468	1·2504388	1·2961850	1·3440492	1·3942131	1·4468796	39
22	1·1653472	1·2073615	1·2511848	1·2969649	1·3448658	1·3950698	1·4477798	38
23	1·1660334	1·2080767	1·2519313	1·2977454	1·3456832	1·3959272	1·4486808	37
24	1·1667200	1·2087924	1·2526784	1·2985265	1·3465011	1·3967852	1·4495825	36
25	1·1674071	1·2095085	1·2534260	1·2993081	1·3473198	1·3976440	1·4504850	35
26	1·1680947	1·2102252	1·2541742	1·3000904	1·3481390	1·3985034	1·4513883	34
27	1·1687827	1·2109424	1·2549229	1·3008733	1·3489589	1·3993636	1·4522923	33
28	1·1694712	1·2116601	1·2556721	1·3016567	1·3497794	1·4002245	1·4531971	32
29	1·1701601	1·2123783	1·2564219	1·3024407	1·3506006	1·4010860	1·4541027	31
30	1·1708496	1·2130970	1·2571723	1·3032254	1·3514224	1 4019483	1·4550090	30
31	1·1715395	1·2138162	1·2579232	1·3040106	1·3522449	1·4028113	1·4559161	20
32	1·1722298	1·2145359	1·2586747	1·3047964	1·3530680	1·4036749	1·4568240	28
33	1·1729207	1·2152562	1·2594267	1·3055828	1·3538918	1·4045393	1·4577326	27
34	1·1736120	1·2159769	1·2601792	1·3063699	1·3547162	1·4054044	1·4586420	26
35	1·1743038	1·2166982	1·2609323	1·3071575	1·3555413	1·4062702	1·4595522	25
36	1·1749960	1·2174199	1·2616860	1·3079457	1·3563670	1·4071367	1·4604632	24
37	1·1756888	1·2181422	1·2624402	1·3087345	1·3571934	1·4080039	1·4613749	23
38	1·1763820	1·2188650	1·2631950	1·3095239	1·3580204	1·4088718	1·4622874	22
39	1·1770756	1·2195883	1·2639503	1·3103140	1·3588481	1·4097405	1·4632007	21
40	1·1777698	1·2203121	1·2647062	1·3111046	1·3596764	1·4106098	1·4641147	20
41	1·1784644	1·2210364	1·2654626	1·3118958	1·3605054	1·4114799	1·4650296	19
42	1·1791595	1·2217613	1·2662196	1·3126876	1·3613350	1·4123506	1·4659452	18
43	1·1798551	1·2224866	1·2669772	1·3134801	1·3621653	1·4132221	1·4668616	17
44	1·1805512	1·2232125	1·2677353	1·3142731	1·3629963	1·4140943	1·4677788	16
46	1·1812477	1·2239389	1·2684940	1·3150668	1·3638279	1·4149673	1·4686967	15
48	1·1819447	1·2246658	1·2692532	1·3158610	1·3646602	1·4158409	1·4696155	14
47	1·1826422	1·2253932	1·2700130	1·3166559	1·3654931	1·4167153	1·4705350	13
48	1·1833402	1·2261211	1·2707733	1·3174513	1·3663267	1·4175904	1·4714553	12
49	1·1840387	1·2268496	1·2715342	1·3182474	1·3671610	1·4184662	1·4723764	11
50	1·1847376	1·2275786	1·2722957	1·3190441	1·3679959	1·4193427	1·4732983	10
51	1·1854370	1·2283081	1·2730578	1·3198414	1·3688315	1·4202200	1·4742210	9
52	1·1861369	1·2290381	1·2738204	1·3206393	1·3696678	1·4210979	1·4751445	8
63	1·1868373	1·2297687	1·2745835	1·3214379	1·3705047	1·4219766	1·4760688	7
54	1·1875382	1·2304997	1·2753473	1·3222370	1·3713423	1·4228561	1·4769938	6
65	1·1882395	1·2312313	1·2761116	1·3230368	1·3721806	1·4237362	1·4779197	5
56	1·1889414	1·2319634	1·2768765	1·3238371	1·3730195	1·4246171	1·4788463	4
67	1·1896437	1·2326961	1·2776419	1·3246381	1·3738591	1·4254988	1·4797738	3
58	1·1903465	1·2334292	1·2784079	1·3254397	1·3746994	1·4263811	1·4807021	2
59	1·1910498	1·2341629	1·2791745	1·3262420	1·3755403	1·4272642	1·4816311	1
60	1·1917536	1·2348972	1·2799416	1·3270448	1·3763819	1·4281480	1·4825610	0
′	40°	39°	38°	37°	36°	35°	34°	′

CO-TANGENTS.

TANGENTS.

′	56°	57°	58°	59°	60°	61°	62°	′
0	1·4825610	1·5398650	1·6003345	1·6642795	1·7320508	1·8040478	1·8807265	60
1	1·4834916	1·5408460	1·6013709	1·6653766	1·7332149	1·8052860	1·8820470	59
2	1·4844231	1·5418280	1·6024082	1·6664748	1·7343803	1·8065256	1·8833690	58
3	1·4853554	1·5428108	1·6034465	1·6675741	1·7355468	1·8077664	1·8846924	57
4	1·4862884	1·5437946	1·6044858	1·6686744	1·7367144	1·8090086	1·8860172	56
5	1·4872223	1·5447792	1·6055260	1·6697758	1·7378833	1·8102521	1·8873436	55
6	1·4881570	1·5457647	1·6065672	1·6708782	1·7390533	1·8114969	1·8886713	54
7	1·4890925	1·5467510	1·6076094	1·6719818	1·7402245	1·8127430	1·8900006	53
8	1·4900288	1·5477383	1·6086525	1·6730864	1·7413969	1·8139904	1·8913313	52
9	1·4909659	1·5487264	1·6096966	1·6741921	1·7425705	1·8152391	1·8926635	51
10	1·4919039	1·5497155	1·6107417	1·6752988	1·7437453	1·8164892	1·8639971	50
11	1·4928426	1·5507054	1·6117878	1·6764067	1·7449213	1·8177405	1·8953322	49
12	1·4937822	1·5516963	1·6128349	1·6775156	1·7460984	1·8189932	1·8966688	48
13	1·4947225	1·5526880	1·6138829	1·6786256	1·7472768	1·8202473	1·8980068	47
14	1·4956637	1·5536806	1·6149320	1·6797367	1·7484564	1·8215026	1·8993464	46
15	1·4966058	1·5546741	1·6159820	1·6808489	1·7496371	1·8227593	1·9006874	45
16	1·4975486	1·5556685	1·6170330	1·6819621	1·7508191	1·8240173	1·9020299	44
17	1·4984923	1·5566639	1·6180850	1·6830765	1·7520023	1·8252767	1·9033738	43
18	1·4994367	1·5576601	1·6191380	1·6841919	1·7531866	1·8265374	1·9047193	42
19	1·5003821	1·5586572	1·6201920	1·6853085	1·7543722	1·8277994	1 9060663	41
20	1·5013282	1·5596552	1·6212469	1·6864261	1·7555590	1·8290628	1·9074147	40
21	1·5022751	1·5606542	1·6223029	1·6875449	1·7567470	1·8303275	1·9087647	39
22	1·5032229	1·5616540	1·6233599	1·6886647	1·7579362	1·8315936	1·9101162	38
23	1·5041716	1·5626548	1·6244178	1·6897856	1·7591267	1·8328610	1·9114691	37
24	1·5051210	1·5636564	1·6254768	1·6909077	1·7603183	1·8341297	1·9128236	36
25	1·5060713	1·5646590	1·6265368	1·6920308	1·7615112	1·8353999	1·9141795	35
26	1·5070224	1·5656625	1·6275977	1·6931550	1·7627053	1·8366713	1·9155370	34
27	1·5079743	1·5666669	1·6286597	1·6942804	1·7639007	1·8379442	1·9168960	33
28	1·5089271	1·5676722	1·6297227	1·6954069	1·7650972	1·8392184	1·9182565	32
29	1·5098807	1·5686784	1·6307867	1·6965344	1·7662950	1·8404940	1·9196186	31
30	1·5108352	1·5696856	1·6318517	1·6976631	1·7674940	1·8417709	1·9209821	30
31	1·5117905	1·5706936	1·6329177	1·6987929	1·7686943	1·8430492	1·9223472	29
32	1·5127466	1·5717026	1·6339847	1·6999238	1·7698958	1·8443289	1·9237138	28
33	1·5137036	1·5727126	1·6350528	1·7010559	1·7710985	1·8456099	1·9250819	27
34	1·5146614	1·5737234	1·6361218	1·7021890	1·7723024	1·8468923	1·9264516	26
35	1·5156201	1·5747352	1·6371919	1·7033233	1·7735076	1·8481761	1·9278228	25
36	1·5165796	1·5757479	1·6382630	1·7044587	1·7747141	1·8494613	1·9291956	24
37	1·5175400	1·5767615	1·6393351	1·7055953	1·7759218	1·8507479	1·9305699	23
38	1·5185012	1·5777760	1·6404082	1·7067329	1·7771307	1·8520358	1·9319457	22
39	1·5194632	1·5787915	1·6414824	1·7078717	1·7783409	1·8533252	1·9333231	21
40	1·5204261	1·5798079	1·6425576	1·7090116	1·7795524	1·8546159	1·9347020	20
41	1·5213899	1·5808253	1·6436338	1·7101527	1·7807651	1·8559080	1·9360825	19
42	1·5223545	1·5818436	1·6447111	1·7112949	1·7819790	1·8572015	1·9374645	18
43	1·5233200	1·5828628	1·6457893	1·7124382	1·7831943	1·8584965	1·9388481	17
44	1·5242863	1·5838830	1·6468687	1·7135827	1·7844107	1·8597928	1·9402333	16
45	1·5252535	1·5849041	1·6479490	1·7147283	1·7856285	1·8610905	1·9416200	15
46	1·5262215	1·5859261	1·6490304	1·7158751	1·7868475	1·8623896	1·9430083	14
47	1·5271904	1·5869491	1·6501128	1·7170230	1·7880678	1·8636902	1·9443981	13
48	1·5281602	1·5879731	1·6511963	1·7181720	1·7892893	1·8649921	1·9457896	12
49	1·5291308	1·5889979	1·6522808	1·7193222	1·7905121	1·8662955	1·9471826	11
50	1·5301023	1·5900238	1·6533663	1·7204736	1·7917362	1·8676003	1·9485772	10
51	1·5310746	1·5910505	1·6544529	1·7216261	1·7929616	1·8689065	1·9499733	9
52	1·5320479	1·5920783	1·6555405	1·7227797	1·7941883	1·8702141	1·9513711	8
53	1·5330219	1·5931070	1·6566292	1·7239346	1·7954162	1·8715231	1·9527704	7
54	1·5339969	1·5941366	1·6577189	1·7250905	1·7966454	1·8728336	1·9541713	6
55	1·5349727	1·5951672	1·6588097	1·7262477	1·7978759	1·8741455	1·9555739	5
56	1·5359494	1·5961987	1·6599016	1·7274060	1·7991077	1·8754588	1·9569780	4
57	1·5369270	1·5972312	1·6609945	1·7285654	1·8003408	1·8767736	1·9583837	3
58	1·5379054	1·5982647	1·6620884	1·7297260	1·8015751	1·8780898	1·9597910	2
59	1·5388848	1·5992991	1·6631834	1·7308878	1·8028108	1·8794074	1·9612000	1
60	1·5398650	1·6003345	1·6642795	1·7320508	1·8040478	1·8807265	1·9626105	0
′	33°	32°	31°	30°	29°	28°	27°	′

CO-TANGENTS.

TANGENTS.

′	63°	64°	65°	66°	67°	68°	69°	′
0	1·9626105	2·0503038	2·1445069	2·2460368	2·3558524	2·4750869	2·6050891	60
1	1·9640227	2·0518185	2·1461366	2·2477962	2·3577590	2·4771612	2·6073558	59
2	1·9654364	2·0533349	2·1477683	2·2495580	2·3596683	2·4792386	2·6096259	58
3	1·9668518	2·0548531	2·1494021	2·2513221	2·3615801	2·4813190	2·6118995	57
4	1·9682688	2·0563732	2·1510378	2·2530885	2·3634946	2·4834023	2·6141766	56
5	1·9696874	2·0578950	2·1526757	2·2548572	2·3654118	2·4854887	2·6164571	55
6	1·9711077	2·0594187	2·1543156	2·2566283	2·3673316	2·4875781	2·6187411	54
7	1·9725296	2·0609442	2·1559575	2·2584016	2·3692540	2·4896706	2·6210286	53
8	1·9739531	2·0624716	2·1576015	2·2601773	2·3711791	2·4917660	2·6233196	52
9	1·9753782	2·0640008	2·1592476	2·2619554	2·3731068	2·4938645	2·6256141	51
10	1·9768050	2·0655318	2·1608958	2·2637357	2·3750372	2·4959661	2·6279121	50
11	1·9782334	2·0670646	2·1625460	2·2655184	2·3769703	2·4980707	2·6302136	49
12	1·9796635	2·0685994	2·1641983	2·2673035	2·3789060	2·5001784	2·6325186	48
13	1·9810952	2·0701359	2·1658527	2·2690909	2·3808444	2·5022891	2·6348271	47
14	1·9825286	2·0716743	2·1675091	2·2708807	2·3827855	2·5044029	2·6371392	46
15	1·9839636	2·0732146	2·1691677	2·2726729	2·3847293	2·5065198	2·6394549	45
16	1·9854003	2·0747567	2·1708283	2·2744674	2·3866758	2·5086398	2·6417741	44
17	1·9868387	2·0763007	2·1724911	2·2762643	2·3886250	2·5107629	2·6440969	43
18	1·9882787	2·0778465	2·1741559	2·2780636	2·3905769	2·5128890	2·6464232	42
19	1·9897204	2·0793942	2·1758229	2·2798653	2·3925316	2·5150183	2·6487531	41
20	1·9911637	2·0809438	2·1774920	2·2816693	2·3944889	2·5171507	2·6510867	40
21	1·9926087	2·0824953	2·1791631	2·2834758	2·3964490	2·5192863	2·6534238	39
22	1·9940554	2·0840487	2·1808364	2·2852846	2·3984118	2·5214249	2·6557645	38
23	1·9955038	2·0856039	2·1825119	2·2870959	2·4003774	2·5235667	2·6581089	37
24	1·9969539	2·0871610	2·1841894	2·2889096	2·4023457	2·5257117	2·6604569	36
25	1·9984056	2·0887200	2·1858691	2·2907257	2·4043168	2·5278598	2·6628085	35
26	1·9998590	2·0902809	2·1875510	2·2925442	2·4062906	2·5300111	2·6651638	34
27	2·0013142	2·0918437	2·1892349	2·2943651	2·4082672	2·5321655	2·6675227	33
28	2·0027710	2·0934085	2·1909210	2·2961885	2·4102465	2·5343231	2·6698853	32
29	2·0042295	2·0949751	2·1926093	2·2980143	2·4122286	2·5364839	2·6722516	31
30	2·0056897	2·0965436	2·1942997	2·2998425	2·4142136	2·5386479	2·6746215	30
31	2·0071516	2·0981140	2·1959923	2·3016732	2·4162013	2·5408151	2·6769951	29
32	2·0086153	2·0996864	2·1976871	2·3035064	2·4181918	2·5429855	2·6793725	28
33	2·0100806	2·1012607	2·1993840	2·3053420	2·4201851	2·5451591	2·6817535	27
34	2·0115477	2·1028369	2·2010831	2·3071801	2·4221812	2·5473359	2·6841383	26
35	2·0130164	2·1044150	2·2027843	2·3090206	2·4241801	2·5495160	2·6865267	25
36	2·0144869	2·1059951	2·2044878	2·3108637	2·4261819	2·5516992	2·6889190	24
37	2·0159592	2·1075771	2·2061934	2·3127092	2·4281864	2·5538858	2·6913149	23
38	2·0174331	2·1091611	2·2079012	2·3145571	2·4301938	2·5560756	2·6937147	22
39	2·0189088	2·1107470	2·2096112	2·3164076	2·4322041	2·5582686	2·6961181	21
40	2·0203862	2·1123348	2·2113234	2·3182606	2·4342172	2·5604649	2·6985254	20
41	2·0218654	2·1139246	2·2130379	2·3201160	2·4362331	2·5626645	2·7009364	19
42	2·0233462	2·1155164	2·2147545	2·3219740	2·4382519	2·5648674	2·7033513	18
43	2·0248289	2·1171101	2·2164733	2·3238345	2·4402736	2·5670735	2·7057699	17
44	2·0263133	2·1187057	2·2181944	2·3256975	2·4422982	2·5692830	2·7081923	16
45	2·0277994	2·1203034	2·2199177	2·3275630	2·4443256	2·5714957	2·7106186	15
46	2·0292873	2·1219030	2·2216432	2·3294311	2·4463559	2·5737118	2·7130487	14
47	2·0307769	2·1235046	2·2233709	2·3313017	2·4483891	2·5759312	2·7154826	13
48	2·0322683	2·1251082	2·2251009	2·3331748	2·4504252	2·5781539	2·7179204	12
49	2·0337615	2·1267137	2·2268331	2·3350505	2·4524642	2·5803800	2·7203620	11
50	2·0352565	2·1283213	2·2285676	2·3369287	2·4545061	2·5826094	2·7228076	10
51	2·0367532	2·1299308	2·2303043	2·3388095	2·4565510	2·5848421	2·7252569	9
52	2·0382517	2·1315423	2·2320433	2·3406928	2·4585987	2·5870782	2·7277102	8
53	2·0397519	2·1331559	2·2337845	2·3425787	2·4606494	2·5893177	2·7301674	7
54	2·0412540	2·1347714	2·2355280	2·3444672	2·4627030	2·5915606	2·7326284	6
55	2·0427578	2·1363890	2·2372738	2·3463582	2·4647596	2·5938068	2·7350934	5
56	2·0442634	2·1380085	2·2390218	2·3482519	2·4668191	2·5960564	2·7375623	4
57	2·0457708	2·1396301	2·2407721	2·3501481	2·4688816	2·5983095	2·7400352	3
58	2·0472800	2·1412537	2·2425247	2·3520469	2·4709470	2·6005659	2·7425120	2
59	2·0487910	2·1428793	2·2442796	2·3539483	2·4730155	2·6028258	2·7449927	1
60	2·0503038	2·1445069	2·2460368	2·3558524	2·4750869	2·6050891	2·7474774	0
′	26°	25°	24°	23°	22°	21°	20°	′

CO-TANGENTS.

TANGENTS.

′	70°	71°	72°	73°	74°	75°	76°	′
0	2·7474774	2·9042109	3·0776835	3·2708526	3·4874144	3·7320508	4·0107809	60
1	2·7499661	2·9069576	3·0807325	3·2742588	3·4912470	3·7363980	4·0157570	59
2	2·7524588	2·9097089	3·0837869	3·2776715	3·4950874	3·7407546	4·0207446	58
3	2·7549554	2·9124649	3·0868468	3·2810907	3·4989356	3·7451207	4·0257440	57
4	2·7574561	2·9152256	3·0899122	3·2845164	3·5027916	3·7494963	4·0307550	56
5	2·7599608	2·9179909	3·0929831	3·2879487	3·5066555	3·7538815	4·0357779	55
6	2·7624695	2·9207610	3·0960596	3·2913876	3·5105273	3·7582763	4·0408125	54
7	2·7649822	2·9235358	3·0991416	3·2948330	3·5144070	3·7626807	4·0458590	53
8	2·7674990	2·9263152	3·1022291	3·2982851	3·5182946	3·7670947	4·0509174	52
9	2·7700199	2·9290995	3·1053223	3·3017438	3·5221902	3·7715185	4·0559877	51
10	2·7725448	2·9318885	3·1084210	3·3052091	3·5260938	3·7759519	4·0610700	50
11	2·7750738	2·9346822	3·1115254	3·3086811	3·5300054	3·7803951	4·0661643	49
12	2·7776069	2·9374807	3·1146353	3·3121598	3·5339251	3·7848481	4·0712707	48
13	2·7801440	2·9402840	3·1177509	3·3156452	3·5378528	3·7893109	4·0763892	47
14	2·7826853	2·9430921	3·1208722	3·3191373	3·5417886	3·7937835	4·0815199	46
15	2·7852307	2·9459050	3·1239991	3·3226362	3·5457325	3·7982661	4·0866627	45
16	2·7877802	2·9487227	3·1271317	3·3261419	3·5496846	3·8027585	4·0918178	44
17	2·7903339	2·9515453	3·1302701	3·3296543	3·5536449	3·8072609	4·0969852	43
18	2·7928917	2·9543727	3·1334141	3·3331736	3·5576133	3·8117733	4·1021649	42
19	2·7954537	2·9572050	3·1365639	3·3366997	3·5615900	3·8162957	4·1073569	41
20	2·7980198	2·9600422	3·1397194	3·3402326	3·5655749	3·8208281	4·1125614	40
21	2·8005901	2·9628842	3·1428807	3·3437724	3·5695681	3·8253707	4·1177784	39
22	2·8031646	2·9657312	3·1460478	3·3473191	3·5735696	3·8299233	4·1230079	38
23	2·8057433	2·9685831	3·1492207	3·3508728	3·5775794	3·8344861	4·1282499	37
24	2·8083263	2·9714399	3·1523994	3·3544333	3·5815975	3·8390591	4·1335046	36
25	2·8109134	2·9743016	3·1555840	3·3580008	3·5856241	3·8436424	4·1387719	35
26	2·8135048	2·9771683	3·1587744	3·3615753	3·5896590	3·8482358	4·1440519	34
27	2·8161004	2·9800400	3·1619706	3·3651568	3·5937024	3·8528396	4·1493446	33
28	2·8187003	2·9829167	3·1651728	3·3687453	3·5977543	3·8574537	4·1546501	32
29	2·8213045	2·9857983	3·1683808	3·3723408	3·6018146	3·8620782	4·1599685	31
30	2·8239129	2·9886850	3·1715948	3·3759434	3·6058835	3·8667131	4·1652998	30
31	2·8265256	2·9915766	3·1748147	3·3795531	3·6099609	3·8713584	4·1706440	29
32	2·8291426	2·9944734	3·1780406	3·3831699	3·6140469	3·8760142	4·1760011	28
33	2·8317639	2·9973751	3·1812724	3·3867938	3·6181415	3·8806805	4·1813713	27
34	2·8343896	3·0002820	3·1845102	3·3904249	3·6222447	3·8853574	4·1867546	26
35	2·8370196	3·0031939	3·1877540	3·3940631	3·6263566	3·8900448	4·1921510	25
36	2·8396539	3·0061109	3·1910039	3·3977085	3·6304771	3·8947429	4·1975606	24
37	2·8422926	3·0090330	3·1942598	3·4013612	3·6346064	3·8994516	4·2029835	23
38	2·8449356	3·0119603	3·1975217	3·4050210	3·6387444	3·9041710	4·2084196	22
39	2·8475831	3·0148926	3·2007897	3·4086882	3·6428911	3·9089011	4·2138690	21
40	2·8502349	3·0178301	3·2040638	3·4123626	3·6470467	3·9136420	4·2193318	20
41	2·8528911	3·0207728	3·2073440	3·4160443	3·6512111	3·9183937	4·2248080	19
42	2·8555517	3·0237207	3·2106304	3·4197333	3·6553844	3·9231563	4·2302977	18
43	2·8582168	3·0266737	3·2139228	3·4234297	3·6595665	3·9279297	4·2358009	17
44	2·8608863	3·0296320	3·2172215	3·4271334	3·6637575	3·9327141	4·2413177	16
45	2·8635602	3·0325954	3·2205263	3·4308446	3·6679575	3·9375094	4·2468482	15
46	2·8662386	3·0355641	3·2238373	3·4345631	3·6721665	3·9423157	4·2523923	14
47	2·8689215	3·0385381	3·2271546	3·4382891	3·6763845	3·9471331	4·2579501	13
48	2·8716088	3·0415173	3·2304780	3·4420226	3·6806115	3·9519615	4·2635218	12
49	2·8743007	3·0445018	3·2338078	3·4457635	3·6848475	3·9568011	4·2691072	11
50	2·8769970	3·0474915	3·2371438	3·4495120	3·6890927	3·9616518	4·2747066	10
51	2·8796979	3·0504866	3·2404860	3·4532679	3·6933469	3·9665137	4·2803199	9
52	2·8824033	3·0534870	3·2438346	3·4570315	3·6976104	3·9713868	4·2859472	8
53	2·8851132	3·0564928	3·2471895	3·4608026	3·7018830	3·9762712	4·2915885	7
54	2·8878277	3·0595038	3·2505508	3·4645813	3·7061648	3·9811669	4·2972440	6
55	2·8905467	3·0625203	3·2539184	3·4683676	3·7104558	3·9860739	4·3029136	5
56	2·8932704	3·0655421	3·2572924	3·4721616	3·7147561	3·9909924	4·3085974	4
57	2·8959986	3·0685694	3·2606728	3·4759632	3·7190658	3·9959223	4·3142955	3
58	2·8987314	3·0716020	3·2640596	3·4797726	3·7233847	4·0008636	4·3200079	2
59	2·9014688	3·0746400	3·2674529	3·4835896	3·7277131	4·0058165	4·3257347	1
60	2·9042109	3·0776835	3·2708526	3·4874144	3·7320508	4·0107809	4·3314759	0
′	19°	18°	17°	16°	15°	14°	13°	′

CO-TANGENTS.

TANGENTS.

′	77°	78°	79°	80°	81°	82°	83°	′
0	4·3314759	4·7046301	5·1445540	5·6712818	6·3137515	7·1153697	8·1443464	60
1	4·3372316	4·7113686	5·1525557	5·6809446	6·3256601	7·1304190	8·1639786	59
2	4·3430018	4·7181256	5·1605813	5·6906394	6·3376126	7·1455308	8·1837041	58
3	4·3487866	4·7249012	5·1686311	5·7003663	6·3496092	7·1607056	8·2035239	57
4	4·3545861	4·7316954	5·1767051	5·7101256	6·3616502	7·1759437	8·2234384	56
5	4·3604003	4·7385083	5·1848035	5·7199173	6·3737359	7·1912456	8·2434485	55
6	4·3662293	4·7453401	5·1929264	5·7297416	6·3858665	7·2066116	8·2635547	54
7	4·3720731	4·7521907	5·2010738	5·7395988	6·3980422	7·2220422	8·2837579	53
8	4·3779317	4·7590603	5·2092459	5·7494889	6·4102633	7·2375378	8·3040586	52
9	4·3838054	4·7659490	5·2174428	5·7594122	6·4225301	7·2530987	8·3244577	51
10	4·3896940	4·7728568	5·2256647	5·7693688	6·4348428	7·2687255	8·3449558	50
11	4·3955977	4·7797837	5·2339116	5·7793588	6·4472017	7·2844184	8·3655536	49
12	4·4015164	4·7867300	5·2421836	5·7893825	6·4596070	7·3001780	8·3862519	48
13	4·4074504	4·7936957	5·2504809	5·7994400	6·4720591	7·3160047	8·4070515	47
14	4·4133996	4·8006808	5·2588035	5·8095315	6·4845581	7·3318989	8·4279531	46
15	4·4193641	4·8076854	5·2671517	5·8196572	6·4971043	7·3478610	8·4489573	45
16	4·4253439	4·8147096	5·2755255	5·8298172	6·5096981	7·3638916	8·4700651	44
17	4·4313392	4·8217536	5·2839251	5·8400117	6·5223396	7·3799909	8·4912772	43
18	4·4373500	4·8288174	5·2923505	5·8502410	6·5350293	7·3961595	8·5125943	42
19	4·4433762	4·8359010	5·3008018	5·8605051	6·5477672	7·4123978	8·5340172	41
20	4·4494181	4·8430045	5·3092793	5·8708042	6·5605538	7·4287064	8·5555468	40
21	4·4554756	4·8501282	5·3177830	5·8811386	6·5733892	7·4450855	8·5771838	39
22	4·4615489	4·8572719	5·3263131	5·8915084	6·5862739	7·4615357	8·5989290	38
23	4·4676379	4·8644359	5·3348696	5·9019138	6·5992080	7·4780576	8·6207833	37
24	4·4737428	4·8716201	5·3434527	5·9123550	6·6121919	7·4946514	8·6427475	36
25	4·4798636	4·8788248	5·3520626	5·9228322	6·6252258	7·5113178	8·6648223	35
26	4·4860004	4·8860499	5·3606993	5·9333455	6·6383100	7·5280571	8·6870088	34
27	4·4921532	4·8932956	5·3693630	5·9438952	6·6514449	7·5448699	8·7093077	33
28	4·4983221	4·9005620	5·3780538	5·9544815	6·6646307	7·5617567	8·7317198	32
29	4·5045072	4·9078491	5·3867718	5·9651045	6·6778677	7·5787179	8·7542461	31
30	4·5107085	4·9151570	5·3955172	5·9757644	6·6911562	7·5957541	8·7768874	30
31	4·5169261	4·9224859	5·4042901	5·9864614	6·7044966	7·6128657	8·7996446	29
32	4·5231601	4·9298358	5·4130906	5·9971957	6·7178891	7·6300533	8·8225186	28
33	4·5294105	4·9372068	5·4219188	6·0079676	6·7313341	7·6473174	8·8455103	27
34	4·5356773	4·9445990	5·4307750	6·0187772	6·7448318	7·6646584	8·8686206	26
35	4·5419608	4·9520125	5·4396592	6·0296247	6·7583826	7·6820769	8·8918505	25
36	4·5482608	4·9594474	5·4485715	6·0405103	6·7719867	7·6995735	8·9152009	24
37	4·5545776	4·9669037	5·4575121	6·0514343	6·7856446	7·7171486	8·9386726	23
38	4·5609111	4·9743817	5·4664812	6·0623967	6·7993565	7·7348028	8·9622668	22
39	4·5672615	4·9818813	5·4754788	6·0733979	6·8131227	7·7525366	8·9859843	21
40	4·5736287	4·9894027	5·4845052	6·0844381	6·8269437	7·7703506	9·0098261	20
41	4·5800129	4·9969459	5·4935604	6·0955174	6·8408196	7·7882453	9·0337933	19
42	4·5864141	5·0045111	5·5026446	6·1066360	6·8547508	7·8062212	9·0578867	18
43	4·5928325	5·0120984	5·5117579	6·1177943	6·8687378	7·8242790	9·0821074	17
44	4·5992680	5·0197078	5·5209005	6·1289923	6·8827807	7·8424191	9·1064564	16
45	4·6057207	5·0273395	5·5300724	6·1402303	6·8968799	7·8606423	9·1309348	15
46	4·6121908	5·0349935	5·5392740	6·1515085	6·9110359	7·8789489	9·1555436	14
47	4·6186783	5·0426700	5·5485052	6·1628272	6·9252489	7·8973396	9·1802838	13
48	4·6251832	5·0503690	5·5577663	6·1741865	6·9395192	7·9158151	9·2051564	12
49	4·6317056	5·0580907	5·5670574	6·1855867	6·9538473	7·9343758	9·2301627	11
50	4·6382457	5·0658352	5·5763786	6·1970279	6·9682335	7·9530224	9·2553035	10
51	4·6448034	5·0736025	5·5857302	6·2085106	6·9826781	7·9717555	9·2805802	9
52	4·6513788	5·0813928	5·5951121	6·2200347	6·9971806	7·9905756	9·3059936	8
53	4·6579721	5·0892061	5·6045247	6·2316007	7·0117441	8·0094835	9·3315450	7
54	4·6645832	5·0970426	5·6139680	6·2432086	7·0263662	8·0284796	9·3572355	6
55	4·6712124	5·1049024	5·6234421	6·2548588	7·0410482	8·0475647	9·3830663	5
56	4·6778595	5·1127855	5·6329474	6·2665515	7·0557905	8·0667394	9·4090384	4
57	4·6845248	5·1206921	5·6424838	6·2782868	7·0705934	8·0860042	9·4351531	3
58	4·6912083	5·1286224	5·6520516	6·2900651	7·0854573	8·1053599	9·4614116	2
59	4·6979100	5·1365763	5·6616509	6·3018866	7·1003826	8·1248071	9·4878149	1
60	4·7046301	5·1445540	5·6712818	6·3137515	7·1153697	8·1443464	9·5143645	0
′	12°	11°	10°	9°	8°	7°	6°	′

CO-TANGENTS.

TANGENTS.

′	84°	85°	86°	87°	88°	89°	′
0	9·5143645	11·430052	14·300666	19·081137	28·636253	57·289962	60
1	9·5410613	11·468474	14·360696	19·187930	28·877089	58·261174	59
2	9·5679068	11·507154	14·421230	19·295922	29·122005	59·265872	58
3	9·5949022	11·546093	14·482273	19·405133	29·371106	60·305820	57
4	9·6220486	11·585294	14·543833	19·515584	29·624499	61·382905	56
5	9·6493475	11·624761	14·605916	19·627296	29·882299	62·499154	55
6	9·6768000	11·664495	14·668529	19·740291	30·144619	63·656741	54
7	9·7044075	11·704500	14·731679	19·854591	30·411580	64·858008	53
8	9·7321713	11·744779	14·795372	19·970219	30·683307	66·105473	52
9	9·7600927	11·785333	14·859616	20·087199	30·959928	67·401854	51
10	9·7881732	11·826167	14·924417	20·205553	31·241577	68·750087	50
11	9·8164140	11·867282	14·989784	20·325308	31·528392	70·153346	49
12	9·8448166	11·908682	15·055723	20·446486	31·820516	71·615070	48
13	9·8733823	11·950370	15·122242	20·569115	32·118099	73·138991	47
14	9·9021125	11·992349	15·189349	20·693220	32·421295	74·729165	46
15	9·9310088	12·034622	15·257052	20·818828	32·730264	76·390009	46
16	9·9600724	12·077192	15·325358	20·945966	33·045173	78·126342	44
17	9·9893050	12·120062	15·394276	21·074664	33·366194	79·943430	43
18	10·018708	12·163236	15·463814	21·204949	33·693509	81·847041	42
19	10·048283	12·206716	15·533981	21·336851	34·027303	83·843507	41
20	10·078031	12·250505	15·604784	21·470401	34·367771	85·939791	40
21	10·107954	12·294609	15·676233	21·605630	34·715115	88·143572	39
22	10·138054	12·339028	15·748337	21·742569	35·069546	90·463336	38
23	10·168332	12·383768	15·821105	21·881251	35·431282	92·908487	37
24	10·198789	12·428831	15·894545	22·021710	35·800553	95·489475	36
25	10·229428	12·474221	15·968667	22·163980	36·177596	98·217943	35
26	10·260249	12·519942	16·043482	22·308097	36·562659	101·10690	34
27	10·291255	12·565997	16·118998	22·454096	36·956001	104·17094	33
28	10·322447	12·612390	16·195225	22·602015	37·357892	107·42648	32
29	10·353827	12·659125	16·272174	22·751892	37·768613	110·89205	31
30	10·385397	12·706205	16·349855	22·903766	38·188459	114·58865	30
31	10·417158	12·753634	16·428279	23·057677	38·617738	118·54018	29
32	10·449112	12·801417	16·507456	23·213666	39·056771	122·77396	28
33	10·481261	12·849557	16·587396	23·371777	39·505895	127·32134	27
34	10·513607	12·898058	16·668112	23·532052	39·965460	132·21851	26
35	10·546151	12·946924	16·749614	23·694537	40·435837	137·50745	25
36	10·578895	12·996160	16·831915	23·859277	40·917412	143·23712	24
37	10·611841	13·045769	16·915025	24·026320	41·410588	149·46502	23
38	10·644992	13·095757	16·998957	24·195714	41·915790	156·25908	22
39	10·678348	13·146127	17·083724	24·367509	42·433464	163·70019	21
40	10·711913	13·196883	17·169337	24·541758	42·964077	171·88540	20
41	10·745687	13·248031	17·255809	24·718512	43·508122	180·93220	19
42	10·779673	13·299574	17·343155	24·897826	44·066113	190·98419	18
43	10·813872	13·351518	17·431385	25·079757	44·638596	202·21875	17
44	10·848288	13·403867	17·520516	25·264361	45·226141	214·85762	16
45	10·882921	13·456625	17·610559	25·451700	45·829351	229·18166	15
46	10·917775	13·509799	17·701529	25·641832	46·448862	245·55198	14
47	10·952850	13·563391	17·793442	25·834823	47·085343	264·44080	13
48	10·988150	13·617409	17·886310	26·030736	47·739501	286·47773	12
49	11·023676	13·671856	17·980150	26·229638	48·412084	312·52137	11
50	11·059431	13·726738	18·074977	26·431600	49·103881	343·77371	10
51	11·095416	13·782060	18·170807	26·636690	49·815726	381·97099	9
52	11·131635	13·837827	18·267654	26·844984	50·548506	429·71757	8
53	11·168089	13·894045	18·365537	27·056557	51·303157	491·10600	7
54	11·204780	13·950719	18·464471	27·271486	52·080673	572·95721	6
55	11·241712	14·007856	18·564473	27·489853	52·882109	687·54887	5
56	11·278885	14·065459	18·665562	27·711740	53·708587	859·43630	4
57	11·316304	14·123536	18·767754	27·937233	54·561300	1145·9153	3
58	11·353970	14·182092	18·871068	28·166422	55·441517	1718·8732	2
59	11·391885	14·241134	18·975523	28·399397	56·350590	3437·7467	1
60	11·430052	14·300666	19·081137	28·636253	57·289962	Infinite.	0
′	5°	4°	3°	2°	1°	0°	′

CO-TANGENTS.

SECANTS.

′	0°	1°	2°	3°	4°	5°	6°	′
0	1.0000000	1·0001523	1·0006095	1·0013723	1·0024419	1·0038198	1·0055083	60
1	1·0000000	1·0001574	1·0006198	1·0013877	1·0024623	1·0038454	1·0055391	59
2	1·0000002	1·0001627	1·0006300	1·0014030	1·0024829	1·0038711	1·0055699	58
3	1·0000004	1·0001679	1·0006404	1·0014185	1·0025035	1·0038969	1·0056009	57
4	1·0000007	1·0001733	1·0006509	1·0014341	1·0025241	1·0039227	1·0056319	56
5	1·0000011	1·0001788	1·0006614	1·0014497	1·0025449	1·0039486	1·0056631	55
6	1·0000015	1·0001843	1·0006721	1·0014655	1·0025658	1·0039747	1·0056943	54
7	1·0000021	1·0001900	1·0006828	1·0014813	1·0025867	1·0040008	1·0057256	53
8	1·0000027	1·0001957	1·0006936	1·0014972	1·0026078	1·0040270	1·0057570	52
9	1·0000034	1·0002015	1·0007045	1·0015132	1·0026289	1·0040533	1·0057885	51
10	1·0000042	1·0002073	1·0007154	1·0015293	1·0026501	1·0040796	1·0058200	50
11	1·0000051	1·0002133	1·0007265	1·0015454	1·0026714	1·0041061	1·0058517	49
12	1·0000061	1·0002194	1·0007376	1·0015617	1·0026928	1·0041326	1·0058834	48
13	1·0000072	1·0002255	1·0007489	1·0015780	1·0027142	1·0041592	1·0059153	47
14	1·0000083	1·0002317	1·0007602	1·0015944	1·0027358	1·0041859	1·0059472	46
15	1·0000095	1·0002380	1·0007716	1·0016109	1·0027574	1·0042127	1·0059792	45
16	1·0000108	1·0002444	1·0007830	1·0016275	1·0027791	1·0042396	1·0060113	44
17	1·0000122	1·0002509	1·0007946	1·0016442	1·0028009	1·0042666	1·0060435	43
18	1·0000137	1·0002575	1·0008063	1·0016609	1·0028228	1·0042937	1·0060757	42
19	1·0000153	1·0002641	1·0008180	1·0016778	1·0028448	1·0043208	1·0061081	41
20	1·0000169	1·0002708	1·0008298	1·0016947	1·0028669	1·0043480	1·0061405	40
21	1·0000187	1·0002776	1·0008417	1·0017117	1·0028890	1·0043753	1·0061731	39
22	1·0000205	1·0002845	1·0008537	1·0017288	1·0029112	1·0044028	1·0062057	38
23	1·0000224	1·0002915	1·0008658	1·0017460	1·0029336	1·0044302	1·0062384	37
24	1·0000244	1·0002986	1·0008779	1·0017633	1·0029560	1·0044578	1·0062712	36
25	1·0000264	1·0003058	1·0008902	1·0017806	1·0029785	1·0044855	1·0063040	35
26	1·0000286	1·0003130	1·0009025	1·0017981	1·0030010	1·0045132	1·0063370	34
27	1·0000308	1·0003203	1·0009149	1·0018156	1·0030237	1·0045411	1·0063701	33
28	1·0000332	1·0003277	1·0009274	1·0018332	1·0030464	1·0045690	1·0064032	32
29	1·0000356	1·0003352	1·0009400	1·0018509	1·0030693	1·0045970	1·0064364	31
30	1·0000381	1·0003428	1·0009527	1·0018687	1·0030922	1·0046251	1·0064697	30
31	1·0000407	1·0003505	1·0009654	1·0018866	1·0031152	1·0046533	1·0065031	29
32	1·0000433	1·0003582	1·0009783	1·0019045	1·0031383	1·0046815	1·0065366	28
33	1·0000461	1·0003660	1·0009912	1·0019225	1·0031615	1·0047099	1·0065702	27
34	1 0000489	1·0003739	1·0010042	1·0019407	1·0031847	1·0047383	1·0066039	26
35	1·0000518	1·0003820	1·0010173	1·0019589	1·0032081	1·0047669	1·0066376	25
36	1·0000548	1·0003900	1·0010305	1·0019772	1·0032315	1·0047955	1·0066714	24
37	1·0000579	1·0003982	1·0010438	1·0019956	1·0032551	1·0048242	1·0067054	23
38	1·0000611	1·0004065	1·0010571	1·0020140	1·0032787	1·0048530	1·0067394	22
39	1·0000644	1·0004148	1·0010705	1·0020326	1·0033024	1·0048819	1·0067735	21
40	1·0000677	1·0004232	1·0010841	1·0020512	1·0033261	1·0049108	1·0068077	20
41	1·0000711	1·0004317	1·0010977	1·0020699	1·0033500	1·0049399	1·0068419	19
42	1·0000746	1·0004403	1·0011114	1·0020887	1 0033740	1·0049690	1·0068763	18
43	1·0000782	1·0004490	1·0011251	1·0021076	1·0033980	1·0049982	1·0069108	17
44	1·0000819	1·0004578	1·0011390	1·0021266	1·0034221	1·0050275	1·0069453	16
45	1·0000857	1·0004666	1·0011529	1·0021457	1·0034463	1·0050569	1·0069799	15
46	1·0000895	1·0004756	1·0011670	1·0021648	1·0034706	1·0050864	1·0070146	14
47	1·0000935	1·0004846	1·0011811	1·0021841	1·0034950	1·0051160	1·0070494	13
48	1·0000975	1·0004937	1·0011953	1·0022034	1·0035195	1·0051456	1·0070843	12
49	1·0001016	1·0005029	1·0012096	1·0022228	1·0035440	1·0051754	1·0071193	11
50	1 0001058	1·0005121	1·0012239	1·0022423	1·0035687	1·0052052	1·0071544	10
51	1·0001101	1·0005215	1·0012384	1·0022619	1·0035934	1·0052351	1·0071895	9
52	1·0001144	1·0005309	1·0012529	1·0022815	1·0036182	1·0052651	1·0072248	8
53	1·0001189	1·0005405	1·0012676	1·0023013	1·0036431	1·0052952	1·0072601	7
54	1·0001234	1·0005501	1·0012823	1·0023211	1·0036681	1·0053254	1·0072955	6
55	1·0001280	1·0005598	1·0012971	1·0023410	1·0036932	1·0053557	1·0073310	5
56	1·0001327	1·0005696	1·0013120	1·0023610	1·0037183	1·0053860	1·0073666	4
57	1·0001375	1·0005794	1·0013269	1·0023811	1·0037436	1·0054164	1·0074023	3
58	1·0001423	1·0005894	1·0013420	1·0024013	1·0037689	1·0054470	1·0074380	2
59	1·0001473	1·0005994	1·0013571	1·0024216	1·0037943	1·0054776	1·0074739	1
60	1·0001523	1·0006095	1·0013723	1·0024419	1·0038198	1·0055083	1·0075098	0
′	89°	88°	87°	86°	85°	84°	83°	′

CO-SECANTS.

SECANTS.

′	7°	8°	9°	10°	11°	12°	13°	′
0	1·0075098	1·0098276	1·0124651	1·0154266	1·0187167	1·0223406	1·0263041	60
1	1·0075459	1·0098689	1·0125118	1·0154787	1·0187743	1·0224039	1·0263731	59
2	1·0075820	1·0099103	1·0125586	1·0155310	1·0188321	1·0224672	1·0264421	58
3	1·0076182	1·0099518	1·0126055	1·0155833	1·0188899	1·0225307	1·0265113	57
4	1·0076545	1·0099934	1·0126524	1·0156357	1·0189478	1·0225942	1·0265806	56
5	1·0076908	1·0100351	1·0126995	1·0156882	1·0190059	1·0226578	1·0266499	55
6	1·0077273	1·0100769	1·0127466	1·0157408	1·0190640	1·0227216	1·0267194	54
7	1·0077639	1·0101187	1·0127939	1·0157934	1·0191222	1·0227854	1·0267889	53
8	1·0078005	1·0101607	1·0128412	1·0158462	1·0191805	1·0228493	1·0268586	52
9	1·0078372	1·0102027	1·0128886	1·0158991	1·0192389	1·0229133	1·0269283	51
10	1·0078741	1·0102449	1·0129361	1·0159520	1·0192973	1·0229774	1·0269982	50
11	1·0079110	1·0102871	1·0129837	1·0160050	1·0193559	1·0230416	1·0270681	49
12	1·0079480	1·0103294	1·0130314	1·0160582	1·0194146	1·0231059	1·0271381	48
13	1·0079851	1·0103718	1·0130791	1·0161114	1·0194734	1·0231703	1·0272082	47
14	1·0080222	1·0104143	1·0131270	1·0161647	1·0195322	1·0232348	1·0272785	46
15	1·0080595	1·0104568	1·0131750	1·0162181	1·0195912	1·0232994	1·0273488	45
16	1·0080968	1·0104995	1·0132230	1·0162716	1·0196502	1·0233641	1·0274192	44
17	1·0081343	1·0105422	1·0132711	1·0163252	1·0197093	1·0234288	1·0274897	43
18	1·0081718	1·0105851	1·0133194	1·0163789	1·0197686	1·0234937	1·0275603	42
19	1·0082094	1·0106280	1·0133677	1·0164327	1·0198279	1·0235587	1·0276310	41
20	1·0082471	1·0106710	1·0134161	1·0164865	1·0198873	1·0236237	1·0277018	40
21	1·0082849	1·0107141	1·0134646	1·0165405	1·0199468	1·0236889	1·0277727	39
22	1·0083228	1·0107573	1·0135132	1·0165946	1·0200064	1·0237541	1·0278437	38
23	1·0083607	1·0108006	1·0135618	1·0166487	1·0200661	1·0238195	1·0279148	37
24	1·0083988	1·0108440	1·0136106	1·0167029	1·0201259	1·0238849	1·0279860	36
25	1·0084369	1·0108875	1·0136595	1·0167573	1·0201858	1·0239504	1·0280573	35
26	1·0084752	1·0109310	1·0137084	1·0168117	1·0202457	1·0240161	1·0281287	34
27	1·0085135	1·0109747	1·0137574	1·0168662	1·0203058	1·0240818	1·0282002	33
28	1·0085519	1·0110184	1·0138066	1·0169208	1·0203660	1·0241476	1·0282717	32
29	1·0085904	1·0110622	1·0138558	1·0169755	1·0204262	1·0242135	1·0283434	31
30	1·0086290	1·0111061	1·0139051	1·0170303	1·0204866	1·0242795	1·0284152	30
31	1·0086676	1·0111501	1·0139545	1·0170851	1·0205470	1·0243456	1·0284871	29
32	1·0087064	1·0111942	1·0140040	1·0171401	1·0206075	1·0244118	1·0285590	28
33	1·0087452	1·0112384	1·0140536	1·0171952	1·0206682	1·0244781	1·0286311	27
34	1·0087842	1·0112827	1·0141032	1·0172503	1·0207289	1·0245445	1·0287033	26
35	1·0088232	1·0113270	1·0141530	1·0173056	1·0207897	1·0246110	1·0287755	25
36	1·0088623	1·0113715	1·0142029	1·0173609	1·0208506	1·0246776	1·0288479	24
37	1·0089015	1·0114160	1·0142528	1·0174163	1·0209116	1·0247442	1·0289203	23
38	1·0089408	1·0114606	1·0143028	1·0174719	1·0209727	1·0248110	1·0289929	22
39	1·0089802	1·0115054	1·0143530	1·0175275	1·0210339	1·0248779	1·0290655	21
40	1·0090196	1·0115502	1·0144032	1·0175832	1·0210952	1·0249448	1·0291383	20
41	1·0090592	1·0115951	1·0144535	1·0176390	1·0211566	1·0250119	1·0292111	19
42	1·0090988	1·0116400	1·0145039	1·0176949	1·0212180	1·0250790	1·0292840	18
43	1·0091386	1·0116851	1·0145544	1·0177509	1·0212796	1·0251463	1·0293571	17
44	1·0091784	1·0117303	1·0146050	1·0178069	1·0213413	1·0252136	1·0294302	16
45	1·0092183	1·0117755	1·0146556	1·0178631	1·0214030	1·0252811	1·0295034	15
46	1·0092583	1·0118209	1·0147064	1·0179194	1·0214649	1·0253486	1·0295768	14
47	1·0092984	1·0118663	1·0147572	1·0179757	1·0215268	1·0254162	1·0296502	13
48	1·0093386	1·0119118	1·0148082	1·0180321	1·0215888	1·0254839	1·0297237	12
49	1·0093788	1·0119575	1·0148592	1·0180887	1·0216510	1·0255518	1·0297973	11
50	1·0094192	1·0120032	1·0149103	1·0181453	1·0217132	1·0256197	1·0298711	10
51	1·0094596	1·0120489	1·0149616	1·0182020	1·0217755	1·0256877	1·0299449	9
52	1·0095001	1·0120948	1·0150129	1·0182588	1·0218379	1·0257558	1·0300188	8
53	1·0095408	1·0121408	1·0150643	1·0183158	1·0219004	1·0258240	1·0300928	7
54	1·0095815	1·0121869	1·0151158	1·0183728	1·0219630	1·0258923	1·0301669	6
55	1·0096223	1·0122330	1·0151673	1·0184298	1·0220257	1·0259607	1·0302411	5
56	1·0096631	1·0122793	1·0152190	1·0184870	1·0220885	1·0260292	1·0303154	4
57	1·0097041	1·0123256	1·0152708	1·0185443	1·0221514	1·0260978	1·0303898	3
58	1·0097452	1·0123720	1·0153226	1·0186017	1·0222144	1·0261665	1·0304643	2
59	1·0097863	1·0124185	1·0153746	1·0186591	1·0222774	1·0262352	1·0305389	1
60	1·0098276	1·0124651	1·0154266	1·0187167	1·0223406	1·0263041	1·0306136	0
′	82°	81°	80°	79°	78°	77°	76°	′

CO-SECANTS.

SECANTS.

′	14°	15°	16°	17°	18°	19°	20°	′
0	1·0306136	1·0352762	1·0402994	1·0456918	1·0514622	1·0576207	1·0641778	60
1	1·0306884	1·0353569	1·0403863	1·0457848	1·0515617	1·0577267	1·0642905	59
2	1·0307633	1·0354378	1·0404732	1·0458780	1·0516612	1·0578328	1·0644033	58
3	1·0308383	1·0355187	1·0405602	1·0459712	1·0517608	1·0579390	1·0645163	57
4	1·0309134	1·0355998	1·0406473	1·0460646	1·0518606	1·0580453	1·0646294	56
5	1·0309886	1·0356809	1·0407346	1·0461581	1·0519605	1·0581517	1·0647425	55
6	1·0310639	1·0357621	1·0408219	1·0462516	1·0520604	1·0582583	1·0648558	54
7	1·0311393	1·0358435	1·0409094	1·0463453	1·0521605	1·0583649	1·0649693	53
8	1·0312147	1·0359249	1·0409969	1·0464391	1·0522607	1·0584717	1·0650828	52
9	1·0312903	1·0360065	1·0410845	1·0465330	1·0523610	1·0585786	1·0651964	51
10	1·0313660	1·0360881	1·0411723	1·0466270	1·0524614	1·0586855	1·0653102	50
11	1·0314418	1·0361699	1·0412601	1·0467211	1·0525619	1·0587926	1·0654240	49
12	1·0315177	1·0362517	1·0413481	1·0468153	1·0526625	1·0588999	1·0655380	48
13	1·0315936	1·0363337	1·0414362	1·0469096	1·0527633	1·0590072	1·0656521	47
14	1·0316697	1·0364157	1·0415243	1·0470040	1·0528641	1·0591146	1·0657663	46
15	1·0317459	1·0364979	1·0416126	1·0470986	1·0529651	1·0592221	1·0658807	45
16	1·0318222	1·0365801	1·0417009	1·0471932	1·0530661	1·0593298	1·0659951	44
17	1·0318985	1·0366625	1·0417894	1·0472879	1·0531673	1·0594376	1·0661097	43
18	1·0319750	1·0367449	1·0418780	1·0473828	1·0532686	1·0595454	1·0662243	42
19	1·0320516	1·0368275	1·0419667	1·0474777	1·0533699	1·0596534	1·0663391	41
20	1·0321282	1·0369101	1·0420554	1·0475728	1·0534714	1·0597615	1·0664540	40
21	1·0322050	1·0369929	1·0421443	1·0476679	1·0535730	1·0598697	1·0665690	39
22	1·0322818	1·0370757	1·0422333	1·0477632	1·0536747	1·0599781	1·0666842	38
23	1·0323588	1·0371587	1·0423224	1·0478586	1·0537765	1·0600865	1·0667994	37
24	1·0324359	1·0372417	1·0424116	1·0479540	1·0538785	1·0601951	1·0669148	36
25	1·0325130	1·0373249	1·0425009	1·0480496	1·0539805	1·0603037	1·0670302	35
26	1·0325903	1·0374082	1·0425903	1·0481453	1·0540826	1·0604125	1·0671458	34
27	1·0326676	1·0374915	1·0426798	1·0482411	1·0541849	1·0605214	1·0672615	33
28	1·0327451	1·0375750	1·0427694	1·0483370	1·0542873	1·0606304	1·0673774	32
29	1·0328227	1·0376585	1·0428591	1·0484330	1·0543897	1·0607395	1·0674933	31
30	1·0329003	1·0377422	1·0429489	1·0485291	1·0544923	1·0608487	1·0676094	30
31	1·0329781	1·0378260	1·0430388	1·0486253	1·0545950	1·0609580	1·0677255	29
32	1·0330559	1·0379098	1·0431289	1·0487217	1·0546978	1·0610675	1·0678418	28
33	1·0331339	1·0379938	1·0432190	1·0488181	1·0548007	1·0611770	1·0679582	27
34	1·0332119	1·0380779	1·0433092	1·0489146	1·0549037	1·0612867	1·0680747	26
35	1·0332901	1·0381621	1·0433995	1·0490113	1·0550068	1·0613965	1·0681914	25
36	1·0333683	1·0382463	1·0434900	1·0491080	1·0551101	1·0615064	1·0683081	24
37	1·0334467	1·0383307	1·0435805	1·0492049	1·0552134	1·0616164	1·0684250	23
38	1·0335251	1·0384152	1·0436712	1·0493019	1·0553169	1·0617265	1·0685420	22
39	1·0336037	1·0384998	1·0437619	1·0493989	1·0554204	1·0618367	1·0686591	21
40	1·0336823	1·0385844	1·0438528	1·0494961	1·0555241	1·0619471	1·0687763	20
41	1·0337611	1·0386692	1·0439437	1·0495934	1·0556279	1·0620575	1·0688936	19
42	1·0338390	1·0387541	1·0440348	1·0496908	1·0557318	1·0621681	1·0690110	18
43	1·0339188	1·0388391	1·0441259	1·0497883	1·0558358	1·0622788	1·0691286	17
44	1·0339979	1·0389242	1·0442172	1·0498859	1·0559399	1·0623896	1·0692463	16
45	1·0340770	1·0390094	1·0443086	1·0499836	1·0560441	1·0625005	1·0693641	15
46	1·0341563	1·0390947	1·0444001	1·0500815	1·0561485	1·0626115	1·0694820	14
47	1·0342356	1·0391800	1·0444917	1·0501794	1·0562529	1·0627227	1·0696000	13
48	1·0343151	1·0392655	1·0445833	1·0502774	1·0563575	1·0628339	1·0697182	12
49	1·0343946	1·0393511	1·0446751	1·0503756	1·0564621	1·0629453	1·0698364	11
50	1·0344743	1·0394368	1·0447670	1·0504738	1·0565669	1·0630568	1·0699548	10
51	1·0345540	1·0395226	1·0448590	1·0505722	1·0566718	1·0631684	1·0700733	9
52	1·0346338	1·0396085	1·0449511	1·0506706	1·0567768	1·0632801	1·0701919	8
53	1·0347138	1·0396945	1·0450433	1·0507692	1·0568819	1·0633919	1·0703106	7
54	1·0347938	1·0397806	1·0451357	1·0508679	1·0569871	1·0635038	1·0704295	6
55	1·0348740	1·0398669	1·0452281	1·0509667	1·0570924	1·0636158	1·0705484	5
56	1·0349542	1·0399532	1·0453206	1·0510656	1·0571978	1·0637280	1·0706675	4
57	1·0350346	1·0400396	1·0454132	1·0511646	1·0573034	1·0638403	1·0707867	3
58	1·0351150	1·0401261	1·0455060	1·0512637	1·0574090	1·0639527	1·0709060	2
59	1·0351955	1·0402127	1·0455988	1·0513629	1·0575148	1·0640652	1·0710254	1
60	1·0352762	1·0402994	1·0456918	1 0514622	1·0576207	1·0641778	1·0711450	0
′	75°	74°	73°	72°	71°	70°	69°	′

CO-SECANTS.

SECANTS.

′	21°	22°	23°	24°	25°	26°	27°	′
0	1·0711450	1·0785347	1·0863604	1·0946363	1·1033779	1·1126019	1·1223262	60
1	1·0712647	1·0786616	1·0864946	1·0947781	1·1035277	1·1127599	1·1224927	59
2	1·0713844	1·0787885	1·0866289	1·0949201	1·1036775	1·1129179	1·1226592	58
3	1·0715043	1·0789156	1·0867634	1·0950622	1·1038275	1·1130761	1·1228259	57
4	1·0716244	1·0790427	1·0868979	1·0952044	1·1039777	1·1132345	1·1229928	56
5	1·0717445	1·0791700	1·0870326	1·0953467	1·1041279	1·1133929	1·1231598	55
6	1·0718647	1·0792975	1·0871675	1·0954892	1·1042783	1·1135516	1·1233269	54
7	1·0719851	1·0794250	1·0873024	1·0956318	1·1044289	1·1137103	1·1234942	63
8	1·0721056	1·0795527	1·0874375	1·0957746	1·1045795	1·1138692	1·1236616	52
9	1·0722262	1·0796805	1·0875727	1·0959174	1·1047303	1·1140282	1·1238292	51
10	1·0723469	1·0798084	1·0877080	1·0960604	1·1048813	1·1141874	1·1239969	50
11	1·0724678	1·0799364	1·0878435	1·0962036	1·1050324	1·1143467	1·1241648	49
12	1·0725887	1·0800646	1·0879791	1·0963468	1·1051836	1·1145062	1·1243328	48
13	1·0727098	1·0801928	1·0881148	1·0964902	1·1053349	1·1146658	1·1245010	47
14	1·0728310	1·0803212	1·0882506	1·0966337	1·1054864	1·1148255	1·1246693	46
15	1·0729523	1·0804497	1·0883866	1·0967774	1·1056380	1·1149854	1·1248377	45
16	1·0730737	1·0805784	1·0885226	1·0969212	1·1057898	1·1151454	1·1250063	44
17	1·0731953	1·0807071	1·0886589	1·0970651	1·1059417	1·1153056	1·1251750	43
18	1·0733170	1·0808360	1·0887952	1·0972091	1·1060937	1·1154659	1·1253439	42
19	1·0734388	1·0809650	1·0889317	1·0973533	1·1062458	1·1156263	1·1255130	41
20	1·0735607	1·0810942	1·0890682	1·0974976	1·1063981	1·1157869	1·1256821	40
21	1·0736827	1·0812234	1·0892050	1·0976420	1·1065506	1·1159476	1·1258514	39
22	1·0738048	1·0813528	1·0893418	1·0977866	1·1067031	1·1161084	1·1260209	38
23	1·0739271	1·0814823	1·0894788	1·0979313	1·1068558	1·1162694	1·1261905	37
24	1·0740495	1·0816119	1·0896159	1·0980761	1·1070087	1·1164306	1·1263603	36
25	1·0741720	1·0817417	1·0897531	1·0982211	1·1071616	1·1165919	1·1265302	35
26	1·0742946	1·0818715	1·0898904	1·0983662	1·1073147	1·1167533	1·1267003	34
27	1·0744173	1·0820015	1·0900279	1·0985114	1·1074680	1·1169148	1·1268705	33
28	1·0745402	1·0821316	1·0901655	1·0986568	1·1076214	1·1170766	1·1270408	32
29	1·0746631	1·0822618	1·0903032	1·0988023	1·1077749	1·1172384	1·1272113	31
30	1·0747862	1·0823922	1·0904411	1·0989479	1·1079285	1·1174004	1·1273819	30
31	1·0749095	1·0825227	1·0905791	1·0990936	1·1080823	1·1175625	1·1275527	29
32	1·0750328	1·0826533	1·0907172	1·0992395	1·1082363	1·1177248	1·1277237	28
33	1·0751562	1·0827840	1·0908554	1·0993855	1·1083903	1·1178872	1·1278948	27
34	1·0752798	1·0829149	1·0909938	1·0995317	1·1085445	1·1180498	1·1280660	26
35	1·0754035	1·0830458	1·0911323	1·0996779	1·1086989	1·1182124	1·1282374	25
36	1·0755273	1·0831769	1·0912709	1·0998243	1·1088533	1·1183753	1·1284089	24
37	1·0756512	1·0833081	1·0914097	1·0999709	1·1090079	1·1185383	1·1285806	23
38	1·0757753	1·0834395	1·0915485	1·1001175	1·1091627	1·1187014	1·1287524	22
39	1·0758995	1·0835709	1·0916876	1·1002644	1·1093176	1·1188647	1·1289244	21
40	1·0760237	1·0837025	1·0918267	1·1004113	1·1094726	1·1190281	1·1290965	20
41	1·0761481	1·0838342	1·0919659	1·1005584	1·1096277	1·1191916	1·1292687	19
42	1·0762727	1·0839661	1·0921053	1·1007056	1·1097830	1·1193553	1·1294412	18
43	1·0763973	1·0840980	1·0922448	1·1008529	1·1099385	1·1195191	1·1296137	17
44	1·0765221	1·0842301	1·0923845	1·1010004	1·1100940	1·1196831	1·1297864	16
45	1·0766470	1·0843623	1·0925243	1·1011480	1·1102498	1·1198472	1·1299593	15
46	1·0767720	1·0844947	1·0926642	1·1012957	1·1104056	1·1200115	1·1301323	14
47	1·0768971	1·0846271	1·0928042	1·1014436	1·1105616	1·1201759	1·1303055	13
48	1·0770224	1·0847597	1·0929444	1·1015916	1·1107177	1·1203405	1·1304788	12
49	1·0771477	1·0848924	1·0930846	1·1017397	1·1108740	1·1205051	1·1306522	11
50	1·0772732	1·0850252	1·0932251	1·1018879	1·1110304	1·1206700	1·1308258	10
51	1·0773988	1·0851582	1·0933656	1·1020363	1·1111869	1·1208350	1·1309996	9
52	1·0775246	1·0852913	1·0935063	1·1021849	1·1113436	1·1210001	1·1311735	8
53	1·0776504	1·0854245	1·0936471	1·1023335	1·1115004	1·1211653	1·1313475	7
54	1·0777764	1·0855578	1·0937880	1·1024823	1·1116573	1·1213308	1·1315217	6
56	1·0779025	1·0856912	1·0939291	1·1026313	1·1118144	1 1214963	1·1316961	5
56	1·0780287	1·0858248	1·0940702	1·1027803	1·1119716	1·1216620	1·1318706	4
57	1·0781550	1·0859585	1·0942116	1·1029295	1·1121290	1·1218278	1·1320452	3
58	1·0782815	1·0860924	1·0943530	1·1030789	1·1122865	1·1219938	1·1322200	2
59	1·0784080	1·0862263	1·0944946	1·1032283	1·1124442	1·1221600	1·1323950	1
60	1·0785347	1·0863604	1·0946363	1·1033779	1·1126019	1·1223262	1·1325701	0
′	68°	67°	66°	65°	64°	63°	62°	′

CO-SECANTS.

SECANTS.

′	28°	29°	30°	31°	32°	33°	34°	′
0	1·1325701	1·1433541	1·1547005	1.1666334	1·1791784	1·1923633	1·2062179	60
1	1·1327453	1·1435385	1·1548945	1·1668374	1·1793928	1·1925886	1·2064547	59
2	1·1329207	1·1437231	1·1550887	1·1670416	1·1796074	1·1928142	1·2066917	58
3	1·1330962	1·1439078	1·1552830	1·1672459	1·1798222	1·1930399	1·2069288	57
4	1·1332719	1·1440927	1·1554775	1·1674504	1·1800372	1·1932658	1·2071662	56
5	1·1334478	1·1442778	1·1556722	1·1676551	1·1802523	1·1934918	1·2074037	55
6	1·1336238	1·1444630	1·1558670	1·1678599	1·1804676	1·1937181	1·2076415	54
7	1·1337999	1·1446484	1·1560620	1·1680649	1·1806831	1·1939446	1·2078794	53
8	1·1339762	1·1448339	1·1562572	1·1682701	1·1808988	1·1941712	1·2081175	52
9	1·1341527	1·1450196	1·1564525	1·1684755	1·1811146	1·1943980	1·2083559	51
10	1·1343293	1·1452055	1·1566480	1·1686810	1·1813307	1·1946251	1·2085944	50
11	1·1345060	1·1453915	1·1568436	1·1688867	1·1815469	1·1948523	1·2088331	49
12	1·1346829	1·1455776	1·1570394	1·1690926	1·1817633	1·1950796	1·2090720	48
13	1·1348600	1·1457639	1·1572354	1·1692986	1.1819798	1·1953072	1·2093112	47
14	1·1350372	1·1459504	1·1574315	1·1695048	1·1821966	1·1955350	1·2095505	46
15	1·1352146	1·1461371	1·1576278	1·1697112	1·1824135	1·1957629	1·2097900	45
16	1·1353921	1·1463238	1·1578243	1·1699178	1·1826306	1·1959911	1·2100297	44
17	1·1355697	1·1465108	1·1580209	1·1701245	1·1828479	1·1962194	1·2102696	43
18	1·1357476	1·1466979	1·1582177	1·1703314	1·1830654	1·1964479	1·2105097	42
19	1·1359255	1·1468852	1·1584146	1·1705385	1·1832830	1·1966767	1·2107500	41
20	1·1361036	1·1470726	1·1586118	1·1707457	1·1835008	1·1969056	1·2109905	40
21	1·1362819	1·1472602	1·1588091	1·1709531	1·1837188	1·1971346	1·2112312	39
22	1·1364603	1·1474479	1·1590065	1·1711607	1·1839370	1·1973639	1·2114721	38
23	1·1366389	1·1476358	1·1592041	1·1713685	1·1841554	1·1975934	1·2117132	37
24	1·1368176	1·1478239	1·1594019	1·1715764	1·1843739	1·1978230	1·2119545	36
25	1·1369965	1·1480121	1·1595999	1·1717845	1·1845927	1·1980529	1·2121960	35
26	1·1371755	1·1482005	1·1597980	1·1719928	1·1848116	1·1982829	1·2124377	34
27	1·1373547	1·1483890	1·1599963	1·1722013	1·1850307	1·1985131	1·2126795	33
28	1·1375341	1·1485777	1·1601947	1·1724099	1·1852500	1·1987435	1·2129216	32
29	1·1377135	1·1487665	1·1603933	1·1726187	1·1854694	1·1989741	1·2131639	31
30	1·1378932	1·1489555	1·1605921	1·1728277	1·1856890	1·1992049	1·2134064	30
31	1·1380730	1·1491447	1·1607911	1·1730368	1·1859089	1·1994359	1·2136491	29
32	1·1382529	1·1493340	1·1609902	1·1732462	1·1861289	1·1996671	1·2138920	28
33	1·1384330	1·1495235	1·1611894	1·1734557	1·1863490	1·1998985	1·2141351	27
34	1·1386133	1·1497132	1·1613889	1·1736653	1·1865694	1·2001300	1·2143784	26
35	1·1387937	1·1499030	1·1615885	1·1738752	1·1867900	1·2003618	1·2146218	25
36	1·1389742	1·1500930	1·1617883	1·1740852	1·1870107	1·2005937	1·2148655	24
37	1·1391550	1·1502831	1·1619882	1·1742954	1·1872316	1·2008258	1·2151094	23
38	1·1393358	1·1504734	1·1621883	1·1745058	1·1874527	1·2010582	1·2153535	22
39	1·1395169	1·1506638	1·1623886	1·1747163	1·1876740	1·2012907	1·2155978	21
40	1·1396980	1·1508544	1·1625891	1·1749270	1·1878954	1·2015234	1·2158423	20
41	1·1398794	1·1510452	1·1627897	1·1751379	1·1881171	1·2017563	1·2160870	19
42	1·1400608	1·1512361	1·1629905	1·1753490	1·1883389	1·2019894	1·2163319	18
43	1·1402425	1·1514272	1·1631914	1·1755603	1·1885609	1·2022226	1·2165770	17
44	1·1404243	1·1516185	1·1633925	1·1757717	1·1887831	1·2024561	1·2168223	16
45	1·1406062	1·1518099	1·1635938	1·1759833	1·1890055	1·2026898	1·2170678	15
46	1·1407883	1·1520015	1·1637953	1·1761951	1·1892280	1·2029236	1·2173135	14
47	1·1409706	1·1521932	1·1639968	1·1764070	1·1894508	1·2031577	1·2175594	13
48	1·1411530	1·1523851	1·1641987	1·1766191	1·1896737	1·2033919	1·2178055	12
49	1·1413356	1·1525772	1·1644007	1·1768314	1·1898968	1·2036264	1·2180518	11
50	1·1415183	1·1527694	1·1646028	1·1770439	1·1901201	1·2038610	1·2182983	10
51	1·1417012	1·1529618	1·1648051	1·1772566	1·1903436	1·2040958	1·2185450	9
52	1·1418842	1·1531543	1·1650076	1·1774694	1·1905673	1·2043308	1·2187919	8
53	1·1420674	1·1533470	1·1652102	1·1776824	1·1907911	1·2045660	1·2190390	7
54	1·1422507	1·1535399	1·1654130	1·1778956	1·1910152	1·2048014	1·2192864	6
55	1·1424342	1·1537329	1·1656160	1·1781089	1·1912394	1·2050370	1·2195339	5
56	1·1426179	1·1539261	1·1658191	1·1783225	1·1914638	1·2052728	1·2197816	4
57	1·1428017	1·1541195	1·1660224	1·1785362	1·1916884	1·2055088	1·2200296	3
58	1·1429857	1·1543130	1·1662259	1·1787501	1·1919132	1·2057450	1·2202777	2
59	1·1431698	1·1545067	1·1664296	1·1789642	1·1921381	1·2059814	1·2205260	1
60	1·1433541	1·1547005	1·1666334	1·1791784	1·1923633	1·2062179	1·2207746	0
′	61°	60°	59°	58°	57°	56°	55°	′

CO-SECANTS.

SECANTS.

′	35°	36°	37°	38°	39°	40°	41°	′
0	1·2207746	1·2360680	1·2521357	1·2690182	1·2867596	1·3054073	1·3250130	60
1	1·2210233	1·2363293	1·2524102	1·2693067	1·2870628	1·3057261	1·3253482	59
2	1·2212723	1·2365909	1·2526850	1·2695955	1·2873663	1·3060451	1·3256837	58
3	1·2215215	1·2368526	1·2529601	1·2698845	1·2876700	1·3063644	1·3260194	57
4	1·2217708	1·2371146	1·2532353	1·2701737	1·2879740	1·3066839	1·3263554	56
5	1·2220204	1·2373768	1·2535108	1.2704632	1·2882782	1·3070038	1·3266918	55
6	1·2222702	1·2376393	1·2537865	1·2707529	1·2885827	1·3073239	1·3270284	54
7	1·2225202	1·2379019	1·2540625	1·2710429	1·2888875	1·3076442	1·3273653	53
8	1·2227703	1·2381647	1·2543387	1·2713331	1·2891925	1·3079649	1·3277024	52
9	1·2230207	1·2384278	1·2546151	1·2716235	1·2894977	1·3082858	1·3280399	51
10	1·2232713	1·2386911	1·2548917	1·2719142	1·2898032	1·3086069	1·3283776	50
11	1·2235222	1·2389546	1·2551685	1·2722052	1·2901090	1·3089284	1·3287156	49
12	1·2237732	1·2392183	1·2554456	1·2724963	1·2904150	1·3092501	1·3290539	48
13	1·2240244	1·2394823	1·2557229	1·2727877	1·2907213	1·3095720	1·3293925	47
14	1·2242758	1·2397464	1·2560005	1·2730794	1·2910278	1·3098943	1·3297314	46
15	1·2245274	1·2400108	1·2562782	1·2733712	1·2913346	1·3102168	1·3300706	45
16	1·2247793	1·2402754	1·2565562	1·2736634	1·2916416	1·3105396	1·3304100	44
17	1.2250313	1·2405402	1·2568345	1·2739557	1·2919489	1·3108626	1·3307497	43
18	1·2252836	1·2408052	1·2571129	1·2742484	1·2922564	1·3111860	1·3310897	42
19	1·2255361	1·2410704	1·2573916	1·2745412	1·2925642	1·3115095	1·3314301	41
20	1·2257887	1·2413359	1·2576705	1·2748343	1·2928723	1·3118334	1·3317707	40
21	1·2260416	1·2416016	1·2579497	1·2751276	1·2931806	1·3121575	1·3321115	39
22	1·2262947	1·2418675	1·2582291	1·2754212	1·2934892	1·3124820	1·3324527	38
23	1·2265480	1·2421336	1·2585087	1·2757151	1·2937980	1·3128066	1·3327942	37
24	1·2268015	1·2423999	1·2587885	1·2760091	1·2941071	1·3131316	1·3331359	36
25	1·2270552	1·2426665	1·2590686	1·2763034	1·2944164	1·3134568	1·3334779	35
26	1·2273091	1·2429333	1·2593489	1·2765980	1·2947260	1·3137823	1·3338203	34
27	1·2275633	1·2432003	1·2596294	1·2768928	1·2950359	1·3141081	1·3341629	33
28	1·2278176	1·2434675	1·2599102	1·2771878	1·2953460	1·3144341	1·3345058	32
29	1·2280722	1·2437349	1·2601912	1·2774831	1·2956564	1·3147604	1·3348489	31
30	1·2283269	1·2440026	1·2604724	1·2777787	1·2959670	1·3150870	1·3351924	30
31	1·2285819	1·2442704	1·2607539	1·2780744	1·2962779	1·3154139	1·3355362	29
32	1·2288371	1·2445385	1·2610356	1·2783705	1·2965890	1·3157410	1·3358802	28
33	1·2290924	1·2448069	1·2613175	1·2786667	1·2969004	1·3160684	1·3362246	27
34	1·2293480	1·2450754	1·2615997	1·2789632	1·2972121	1·3163961	1·3365692	26
35	1·2296039	1·2453442	1·2618820	1·2792600	1·2975240	1·3167240	1·3369141	25
36	1·2298599	1·2456131	1·2621647	1·2795570	1·2978362	1·3170523	1·3372594	24
37	1·2301161	1·2458823	1·2624475	1·2798543	1·2981487	1·3173808	1·3376049	23
38	1·2303725	1·2461518	1·2627306	1·2801518	1·2984614	1·3177096	1·3379507	22
39	1·2306292	1·2464214	1·2630140	1·2804495	1·2987743	1·3180386	1·3382968	21
40	1·2308861	1·2466913	1·2632975	1·2807475	1·2990876	1·3183680	1·3386432	20
41	1·2311432	1·2469614	1·2635813	1·2810457	1·2994011	1·3186976	1·3389898	19
42	1·2314004	1·2472317	1·2638653	1·2813442	1·2997148	1·3190274	1·3393368	18
43	1·2316579	1·2475022	1·2641496	1·2816430	1·3000288	1·3193576	1·3396841	17
44	1·2319156	1·2477730	1·2644341	1·2819419	1·3003431	1·3196881	1·3400316	16
45	1·2321736	1·2480440	1·2647188	1·2822412	1·3006576	1·3200188	1·3403795	15
46	1·2324317	1·2483152	1·2650038	1·2825407	1·3009724	1·3203498	1·3407276	14
47	1·2326900	1·2485866	1·2652890	1·2828404	1·3012875	1·3206810	1·3410761	13
48	1·2329486	1·2488583	1·2655745	1·2831404	1·3016028	1·3210126	1·3414248	12
49	1·2332074	1·2491302	1·2658601	1·2834406	1·3019184	1·3213444	1·3417738	11
50	1·2334664	1·2494023	1·2661460	1·2837411	1·3022343	1·3216765	1·3421232	10
51	1·2337256	1·2496746	1·2664322	1·2840418	1·3025504	1·3220089	1·3424728	9
52	1·2339850	1·2499471	1·2667186	1·2843428	1·3028667	1·3223416	1·3428227	8
53	1·2342446	1·2502199	1·2670052	1·2846440	1·3031834	1·3226745	1·3431729	7
54	1·2345044	1·2504929	1·2672921	1·2849455	1·3035003	1·3230078	1·3435234	6
55	1·2347645	1·2507661	1·2675792	1·2852472	1·3038175	1·3233413	1·3438742	5
56	1·2350248	1·2510396	1·2678665	1·2855492	1·3041349	1·3236750	1·3442253	4
57	1·2352852	1·2513133	1·2681541	1·2858514	1·3044526	1·3240091	1·3445767	3
58	1·2355459	1·2515872	1·2684419	1·2861539	1·3047706	1·3243435	1·3449284	2
59	1·2358069	1·2518613	1·2687299	1·2864566	1·3050888	1·3246781	1·3452804	1
60	1·2360680	1·2521357	1·2690182	1·2867596	1·3054073	1·3250130	1·3456327	0
′	54°	53°	52°	51°	50°	49°	48°	′

CO-SECANTS.

SECANTS.

′	42°	43°	44°	45°	46°	47°	48°	′
0	1·3456327	1·3673275	1·3901636	1·4142136	1·4395565	1·4662792	1·4944765	60
1	1·3459853	1·3676985	1·3905543	1·4146251	1·4399904	1·4667368	1·4949596	59
2	1·3463382	1·3680699	1·3909453	1·4150370	1·4404246	1·4671948	1·4954431	58
3	1·3466914	1·3684416	1·3913366	1·4154493	1·4408592	1·4676532	1·4959270	57
4	1·3470449	1·3688136	1·3917283	1·4158619	1·4412941	1·4681120	1·4964113	56
5	1·3473987	1·3691859	1·3921203	1·4162749	1·4417295	1·4685713	1·4968961	55
6	1·3477528	1·3695586	1·3925127	1·4166883	1·4421652	1·4690309	1·4973813	54
7	1·3481072	1·3699315	1·3929054	1·4171020	1·4426013	1·4694910	1·4978670	53
8	1·3484619	1·3703048	1·3932985	1·4175161	1·4430379	1·4699514	1·4983531	52
9	1·3488168	1·3706784	1·3936918	1·4179306	1·4434748	1·4704123	1·4988397	51
10	1·3491721	1·3710523	1·3940856	1·4183454	1·4439120	1·4708736	1·4993267	50
11	1·3495277	1·3714266	1·3944796	1·4187605	1·4443497	1·4713354	1·4998141	49
12	1·3498836	1·3718011	1·3948740	1·4191761	1·4447878	1·4717975	1·5003020	48
13	1·3502398	1·3721760	1·3952688	1·4195920	1·4452262	1·4722600	1·5007903	47
14	1·3505963	1·3725512	1·3956639	1·4200082	1·4456651	1·4727230	1·5012791	46
15	1·3509531	1·3729268	1·3960593	1·4204248	1·4461043	1·4731864	1·5017683	45
16	1·3513102	1·3733026	1·3964551	1·4208418	1·4465439	1·4736502	1·5022580	44
17	1·3516677	1·3736788	1·3968512	1·4212592	1·4469839	1·4741144	1·5027481	43
18	1·3520254	1·3740553	1·3972477	1·4216769	1·4474243	1·4745790	1·5032387	42
19	1·3523834	1·3744321	1·3976445	1·4220950	1·4478651	1·4750440	1·5037297	41
20	1·3527417	1·3748092	1·3980416	1·4225134	1·4483063	1·4755095	1·5042211	40
21	1·3531003	1·3751867	1·3984391	1·4229323	1·4487478	1·4759754	1·5047131	39
22	1·3534593	1·3755645	1·3988369	1·4233514	1·4491898	1·4764417	1·5052054	38
23	1·3538185	1·3759426	1·3992351	1·4237710	1·4496322	1·4769084	1·5056982	37
24	1·3541780	1·3763210	1·3996336	1·4241909	1·4500749	1·4773755	1·5061915	36
25	1·3545379	1·3766998	1·4000325	1·4246112	1·4505181	1·4778431	1·5066852	35
26	1·3548980	1·3770789	1·4004317	1·4250319	1·4509616	1·4783111	1·5071793	34
27	1·3552585	1·3774583	1·4008313	1·4254529	1·4514055	1·4787795	1·5076739	33
28	1·3556193	1·3778380	1·4012312	1·4258743	1·4518498	1·4792483	1·5081690	32
29	1·3559803	1·3782181	1·4016315	1·4262961	1·4522946	1·4797176	1·5086645	31
30	1·3563417	1·3785985	1·4020321	1·4267182	1·4527397	1·4801872	1·5091605	30
31	1·3567034	1·3789792	1·4024330	1·4271407	1·4531852	1·4806573	1·5096569	29
32	1·3570654	1·3793602	1·4028343	1·4275636	1·4536311	1·4811278	1·5101538	28
33	1·3574277	1·3797416	1·4032360	1·4279868	1·4540774	1·4815988	1·5106511	27
34	1·3577903	1·3801233	1·4036380	1·4284105	1·4545241	1·4820702	1·5111489	26
35	1·3581532	1·3805053	1·4040403	1·4288345	1·4549712	1·4825420	1·5116472	25
36	1·3585164	1.3808877	1·4044430	1·4292588	1·4554187	1·4830142	1·5121459	24
37	1·3588800	1·3812704	1·4048461	1·4296836	1·4558666	1·4834868	1·5126450	23
38	1·3592438	1·3816534	1·4052494	1·4301087	1·4563149	1·4839599	1·5131446	22
39	1·3596080	1·3820367	1·4056532	1·4305342	1·4567636	1·4844334	1·5136447	21
40	1·3599725	1·3824204	1·4060573	1·4309600	1·4572127	1.4849073	1·5141452	20
41	1·3603372	1·3828044	1·4064617	1·4313863	1·4576621	1·4853817	1·5146462	19
42	1·3607023	1·3831887	1·4068665	1·4318129	1·4581120	1·4858565	1·5151477	18
43	1·3610677	1·3835734	1·4072717	1·4322399	1·4585623	1·4863317	1·5156496	17
44	1·3614334	1·3839584	1·4076772	1·4326672	1·4590130	1·4868073	1·5161520	16
45	1·3617995	1·3843437	1·4080831	1·4330950	1·4594641	1·4872834	1·5166548	15
46	1·3621658	1·3847294	1·4084893	1·4335231	1·4599156	1·4877599	1·5171581	14
47	1·3625324	1·3851153	1·4088958	1·4339516	1·4603675	1·4882369	1·5176619	13
48	1·3628994	1·3855017	1·4093028	1·4343805	1·4608198	1·4887142	1·5181661	12
49	1·3632667	1·3858883	1·4097100	1·4348097	1·4612726	1·4891920	1·5186708	11
50	1·3636343	1·3862753	1·4101177	1·4352393	1·4617257	1·4896703	1·5191759	10
51	1·3640022	1·3866626	1·4105257	1·4356693	1·4621792	1·4901489	1·5196815	9
52	1·3643704	1·3870503	1·4109340	1·4360997	1·4626321	1·4906280	1·5201876	8
53	1·3647389	1·3874383	1·4113427	1·4365305	1·4630875	1·4911076	1·5206942	7
54	1·3651078	1·3878266	1·4117517	1·4369616	1·4635422	1·4915876	1·5212012	6
55	1·3654770	1·3882153	1·4121612	1·4373932	1·4639973	1·4920680	1·5217087	5
56	1·3658464	1·3886043	1·4125709	1·4378251	1·4644529	1·4925488	1·5222166	4
57	1·3662162	1·3889936	1·4129810	1·4382574	1·4649089	1·4930301	1·5227250	3
58	1·3665863	1·3893832	1·4133915	1·4386900	1·4653652	1·4935118	1·5232339	2
59	1·3669567	1·3897733	1·4138024	1·4391231	1·4658220	1·4939940	1·5237433	1
60	1·3673275	1·3901636	1·4142136	1·4395565	1·4662792	1·4944765	1·5242531	0
′	47°	46°	45°	44°	43°	42°	41°	′

CO-SECANTS.

SECANTS.

′	49°	50°	51°	52°	53°	54°	55°	′
0	1·5242531	1·5557238	1·5890157	1·6242692	1·6616401	1·7013016	1·7434468	60
1	1·5247634	1·5562634	1·5895868	1·6248743	1·6622819	1·7019831	1·7441715	59
2	1·5252741	1·5568035	1·5901584	1·6254799	1·6629243	1·7026653	1·7448969	58
3	1·5257854	1·5573441	1·5907306	1·6260861	1·6635673	1·7033482	1·7456230	57
4	1·5262971	1·5578852	1·5913033	1·6266929	1·6642110	1·7040318	1·7463499	56
5	1·5268093	1·5584268	1·5918766	1·6273003	1·6648553	1·7047160	1·7470776	55
6	1·5273219	1·5589689	1·5924504	1·6279083	1·6655002	1·7054010	1·7478060	54
7	1·5278351	1·5595115	1·5930247	1·6285169	1·6661458	1·7060867	1·7485352	53
8	1·5283487	1·5600546	1·5935996	1·6291261	1·6667920	1·7067730	1·7492651	52
9	1·5288627	1·5605982	1·5941751	1·6297359	1·6674389	1·7074601	1·7499958	51
10	1·5293773	1·5611424	1·5947511	1·6303462	1·6680864	1·7081478	1·7507273	50
11	1·5298923	1·5616871	1·5953276	1·6309572	1·6687345	1·7088362	1·7514595	49
12	1·5304078	1·5622322	1·5959048	1·6315688	1·6693833	1·7095254	1·7521924	48
13	1·5309238	1·5627779	1·5964824	1·6321809	1·6700328	1·7102152	1·7529262	47
14	1·5314403	1·5633241	1·5970606	1·6327937	1·6706828	1·7109058	1·7536607	46
15	1·5319572	1·5638708	1·5976394	1·6334070	1·6713336	1·7115970	1·7543959	45
16	1·5324746	1·5644181	1·5982187	1·6340210	1·6719850	1·7122890	1·7551320	44
17	1·5329925	1·5649658	1·5987986	1·6346355	1·6726370	1·7129817	1·7558687	43
18	1·5335109	1·5655141	1·5993790	1·6352507	1·6732897	1·7136750	1·7566063	42
19	1·5340297	1·5660628	1·5999600	1·6358664	1·6739430	1·7143691	1·7573446	41
20	1·5345491	1·5666121	1·6005416	1·6364828	1·6745970	1·7150639	1·7580837	40
21	1·5350689	1·5671619	1·6011237	1·6370997	1·6752517	1·7157594	1·7588236	39
22	1·5355892	1·5677123	1·6017064	1·6377173	1·6759070	1·7164556	1·7595642	38
23	1·5361100	1·5682631	1·6022896	1·6383355	1·6765629	1·7171525	1·7603057	37
24	1·5366313	1·5688145	1·6028734	1·6389542	1·6772195	1·7178501	1·7610478	36
25	1·5371530	1·5693664	1·6034577	1·6395736	1·6778768	1·7185484	1·7617908	35
26	1·5376752	1·5699188	1·6040426	1·6401936	1·6785347	1·7192475	1·7625345	34
27	1·5381980	1·5704717	1·6046281	1·6408142	1·6791933	1·7199472	1·7632791	33
28	1·5387212	1·5710252	1·6052142	1·6414354	1·6798525	1·7206477	1·7640244	32
29	1·5392449	1·5715792	1·6058008	1·6420572	1·6805124	1·7213489	1·7647704	31
30	1·5397690	1·5721337	1·6063879	1·6426796	1·6811730	1·7220508	1·7655173	30
31	1·5402937	1·5726887	1·6069757	1·6433027	1·6818342	1·7227534	1·7662649	29
32	1·5408189	1·5732443	1·6075640	1·6439263	1·6824961	1·7234568	1·7670133	28
33	1·5413445	1·5738004	1·6081528	1·6445506	1·6831586	1·7241609	1·7677625	27
34	1·5418706	1·5743570	1·6087423	1·6451754	1·6838219	1·7248657	1·7685125	26
35	1·5423973	1·5749141	1·6093323	1·6458009	1·6844857	1·7255712	1·7692633	25
36	1·5429244	1·5754718	1·6099228	1·6464270	1·6851503	1·7262774	1·7700149	24
37	1·5434520	1·5760300	1·6105140	1·6470537	1·6858155	1·7269844	1·7707672	23
38	1·5439801	1·5765887	1·6111057	1·6476811	1·6864814	1·7276921	1·7715204	22
39	1·5445087	1·5771479	1·6116980	1·6483090	1·6871479	1·7284005	1·7722743	21
40	1·5450378	1·5777077	1·6122908	1·6489376	1·6878151	1·7291096	1·7730290	20
41	1·5455673	1·5782680	1·6128843	1·6495668	1·6884830	1·7298195	1·7737845	19
42	1·5460974	1·5788289	1·6134783	1·6501966	1·6891516	1·7305301	1·7745409	18
43	1·5466280	1·5793902	1·6140728	1·6508270	1·6898202	1·7312414	1·7752980	17
44	1·5471590	1·5799521	1·6146680	1·6514581	1·6904907	1·7319535	1·7760559	16
45	1·5476906	1·5805146	1·6152637	1·6520898	1·6911613	1·7326663	1·7768146	15
46	1·5482226	1·5810776	1·6158600	1·6527221	1·6918326	1·7333798	1·7775741	14
47	1·5487552	1·5816411	1·6164569	1·6533550	1·6925045	1·7340941	1·7783344	13
48	1·5492882	1·5822051	1·6170544	1·6539885	1·6931771	1·7348091	1·7790955	12
49	1·5498218	1·5827697	1·6176524	1·6546227	1·6938504	1·7355248	1·7798574	11
50	1·5503558	1·5833348	1·6182510	1·6552575	1·6945244	1·7362413	1·7806201	10
51	1·5508904	1·5839005	1·6188502	1·6558929	1·6951990	1·7369585	1·7813836	9
52	1·5514254	1·5844667	1·6194500	1·6565290	1·6958744	1·7376764	1·7821479	8
53	1·5519610	1·5850334	1·6200504	1·6571657	1·6965504	1·7383951	1·7829131	7
54	1·5524970	1·5856007	1·6206513	1·6578030	1·6972271	1·7391145	1·7836790	6
55	1·5530335	1·5861685	1·6212528	1·6584409	1·6979044	1·7398347	1·7844457	5
56	1·5535706	1·5867369	1·6218549	1·6590795	1·6985825	1·7405556	1·7852133	4
57	1·5541081	1·5873058	1·6224576	1·6597187	1·6992612	1·7412773	1·7859817	3
58	1·5546462	1·5878752	1·6230609	1·6603586	1·6999407	1·7419997	1·7867508	2
59	1·5551848	1·5884452	1·6236648	1·6609990	1·7006208	1·7427229	1·7875208	1
60	1·5557238	1·5890157	1·6242692	1·6616401	1·7013016	1·7434468	1·7882916	0
′	40°	39°	38°	37°	36°	35°	34°	′

CO-SECANTS.

SECANTS.

′	56°	57°	58°	59°	60°	61°	62°	′
0	1·7882916	1·8360785	1·8870799	1·9416040	2·0000000	2·0626653	2·1300545	60
1	1·7890633	1·8369013	1·8879589	1·9425445	2·0010083	2·0637484	2·1312205	59
2	1·7898357	1·8377251	1·8888388	1·9434861	2·0020177	2·0648328	2·1323880	58
3	1·7906090	1·8385498	1·8897197	1·9444288	2·0030283	2·0659186	2·1335570	57
4	1·7913831	1·8393753	1·8906016	1·9453725	2·0040402	2·0670056	2·1347274	56
5	1·7921580	1·8402018	1·8914845	1·9463173	2·0050532	2·0680940	2·1358993	55
6	1·7929337	1·8410292	1·8923684	1·9472632	2·0060674	2·0691836	2·1370726	54
7	1·7937102	1·8418574	1·8932532	1·9482102	2·0070828	2·0702746	2·1382475	53
8	1·7944876	1·8426866	1·8941391	1·9491583	2·0080994	2·0713670	2·1394238	52
9	1·7952658	1·8435166	1·8950259	1·9501075	2·0091172	2·0724606	2·1406015	51
10	1·7960449	1·8443476	1·8959138	1·9510577	2·0101362	2·0735556	2·1417808	50
11	1·7968247	1·8451795	1·8968026	1·9520091	2·0111564	2·0746519	2·1429615	49
12	1·7976054	1·8460123	1·8976924	1·9529615	2·0121779	2·0757496	2·1441438	48
13	1·7983869	1·8468460	1·8985832	1·9539150	2·0132005	2·0768486	2·1453275	47
14	1·7991693	1·8476806	1·8994750	1·9548697	2·0142243	2·0779489	2·1465127	46
15	1·7999524	1·8485161	1·9003678	1·9558254	2·0152494	2·0790506	2·1476993	45
16	1·8007365	1·8493525	1·9012616	1·9567822	2·0162756	2·0801536	2·1488875	44
17	1·8015213	1·8501898	1·9021564	1·9577402	2·0173031	2·0812580	2·1500772	43
18	1·8023070	1·8510281	1·9030522	1·9586992	2·0183318	2·0823637	2·1512684	42
19	1·8030935	1·8518672	1·9039491	1·9596593	2·0193618	2·0834708	2·1524611	41
20	1·8038809	1·8527073	1·9048469	1·9606206	2·0203929	2·0845792	2·1536553	40
21	1·8046691	1·8535483	1·9057457	1·9615829	2·0214253	2·0856890	2·1548510	39
22	1·8054582	1·8543903	1·9066456	1·9625464	2·0224589	2·0868002	2·1560482	38
23	1·8062481	1·8552331	1·9075464	1·9635110	2·0234937	2·0879127	2·1572469	37
24	1·8070388	1·8560769	1·9084483	1·9644767	2·0245297	2·0890265	2·1584471	36
25	1·8078304	1·8569216	1·9093512	1·9654435	2·0255670	2·0901418	2·1596489	35
26	1·8086228	1·8577672	1·9102551	1·9664114	2·0266056	2·0912584	2·1608522	34
27	1·8094161	1·8586138	1·9111600	1·9673805	2·0276453	2·0923764	2·1620570	33
28	1·8102102	1·8594612	1·9120659	1·9683507	2·0286863	2·0934957	2·1632633	32
29	1·8110052	1·8603097	1·9129729	1·9693220	2·0297286	2·0946164	2·1644712	31
30	1·8118010	1·8611590	1·9138809	1·9702944	2·0307720	2·0957385	2·1656806	30
31	1·8125977	1·8620093	1·9147899	1·9712680	2.0318168	2·0968620	2·1668915	29
32	1·8133953	1·8628605	1·9156999	1·9722427	2·0328628	2·0979869	2·1681040	28
33	1·8141937	1·8637126	1·9166110	1·9732185	2·0339100	2·0991131	2·1693180	27
34	1·8149929	1·8645657	1·9175230	1·9741954	2·0349585	2·1002408	2·1705335	26
35	1·8157930	1·8654197	1·9184362	1·9751735	2·0360082	2·1013698	2·1717506	25
36	1·8165940	1·8662747	1·9193503	1·9761527	2·0370592	2·1025002	2·1729693	24
37	1·8173958	1·8671306	1·9202655	1·9771331	2·0381114	2·1026320	2·1741895	23
38	1·8181985	1·8679875	1·9211817	1·9781146	2·0391649	2·1047652	2·1754113	22
39	1·8190021	1·8688453	1·9220990	1·9790972	2·0402197	2·1058998	2·1766346	21
40	1·8198065	1·8699040	1·9230173	1·9800810	2·0412757	2·1070359	2·1778595	20
41	1·8206118	1·8705637	1·9239366	1·9810659	2·0423330	2·1081733	2·1790859	19
42	1·8214179	1·8714244	1·9248570	1·9820520	2·0433916	2·1093121	2·1803139	18
43	1·8222249	1·8722859	1·9257784	1·9830393	2·0444515	2·1104523	2·1815435	17
44	1·8230328	1·8731485	1·9267009	1·9840276	2·0455126	2·1115940	2·1827746	16
45	1·8238416	1·8740120	1·9276244	1·9850172	2·0465750	2·1127371	2·1840074	15
46	1·8246512	1·8748764	1·9285490	1·9860080	2·0476386	2·1138815	2·1852417	14
47	1·8254617	1·8757419	1·9294746	1·9869997	2·0487036	2·1150274	2·1864775	13
48	1·8262731	1·8766082	1·9304013	1·9879927	2·0497698	2·1161748	2·1877150	12
49	1·8270854	1·8774755	1·9313290	1·9889869	2·0508373	2·1173235	2·1889541	11
50	1·8278985	1·8783438	1·9322578	1·9899822	2·0519061	2·1184737	2·1901947	10
51	1·8287125	1·8792131	1·9331876	1·9909787	2·0529762	2·1196253	2·1914370	9
52	1·8295274	1·8800833	1·9341185	1·9919764	2·0540476	2·1207783	2·1926808	8
53	1·8303432	1·8809545	1·9350505	1·9929752	2·0551203	2·1219328	2·1939262	7
54	1·8311599	1·8818266	1·9359835	1·9939753	2·0561942	2·1230887	2·1951733	6
55	1·8319774	1·8826998	1·9369176	1·9949764	2·0572695	2·1242460	2·1964219	5
56	1·8327959	1·8835738	1·9378527	1·9959788	2·0583460	2·1254048	2·1976721	4
57	1·8336152	1·8844489	1·9387889	1·9969823	2·0594239	2·1265651	2·1989240	3
58	1·8344354	1·8853249	1·9397262	1·9979870	2·0605031	2·1277267	2·2001775	2
59	1·8352565	1·8862019	1·9406646	1·9989929	2·0625836	2·1288899	2·2014326	1
60	1·8360785	1·8870799	1·9416040	2·0000000	2·0626653	2·1300545	2·2026893	0
′	33°	32°	31°	30°	29°	28°	27°	′

CO-SECANTS.

SECANTS.

′	63°	64°	65°	66°	67°	68°	69°	′
0	2·2026893	2·2811720	2·3662016	2·4585933	2·5593047	2·6694672	2·7904281	60
1	2·2039476	2·2825335	2·3676787	2·4602008	2·5610599	2·6713906	2·7925444	59
2	2·2052075	2·2838967	2·3691578	2·4618106	2·5628176	2·6733171	2·7946641	58
3	2·2064691	2·2852618	2·3706390	2·4634227	2·5645781	2·6752465	2·7967873	57
4	2·2077323	2·2866286	2·3721222	2·4650371	2·5663412	2·6771790	2·7989140	56
5	2·2089972	2·2879974	2.3736075	2·4666538	2·5681069	2·6791145	2·8010441	55
6	2·2102637	2·2893679	2·3750949	2·4682729	2·5698752	2·6810530	2·8031777	54
7	2·2115318	2·2907403	2·3765843	2·4698943	2·5716462	2·6829945	2·8053148	53
8	2·2128016	2·2921145	2·3780758	2·4715181	2·5734199	2·6849391	2·8074554	52
9	2·2140730	2·2934906	2·3795694	2·4731442	2·5751963	2·6868867	2·8095995	51
10	2·2153460	2·2948685	2·3810650	2·4747726	2·5769753	2·6888374	2·8117471	50
11	2·2166208	2·2962483	2·3825627	2·4764034	2·5787570	2·6907912	2·8138982	49
12	2·2178971	2·2976299	2·3840625	2·4780366	2·5805414	2·6927480	2·8160529	48
13	2·2191752	2·2990134	2·3855645	2·4796721	2·5823284	2·6947079	2·8182111	47
14	2·2204548	2·3003988	2·3870685	2·4813100	2·5841182	2·6966709	2·8203729	46
15	2·2217362	2·3017860	2·3885746	2·4829503	2·5859107	2·6986370	2·8225382	45
16	2·2230192	2·3031751	2·3900828	2·4845929	2·5877058	2·7006061	2·8247071	44
17	2·2243039	2·3045660	2·3915931	2·4862380	2·5895037	2·7025784	2·8268796	43
18	2·2255903	2·3059588	2·3931055	2·4878854	2·5913043	2·7045538	2·8290556	42
19	2·2268783	2·3073536	2·3946201	2·4895352	2·5931077	2·7065323	2·8312353	41
20	2·2281681	2·3087501	2·3961367	2·4911874	2·5949137	2·7085139	2·8334185	40
21	2·2294595	2·3101486	2·3976555	2·4928421	2·5967225	2·7104987	2·8356054	39
22	2·2307526	2·3115490	2·3991764	2·4944991	2·5985341	2·7124866	2·8377958	38
23	2·2320474	2·3129513	2·4006995	2·4961586	2·6003484	2·7144777	2·8399899	37
24	2·2333438	2·3143554	2·4022247	2·4978204	2·6021654	2·7164719	2·8421877	36
25	2·2346420	2·3157615	2·4037520	2·4994848	2·6039852	2·7184693	2·8443891	35
26	2·2359419	2·3171695	2·4052815	2·5011515	2·6058078	2·7204698	2·8465941	34
27	2·2372435	2·3185794	2·4068132	2·5028207	2·6076332	2·7224735	2·8488028	33
28	2·2385468	2·3199912	2·4083469	2·5044923	2·6094613	2·7244804	2·8510152	32
29	2·2398517	2·3214049	2·4098829	2·5061663	2·6112922	2·7264905	2·8532312	31
30	2·2411585	2·3228205	2·4114210	2·5078428	2·6131259	2·7285038	2·8554510	30
31	2·2424669	2·3242381	2·4129613	2·5095218	2·6149624	2·7305203	2·8576744	29
32	2·2437770	2·3256575	2·4145038	2·5112032	2·6168018	2·7325400	2·8599015	28
33	2·2450889	2·3270790	2·4160484	2·5128871	2·6186439	2·7345630	2·8621324	27
34	2·2464025	2·3285023	2·4175952	2·5145735	2·6204888	2·7365892	2·8643670	26
35	2·2477178	2·3299276	2·4191442	2·5162624	2·6223366	2·7386186	2·8666053	25
36	2·2490348	2·3313548	2·4206954	2·5179537	2·6241872	2·7406512	2·8688474	24
37	2·2503536	2·3327840	2·4222488	2·5196475	2·6260406	2·7426871	2·8710932	23
38	2·2516741	2·3342152	2·4238044	2·5213438	2·6278969	2·7447263	2·8733428	22
39	2·2529964	2·3356482	2·4253622	2·5230426	2·6297560	2·7467687	2·8755961	21
40	2·2543204	2·3370833	2·4269222	2·5247440	2·6316180	2·7488144	2·8778532	20
41	2·2556461	2·3385203	2·4284844	2·5264478	2·6334828	2·7508634	2·8801142	19
42	2·2569736	2·3399593	2·4300489	2·5281541	2·6353506	2·7529157	2·8823789	18
43	2·2583029	2·3414002	2·4316155	2·5298630	2·6372211	2·7549712	2·8846474	17
44	2·2596339	2·3428432	2·4331844	2·5315744	2·6390946	2·7570301	2·8869198	16
45	2·2609607	2·3442881	2·4347555	2·5332883	2·6409710	2·7590923	2·8891960	15
46	2·2623012	2·3457349	2·4363289	2·5350048	2·6428502	2·7611578	2·8914760	14
47	2·2636376	2·3471838	2·4379045	2·5367238	2·6447323	2·7632267	2·8937598	13
48	2·2649756	2·3486347	2·4394823	2·5384453	2·6466174	2·7652988	2·8960475	12
49	2·2663155	2·3500875	2·4410624	2·5401694	2·6485054	2·7673744	2·8983391	11
50	2·2676571	2·3515424	2·4426448	2·5418961	2·6503962	2·7694532	2·9006346	10
51	2·2690005	2·3529992	2·4442294	2·5436253	2·6522901	2·7715355	2·9029339	9
52	2·2703457	2·3544581	2·4458163	2·5453571	2·6541868	2·7736211	2·9052372	8
53	2·2716927	2·3559189	2·4474054	2·5470915	2·6560865	2·7757100	2·9075443	7
54	2·2730415	2·3573818	2·4489968	2·5488284	2·6579891	2·7778024	2·9098553	6
55	2·2743921	2·3588467	2·4505905	2·5505680	2·6598947	2·7798982	2·9121703	5
56	2·2757445	2·3603136	2·4521865	2·5523101	2·6618033	2·7819973	2·9144892	4
57	2·2770987	2·3617826	2·4537848	2·5540548	2·6637148	2·7840999	2·9168121	3
58	2·2784546	2·3632535	2·4553853	2·5558022	2·6656292	2·7862059	2·9191389	2
59	2·2798124	2·3647265	2·4569882	2·5575521	2·6675467	2·7883153	2·9214697	1
60	2·2811720	2·3662016	2·4585933	2·5593047	2·6694672	2·7904281	2·9238044	0
′	26°	25°	24°	23°	22°	21°	20°	′

CO-SECANTS.

SECANTS.

′	70°	71°	72°	73°	74°	75°	76°	′
0	2·9238044	3·0715535	3·2360680	3·4203036	3·6279553	3·8637033	4·1335655	60
1	2·9261431	3·0741507	3·2389678	3·4235611	3·6316395	3·8679025	4·1383939	59
2	2·9284858	3·0767525	3·2418732	3·4268251	3·6353316	3·8721112	4·1432339	58
3	2·9308326	3·0793590	3·2447840	3·4300956	3·6390315	3·8763293	4·1480856	57
4	2·9331833	3·0819702	3·2477003	3·4333727	3·6427392	3·8805570	4·1529491	56
5	2·9355380	3·0845860	3·2506222	3·4366563	3·6464548	3·8847943	4·1578243	55
6	2·9378968	3·0872066	3·2535496	3·4399465	3·6501783	3·8890411	4·1627114	54
7	2·9402597	3·0898319	3·2564825	3·4432433	3·6539097	3·8932976	4·1676102	53
8	2·9426265	3·0924620	3·2594211	3·4465467	3·6576491	3·8975637	4·1725210	52
9	2·9449975	3·0950967	3·2623652	3·4498568	3·6613964	3·9018395	4·1774438	51
10	2·9473725	3·0977363	3·2653149	3·4531735	3·6651518	3·9061250	4·1823785	50
11	2·9497516	3·1003805	3·2682702	3·4564969	3·6689151	3·9104203	4·1873252	49
12	2·9521348	3·1030296	3·2712311	3·4598269	3·6726865	3·9147254	4·1922840	48
13	2·9545221	3·1056835	3·2741977	3·4631637	3·6764660	3·9190403	4·1972549	47
14	2·9569135	3·1083422	3·2771700	3·4665073	3·6802536	3·9233651	4·2022380	46
15	2·9593090	3·1110057	3·2801479	3·4698576	3·6840493	3·9276997	4·2072333	45
16	2·9617087	3·1136740	3·2831316	3·4732146	3·6878532	3·9320443	4·2122408	44
17	2·9641125	3·1163472	3·2861209	3·4765785	3·6916652	3·9363988	4·2172606	43
18	2·9665205	3·1190252	3·2891160	3·4799492	3·6954854	3·9407633	4·2222928	42
19	2·9689327	3·1217081	3·2921168	3·4833267	3·6993139	3·9451379	4·2273373	41
20	2·9713490	3·1243959	3·2951234	3·4867110	3·7031506	3·9495224	4·2323943	40
21	2·9737695	3·1270886	3·2981357	3·4901023	3·7069956	3·9539171	4·2374637	39
22	2·9761942	3·1297862	3·3011539	3·4935004	3·7108489	3·9583219	4·2425457	38
23	2·9786231	3·1324887	3·3041778	3·4969055	3·7147105	3·9627369	4·2476402	37
24	2·9810563	3·1351962	3·3072076	3·5003175	3·7185805	3·9671621	4·2527474	36
25	2·9834936	3·1379086	3·3102432	3·5037365	3·7224589	3·9715975	4·2578671	35
26	2·9859352	3·1406259	3·3132847	3·5071625	3·7263457	3·9760431	4·2629996	34
27	2·9883811	3·1433483	3·3163320	3·5105954	3·7302409	3·9804991	4·2681449	33
28	2·9908312	3·1460756	3·3193853	3·5140354	3·7341446	3·9849654	4·2733029	32
29	2·9932856	3·1488079	3·3224444	3·5174824	3·7380568	3·9894421	4·2784738	31
30	2·9957443	3·1515453	3·3255095	3·5209365	3·7419775	3·9939292	4·2836576	30
31	2·9982073	3·1542877	3·3285805	3·5243977	3·7459068	3·9984267	4·2888543	29
32	3·0006746	3·1570351	3·3316575	3·5278660	3·7498447	4·0029347	4·2940640	28
33	3·0031462	3·1597876	3·3347405	3·5313414	3·7537911	4·0074532	4·2992867	27
34	3·0056221	3·1625452	3·3378294	3·5348240	3·7577462	4·0119823	4·3045225	26
35	3·0081024	3·1653078	3·3409244	3·5383138	3·7617100	4·0165219	4·3097715	25
36	3·0105870	3·1680756	3·3440254	3·5418107	3·7656824	4·0210722	4·3150336	24
37	3·0130760	3·1708484	3·3471324	3·5453149	3·7696636	4·0256332	4·3203090	23
38	3·0155694	3·1736264	3·3502455	3·5488263	3·7736535	4·0302048	4·3255977	22
39	3·0180672	3·1764095	3·3533647	3·5523450	3·7776522	4·0347872	4·3308996	21
40	3·0205693	3·1791978	3·3564900	3·5558710	3·7816596	4·0393804	4·3362150	20
41	3·0230759	3·1819913	3·3596214	3·5594042	3·7856760	4·0439844	4·3415438	19
42	3·0255868	3·1847899	3·3627589	3·5629448	3·7897011	4·0485992	4·3468861	18
43	3·0281023	3·1875937	3·3659026	3·5664928	3·7937352	4·0532249	4·3522419	17
44	3·0306221	3·1904028	3·3690524	3·5700481	3·7977782	4·0578615	4·3576113	16
45	3·0331464	3·1932170	3·3722084	3·5736108	3·8018301	4·0625091	4·3629943	15
46	3·0356752	3·1960365	3·3753707	3·5771810	3·8058911	4·0671677	4·3683910	14
47	3·0382084	3·1988613	3·3785391	3·5807586	3·8099610	4·0718374	4·3738015	13
48	3·0407462	3·2016913	3·3817138	3·5843437	3·8140399	4·0765181	4·3792257	12
49	3·0432884	3·2045266	3·3848948	3·5879362	3·8181280	4·0812100	4·3846638	11
50	3·0458352	3·2073673	3·3880820	3·5915363	3·8222251	4·0859130	4·3901158	10
51	3·0483864	3·2102132	3·3912755	3·5951439	3·8263313	4·0906272	4·3955817	9
52	3·0509423	3·2130644	3·3944754	3·5987590	3·8304467	4·0953526	4·4010616	8
53	3·0535026	3·2159210	3·3976816	3·6023818	3·8345713	4·1000893	4·4065556	7
54	3·0560675	3·2187830	3·4008941	3·6060121	3·8387052	4·1048374	4·4120637	6
55	3·0586370	3·2216503	3·4041130	3·6096501	3·8428482	4·1095967	4·4175859	5
56	3·0612111	3·2245230	3·4073382	3·6133957	3·8470006	4·1143675	4·4231224	4
57	3·0637898	3·2274011	3·4105699	3·6169490	3·8511622	4·1191498	4·4286731	3
58	3·0663731	3·2302846	3·4138080	3·6206101	3·8553332	4·1239435	4·4342382	2
59	3·0689610	3·2331736	3·4170526	3·6242788	3·8595135	4·1287487	4·4398176	1
60	3·0715535	3·2360680	3·4203036	3·6279553	3·8637033	4·1335655	4·4454115	0
′	19°	18°	17°	16°	15°	14°	13°	′

CO-SECANTS.

SECANTS.

′	77°	78°	79°	80°	81°	82°	83°	′
0	4·4454115	4·8097343	5·2408431	5·7587705	6·3924532	7·1852965	8·2055090	60
1	4·4510198	4·8163258	5·2486979	5·7682867	6·4042154	7·2001996	8·2249952	59
2	4·4566428	4·8229357	5·2565768	5·7778350	6·4160216	7·2151653	8·2445748	58
3	4·4622803	4·8295643	5·2644798	5·7874153	6·4278719	7·2301940	8·2642485	57
4	4·4679324	4·8362114	5·2724070	5·7970280	6·4397666	7·2452859	8·2840171	56
5	4·4735993	4·8428774	5·2803587	5·8066732	6·4517059	7·2604417	8·3038812	55
6	4·4792810	4·8495621	5·2883347	5·8163510	6·4636901	7·2756616	8·3238415	54
7	4·4849775	4·8562657	5·2963354	5·8260617	6·4757195	7·2909460	8·3438986	53
8	4·4906889	4·8629883	5·3043608	5·8358053	6·4877944	7·3062954	8·3640534	52
9	4·4964152	4·8697299	5·3124109	5·8455820	6·4999148	7·3217102	8·3843065	51
10	4·5021565	4·8764907	5·3204860	5·8553921	6·5120812	7·3371909	8·4046586	50
11	4·5079129	4·8832707	5·3285861	5·8652356	6·5242938	7·3527377	8·4251105	49
12	4·5136844	4·8900700	5·3367114	5·8751128	6·5365528	7·3683512	8·4456629	48
13	4·5194711	4·8968886	5·3448620	5·8850238	6·5488586	7·3840318	8·4663165	47
14	4·5252730	4·9037267	5·3530379	5·8949688	6·5612113	7·3997798	8·4870721	46
15	4·5310903	4·9105844	5·3612393	5·9049479	6·5736112	7·4155959	8·5079304	45
16	4·5369229	4·9174616	5·3694664	5·9149614	6·5860587	7·4314803	8·5288923	44
17	4·5427709	4·9243586	5·3777192	5·9250095	6·5985540	7·4474335	8·5499584	43
18	4·5486344	4·9312754	5·3859979	5·9350922	6·6110973	7·4634560	8·5711295	42
19	4·5545134	4·9382120	5·3943026	5·9452098	6·6236890	7·4795482	8·5924065	41
20	4·5604080	4·9451687	5·4026333	5·9553625	6·6363293	7·4957106	8·6137901	40
21	4·5663183	4·9521453	5·4109903	5·9655504	6·6490184	7·5119437	8·6352812	39
22	4·5722444	4·9591421	5·4193737	5·9757737	6·6617568	7·5282478	8·6568805	38
23	4·5781862	4·9661591	5·4277835	5·9860326	6·6745446	7·5446236	8·6785889	37
24	4·5841439	4·9731964	5·4362199	5·9963274	6·6873822	7·5610713	8·7004071	36
25	4·5901174	4·9802541	5·4446831	6·0066581	6·7002699	7·5775916	8·7223361	35
26	4·5961070	4·9873323	5·4531731	6·0170250	6·7132079	7·5941849	8·7443766	34
27	4·6021126	4·9944311	5·4616901	6·0274282	6·7261965	7·6108516	8·7665295	33
28	4·6081343	5·0015505	5·4702342	6·0378680	6·7392360	7·6275923	8·7887957	32
29	4·6141722	5·0086907	5·4788056	6·0483445	6·7523268	7·6444075	8·8111761	31
30	4·6202263	5·0158517	5·4874043	6·0588580	6·7654691	7·6612976	8·8336715	30
31	4·6262967	5·0230337	5·4960305	6·0694085	6·7786632	7·6782631	8·8562828	29
32	4·6323835	5·0302367	5·5046843	6·0799964	6·7919095	7·6953047	8·8790109	28
33	4·6384867	5·0374607	5·5133659	6·0906219	6·8052082	7·7124227	8·9018567	27
34	4·6446064	5·0447060	5·5220754	6·1012850	6·8185597	7·7296176	8·9248211	26
35	4·6507427	5·0519726	5·5308129	6·1119861	6·8319642	7·7468901	8·9479051	25
36	4·6568956	5·0592606	5·5395786	6·1227253	6·8454222	7·7642406	8·9711095	24
37	4·6630652	5·0665701	5·5483726	6·1335028	6·8589338	7·7816697	8·9944354	23
38	4·6692516	5·0739012	5·5571951	6·1443189	6·8724995	7·7991778	9·0178837	22
39	4·6754548	5·0812539	5·5660460	6·1551736	6·8861195	7·8167656	9·0414553	21
40	4·6816748	5·0886284	5·5749258	6·1660674	6 8997942	7·8344335	9·0651512	20
41	4·6879119	5·0960248	5·5838343	6·1770003	6·9135239	7·8521821	9·0889725	19
42	4·6941660	5·1034431	5·5927719	6·1879725	6·9273089	7·8700120	9·1129200	18
43	4·7004372	5·1108835	5·6017386	6·1989843	6·9411496	7·8879238	9·1369949	17
44	4·7067256	5·1183461	5·6107345	6·2100359	6·9550464	7·9059179	9·1611980	16
45	4·7130313	5·1258309	5·6197599	6·2211275	6·9689994	7·9239950	9·1855305	15
46	4·7193542	5·1333381	5·6288148	6·2322594	6·9830092	7·9421556	9·2099934	14
47	4·7256945	5·1408677	5·6378995	6·2434316	6·9970760	7·9604003	9·2345877	13
48	4·7320524	5·1484199	5·6470140	6·2546446	7·0112001	7·9787298	9·2593145	12
49	4·7384277	5·1559948	5·6561584	6·2658984	7·0253820	7·9971445	9·2841749	11
50	4·7448206	5·1635924	5·6653331	6·2771933	7·0396220	8·0156450	9·3091699	10
51	4·7512312	5·1712128	5·6745380	6·2885295	7·0539205	8·0342321	9·3343006	9
52	4·7576596	5·1788563	5·6837734	6·2999073	7·0682777	8·0529062	9·3595682	8
53	4·7641058	5·1865228	5·6930393	6·3113269	7·0826941	8·0716681	9·3849738	7
54	4·7705699	5·1942125	5·7023360	6·3237884	7·0971700	8·0905182	9·4105184	6
55	4·7770519	5·2019254	5·7116636	6·3342923	7·1117059	8·1094573	9·4362033	5
56	4·7835520	5·2096618	5·7210223	6·3458386	7·1263019	8·1284860	9·4620296	4
57	4·7900702	5·2174216	5·7304121	6·3574276	7·1409587	8·1476048	9·4879984	3
58	4·7966066	5·2252050	5·7398333	6·3690595	7·1556764	8·1668145	9·5141110	2
59	4·8031613	5·2330121	5·7492861	6·3807347	7·1704556	8·1861157	9·5403686	1
60	4·8097343	5·2408431	5·7587705	6·3924532	7·1852965	8·2055090	9·5667722	0
′	12°	11°	10°	9°	8°	7°	6°	′

CO-SECANTS.

SECANTS.

′	84°	85°	86°	87°	88°	89°	′
0	9·5667722	11·473713	14·335587	19·107323	28·653708	57·298688	60
1	9·5933233	11·511990	14·395471	19·213970	28·894398	58·269755	59
2	9·6200229	11·550523	14·455859	19·321816	29·139169	59·274308	58
3	9·6468724	11·589316	14·516757	19·430882	29·388124	60·314110	57
4	9·6738730	11·628372	14·578172	19·541187	29·641373	61·391050	56
5	9·7010260	11·667693	14·640109	19·652754	29·899026	62·507153	55
6	9·7283327	11·707282	14·702576	19·765604	30·161201	63·664595	54
7	9·7557944	11·747141	14·765580	19·879758	30·428017	64·865716	53
8	9·7834124	11·787274	14·829128	19·995241	30·099598	66·113036	52
9	9·8111880	11·827683	14·893226	20·112075	30·976074	67·409272	51
10	9·8391227	11·868370	14·957882	20·230284	31·257577	68·757360	50
11	9·8672176	11·909340	15·023103	20·349893	31·544246	70·160474	49
12	9·8954744	11·950595	15·088896	20·470926	31·836225	71·622052	48
13	9·9238943	11·992137	15·155270	20·593409	32·133663	73·145827	47
14	9·9524787	12·033970	15·222231	20·717368	32·436713	74·735856	46
15	9·9812291	12·076098	15·289788	20·842830	32·745537	76·396554	45
16	10·010147	12·118522	15·357949	20·969824	33·060300	78·132742	44
17	10·039234	12·161246	15·426721	21·098376	33·381176	79·949684	43
18	10·068491	12·204274	15·496114	21·228515	33·708345	81·853150	42
19	10·097920	12·247608	15·566135	21·360272	34·041994	83·849470	41
20	10·127522	12·291252	15·636793	21·493676	34·382316	85·945609	40
21	10·157300	12·335210	15·708096	21·628759	34·729515	88·149244	39
22	10·187254	12·379484	15·780054	21·765553	35·083800	90·468863	38
23	10·217386	12·424078	15·852676	21·904090	35·445391	92·913869	37
24	10·247697	12·468995	15·925971	22·044403	35·814517	95·494711	36
25	10·278190	12·514240	15·999948	22·186528	36·191414	98·223033	35
26	10·308866	12·559815	16·074617	22·330499	36·576332	101·11185	34
27	10·339726	12·605724	16·149987	22·476353	36·969528	104·17574	33
28	10·370772	12·651971	16·226069	22·624126	37·371273	107·43114	32
29	10·402007	12·698560	16·302873	22·773857	37·781849	110·89656	31
30	10·433431	12·745495	16·380408	22·925586	38·201550	114·59301	30
31	10.465046	12·792779	16·458686	23·079351	38·630683	118·54440	29
32	10·496854	12·840416	16·537717	23·235196	39·069571	122·77803	28
33	10·528857	12·888410	16·617512	23·393161	39·518549	127·32526	27
34	10·561057	12·936765	16·698082	23·553291	39·977969	132·22229	26
35	10·593455	12·985486	16·779439	23·715630	40·448201	137·51108	25
36	10·626054	13·034576	16·861594	23·880224	40·929630	143·24061	24
37	10·658854	13·084040	16·944559	24·047121	41·422660	149·46837	23
38	10·691859	13·133882	17·028346	24·216370	41·927717	156·26228	22
39	10·725070	13·184106	17·112966	24·388020	42·445245	163·70325	21
40	10·758488	13·234717	17·198434	24·562123	42·975713	171·88831	20
41	10·792117	13·285719	17·284761	24·738731	43·519612	180·93496	19
42	10·825957	13·337116	17·371960	24·917900	44·077458	190·98680	18
43	10·860011	13·388914	17·460046	25·099685	44·649795	202·22122	17
44	10·894281	13·441118	17·549030	25·284144	45·237195	214·85995	16
45	10·928768	13·493731	17·638928	25·471337	45·840260	229·18385	15
46	10·963476	13·546758	17·729753	25·661324	46·459625	245·55402	14
47	10·998406	13·600205	17·821520	25·854169	47·095961	264·44269	13
48	11·033560	13·654077	17·914243	26·049937	47·749974	286·47948	12
49	11·068940	13·708379	18·007937	26·248694	48·422411	312·52297	11
50	11·104549	13·763115	18·102619	26·450510	49·114062	343·77516	10
51	11·140389	13·818291	18·198303	26·655455	49·825762	381·97230	9
52	11·176462	13·873913	18·295005	26·863603	50·558396	429·71873	8
53	11·212770	13·929985	18·392742	27·075030	51·312902	491·10702	7
54	11·249316	13·986514	18·491530	27·289814	52·090272	572·95809	6
55	11·286101	14·043504	18·591387	27·508035	52·891564	687·54960	5
56	11·323129	14·100963	18·692330	27·729777	53·717896	859.43689	4
57	11·360402	14·158894	18·794377	27·955125	54·570464	1145·9157	3
58	11·397922	14·217304	18·897545	28·184168	55·450534	1718·8735	2
59	11·435692	14·276200	19·001854	28·416997	56·359462	3437·7468	1
60	11·473715	14·335587	19·107323	28·653708	57·298688	Infinite.	0
′	5°	4°	3°	2°	1°	0°	′

CO-SECANTS.

INDEX.

ACRE of land, 2, 269
Acute angle, 103
Acute-angled triangle, 102, 103
Adjustment of the eidograph, 276
—— of the level, 74, 199
—— of the pantagraph, 273
—— for parallax, 76, 200
—— of the theodolite, 41
—— of the vertical axis, 41
Allowance for slope, adjustment of, 22
—— for collimation, 74
—— for curvature, 197, 219
Altazimuth theodolite, 98
Aneroid barometer, 79
—— levelling with, 226
Angle, acute, 103
—— of buildings, 16
—— chain, 25
—— complemental, 109, 110, 116
—— "defect" of, 117
—— of deflection of curve, 239, 241
—— of fences, 17
—— of intersection, 237, 241
—— obtuse, 103
—— plane, 102
—— plane rectilineal, 103
—— of reference, 113, 121
—— right, 103
—— of slope, 21
—— of streets, 187
Angles, when to take, 171, 192
Apex, inaccessible, 239
Arc of circle, 122
—— vertical, of theodolite, 50
Areas by different scales, 290
Arithmetical complement, 139
—— computation, 142
Arrows, 3
—— how to use them, 12
—— necessity for booking change of, 12

BACK sight, 200, 207
Ball and socket arrangement, 46
Barometer, aneroid, 79
Base lines, 23, 154
—— best form of, 166
Beam compass, 259
—— striking an arc with, 259
Bench-marks, 204, 206
Boning lines with laths, 164
Borders of plans, 270
Boundaries, how shown, 13
—— of different properties, 13, 268
Box sextant, 59
—— how to use it, 61
Box-wood scales, 254, 256
Bridle-path, how shown, 158
Broken ground, how shown, 160
Brushwood, how shown, 159
Buildings, how to measure, 16
—— description of, 194

CAP and spanner, 45
Cardiff, survey at, 166
Care in checking, 185
Cart-tracks, how shown, 158, 168
Centering-plates of theodolite, 43
Centre of circle, 103

Chain, how to hold it, 11
—— how to use it, 12
—— angles, 25
—— advantages of 100-feet, 2
—— décamètre, 3
—— divisions of Gunter's, 3
—— Gunter's 66-feet, 2
—— testing, 9
Chain and arrows, 2, 10
Chaining a hedge, 14
Chain-men, 10
—— should be instructed in their duties, 164
Chain-surveying, 157
Check-lines, 24
—— obviated, 171
—— check measurements in plotting, 254
Chord of an arc, 110
Circle, 103
—— centre of, 103
—— diameter of, 103
—— segment of, 103
—— semi, 101
Circumferentor, 40
Clamps, 48, 49
Clinometer, 31
—— rule, 32
—— scale, 35
Clips, 50
Close-paling, how shown, 158
Closing a traverse, 184
Collimation, adjustment of, 74, 75
—— line of, 74, 200
—— method of keeping level-book, 213
Colonial surveyors, licensing regulations, 291
Colour saucers, 265
Colouring, precautions in, 268
Colours, 266
Compass, prismatic, 38
—— of theodolite, 50
Compound levelling, 201
Computation scale, 287
—— various kinds of, 289
Contouring, 215
Contours, interpolation of, 236
Conventional signs, 158, 266
Copying a plan, 279
Corroboration of observations, 176, 208
Co-secant, definition of, 110
Cosine, definition of, 109
Cotangent, definition of, 109
County boundaries, how shown, 160
Coversed sine, definition of, 110
Cross-sections, 229
Cross-staff, 29
Curvature of earth, 197
—— allowance for, 197, 199, 219
Curve, length of, 241
—— impeded point in, 240, 243, 244
Curve-rangers, 92, 94
Curves, setting-out, 237
—— formulas for, 241
—— setting-out with theodolite, 239
—— with two theodolites, 245
—— with offsets, 248
—— with ordinates, 251
—— of different radii, 247
—— limit of radii, 238
—— setting out from same tangent, 250
—— of contraflexure, 248
—— for office use, 261

DATUM, 202
—— ordnance, 202
Décamètre chain, 3
Decimals converted into thirds, 242
Declination or variation, magnetic, 185
Diameter of circle, 103
Diaphragm of level, 73
—— theodolite, 51
Distances, inaccessible, 25, 152
—— in levelling, 217
Ditch and hedge, 13, 158
Dividers, 263
Double sextant, 62
Drawing instruments, 256–265
Drawing-pen, 261
—— how to hold, 261
Drawing-tables, 255

EARTH'S curvature, 197
—— allowance for, 197, 219

Earth's radius, 156
Eidograph, 273
—— to adjust, 276
Enlarging and reducing plan, 271
—— by squares, 278
Equilateral triangle, 104
Equipment of office, 255
—— of surveyor, 5
Eye-piece, 51, 73, 100

FENCES, how to measure, 13
—— how shown, 158
—— corners of, 17
——intersection of, 17
—— not to be cut or crossed unnecessarily, 168
Field-book, 6, 157, 160
—— ordnance, 157
—— best size of, 162
Figures not to be erased, 194
Flags, 4
Follower, duties of, 11
Footpaths and cart-tracks, 158, 168
Footpaths, how shown, 158, 168
Footplates, 209
Foot-set hedge, 14, 158
Foresight, 200, 208

GATES, how shown, 158
—— position of, 168
Gauge, test, 9
Glass, object, 73
Ground should be cleared after surveyor, 169
Gullies in towns, 194
Gunter's chain, 2
—— divisions of, 3

HEAD-stock of level and theodolite, 45
Heath or gorse, how shown, 159
Hedge and ditch, as boundary of property, 13
—— how to mark, 158, 168
—— foot-set, 14, 158
Hedges, crossing, 12
Heights and distances, 147
Hints on use of theodolite, 169
Hints and cautions, general, 5, 169
Horizontal equivalents, 231
—— table of, 233
Horn centre, 265
Hypotenusal allowance for sloping ground, 22
Hypotenuse, in relation to other sides of right-angled triangles, 110
Hypsometer, 226
—— levelling with, 228, 307

INACCESSIBLE apex, 239
—— distances, 25, 152
—— height, 149
India-rubber, 265
Indian ink, 265
—— to prevent running, 267
Instruments, spare parts, 100
—— novelties in, 100
Intermediate sights, 200, 208
Intersection of fences, 17
Isosceles triangle, 104

LAMP-posts in towns, 194
—— objections to, as stations, 179
Land quantities, 281
—— by triangles, 283
—— by computation scale, 287
Laths, bundle of, 5
Laying out a base for telemeter, 66
—— down survey lines on paper, 254
Leader, duties of, 10
Level "Abney," 33
—— adjusting, 74, 199
—— book, 206, 210, 214
—— collimation method, 213
—— dumpy, 71
—— reflecting, 33, 77
—— tube, to replace, 100
—— "Y," 69, 70
Level ground, what is, 20
Levelling, 197
—— different kinds of, 200
—— collimation method, 213
—— compound, 201
—— the plates, 49
—— "precise," 230
—— simple, 200
—— staff, 77, 215

Levelling with aneroid, 226
—— with hypsometer, 228
—— —— tables, Appendix
—— with theodolite, 222
Limb or lower plate, 46
Limits of offset, 18
—— radii, 238
Line, direction of, how shown, 160
Lines, base, 23, 154
—— of collimation, 74, 200
—— ranger, 31
—— ranging out, 19
Logarithmic sines etc., 141, *and* Appendix
Logarithms, 137
—— division by, 139
—— evolution by, 140
—— involution by, 140
—— multiplication by, 139
—— proportion by, 139
Lower plate or limb, 46

MAGNETIC variation, 185
Marshy ground, how shown, 159
Measure, standards of, 2
Measuring across stream, 27, 28, 69, 218
Minus signs in trigonometry, 118
Mirror, reflecting, 77
Multilateral figures, 102

NATURAL sines etc., 141, 311; tables of, 312
North points, 269
Northings and southings, 183
Novelties in instruments, 100

OBJECT-GLASS, 50, 73
Oblique-angled triangles, solution of, 144
Obtuse-angled triangle, 104
Office work, 253
Offsets, 14
—— limits of, 18
—— plotting of, 261
—— staff, 3, 19
—— objections to tapes for, 18
Omnimeter, 82
Omni-telemeter, 84

Optical square, 30
—— how to use it, 15, 30
Orchards, how shown, 159
Ordnance bench-mark, 205
—— field-book, 157

PACING, 18
Paint brushes, 268
Pantagraph, 272
Paper to be perfectly flat, 254
—— well seasoned, 253
Parallax, adjustment of, 76, 200
Parallel lines, 106
—— plates and their screws, 45, 46
—— rules, 260
Parish boundaries, how shown, 160
Pencil lines, no plotting from, 254
Pencils, 256
Perpendicular, 103
—— relation to hypotenuse and base, 111
Photographic surveying, 195
Plane angle, 102
—— rectilineal angle, 103
—— surface, 102
—— table, 63, 97
Planimeter, 290
Plans, enlarging or reducing, 271, 278
—— copying, 279
Plotting, 253, 254
—— long lines, 254
—— to be north and south, 254
—— survey lines first, 254
—— each day's work, 254
Plus and minus signs in trigonometry, 118
Poles, 4
Post-and-rail fence, how shown, 158
"Precise" levelling, 230
Preliminary operations in setting-out curve, 239
Pricker, 256, 260
Printing and writing on plan, 271
Prismatic compass, 38
—— and combined clinometer, 35
Proportional compasses, 264
—— how to use them, 265
Protractor, 256

QUADRANT of circle, 108, 111
—— improved, 32
Quadrilateral figures, 104
Quantities, land, 281, 283

RADIUS of circle, 103
—— of curve, 238, 241
—— of earth, 156
—— ratio of, 127
—— unity, 122
Railways, how shown, 160
Ranging rods, 4
Ray shade, 96
Reconnoitring, necessity of, 7, 157
Reflecting clinometer scale, 35
—— level, 33, 77
—— mirror, 77
Refraction, 198
River, how to measure breadth of, 27, 28, 69, 218
—— survey of, 176
Roads, how shown, 159

SAUCERS for colours, 266
Scale to be drawn on plan, 253
Scalene triangle, 104
Scales, 254, 256, 271
Screws, parallel plate, 46
Secant, 109
Sections, cross, 229
Segment of circle, 103
Set-square, 260
Setting-out curves, 237
—— by offsets, 248
—— by ordinates, 251
—— with two theodolites, 245
—— from same tangent, 250
Sextant, box, 59
—— how to use it, 61
—— double, 62
Signalling against wind, 6
Signs in use on plans, 158, 266
Sine A in terms of Cos A, 115
—— Tan A, 115
—— Sec A, 115
Sines, etc., of half an angle, 132
—— of twice an angle, 132
—— of three times an angle, 133
Sines, in terms of sides, 136
—— of semi-angles, 136
—— for 18 degrees, 125
—— for 30 degrees, 124, 125
—— for 45 degrees, 123
—— for 60 degrees, 124, 125
—— for 120 degrees, 126
—— for 225 degrees, 126
Sketch map, 7
Skew chaining, 14
Slope, observing angle of, 21
—— adjusting allowance for, 22
—— staff, 21
—— table of allowances for, 22
Solution of triangles. *See* Triangles
Southings and northings, 183
Spanner and cap, 45
Spare parts of instruments, 100
Spherical excess, 155
Spirit-level tube, to replace, 100
Spring bows, 264
Square, optical, 30
—— how to use it, 15, 30
Stadiometer, 81
Staff, cross, 29
—— holder, instructions to, 216, 221
—— levelling, 77, 215
—— offset, 3, 19
—— slope, 21
Standards of measure, 2
Station marks, 8
—— pegs, 8
Stations, fixed, 7
—— main, 7
—— subsidiary, 8
—— marking of, 160, 254
Stepping, 23
Stock head or tripod head, 42, 45, 97
Straight-edge, 254
Superficial land-measures, 281
Supplemental angles, 118
Surface, plane, 102
Survey lines to be numbered consecutively, 157
Survey, trigonometrical, 154
Surveying with chain only, 157
—— with theodolite, 171
—— photographic, 195

Surveying a river, 176
—— a town, 186
—— stations, how shown, 160
Surveyors' Institution, examination of, 173
System, necessity for, 253

TABLE, plane, 63, 97
Tables of natural sines etc., 311
Tacheometer, 83
Tan A in terms of sine A, 115
—— of cos A, 115
Tangent of an angle, 109
—— of curve, 241
—— points, 239
—— screw, 48, 49
Tangential angle, 241, 243
Tape, 4
—— not to be used for offsets, 18, 155
Telemeter, 65
—— how to use it, 66
Telescope of level, 72
—— theodolite, 50
Test gauge, 9
Testing chain, 9
Theodolite, adjustment of, 41
—— "Everest," 58
—— levelling with, 211
—— stand, 41
—— transit, 54, 91
—— use of, not to be spared, 222
—— "Y," 47
Theorems, geometrical, 104
Ticketing, 230
Town surveying, 186
Transit theodolite, 54, 91
Traversing, 180
—— closing, 184
Trees, how shown, 159
—— not to be cut down, 169
Triangle, acute, 102, 103
—— equilateral, 104
—— isosceles, 104
—— obtuse-angled, 104
—— oblique-angled, solution of, 144
—— right-angled, 104
—— —— solution of, 142
—— scalene, 104
Triangles, solution of, 142–147
Triangular plates of levels, 69
Trigonometrical canon, 108
—— ratios, 109
—— ratio of two angles, 128
—— survey, 154
Trigonometry as applied to surveying, 102
Tripod head or stock head, 42, 45, 97

UPPER or vernier plate, 47

VARIATION, magnetic, 185
Vernier, 52
—— or upper plate, 47
Versed sine, 110
Vertical arc of theodolite, 50
—— intervals and horizontal equivalents, 231

WALLS, how shown, 158
Water, taking level of, 222
Whites, 5
Wimbledon Park, survey of, 163
Woods, how shown, 159
Work in office, hints as to, 245
Writing on plans, 271

THE END.

PRINTED BY WILLIAM CLOWES AND SONS, LIMITED, LONDON AND BECCLES.

CATALOGUE OF

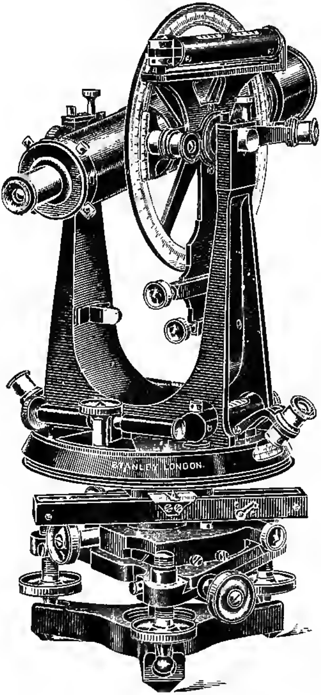
SURVEYING INSTRUMENTS

MADE BY

W. F. STANLEY & Co., Ltd.,

4 & 5, GREAT TURNSTILE, HOLBORN, LONDON.

THEODOLITES.


STANLEY LONDON.

	£	s.	d.
Stanley's New Model Telemetrical Theodolite, patent, Telescope supports and axis in one casting, tribrach mechanical stage, trough compass, with case and tripod, 4-in., £31; 5-in., £34; 6-in., £37; 8-in.	60	0	0
If without Compass, £1 less (except 8-in.).			
Bubble on Verniers (as illustrated) or on Telescope. If on *both*, extra	1	0	0

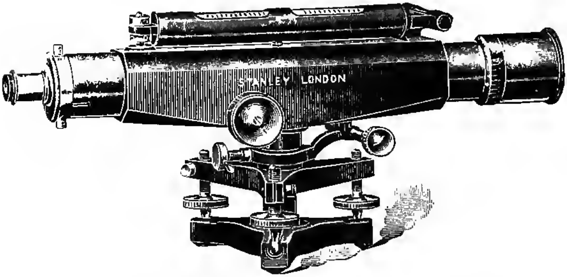
	£	s.	d.
Telescopic (instead of trough) compass to either of above, extra	1	0	0
New Model Transit, as foregoing, but without mechanical stage, with case and tripod, 4-in., £26 10*s.*; 5-in., £28; 6-in. ...	30	10	0
If without compass, £1 less.			
New Transit, with circular compass and old four-screw adjustment, but made of solid construction, 5-in., £27; 6-in.	29	10	0
Either of the foregoing Instruments, with illuminated axis and lamp, striding level, erecting and diagonal eye-pieces, extra	5	0	0
Framed stand (in place of solid) or sliding adjustable legs, extra	1	0	0
Transit Theodolite, 8-in., reading to 10″, etc., double clamping arc, framed stand, diagonal eye-piece, illuminated axis, lamp, striding level, etc.; suitable for astronomical and geodetic work ...	50	0	0
Lister's Inclinometer-Theodolite, Stanley model, perfect as a theodolite, and with supplemental axis to set out railway cuttings, 5-in.	30	0	0
Stanley's Railway Engineers' Theodolite (new model), telescope transiting eye end, divided on silver; platinum-iridium point stadia reading diaphragm; a very strong, light instrument; 4-in., £21; 5-in., £23; 6-in.	25	0	0
Trough compass extra, £1			
Stanley's Hedley's Miners' Dial, with movable sights, fitted also with telescope in Y's with limb circle to read outside as a theodolite, fitted with combined ball and socket and parallel plates. Subtense points for reading distances; 6-in. only, ...	20	10	0
Same as above, but with cranked gimbal, *patented*, enabling telescope or sights to read 90° both ways. Vernier reading circle with clamp and tangent and Hoffmann head; 6-in. only,	25	0	0

SEXTANTS, Etc.

	£	s.	d.
Nautical Sextant, 7-in., metal bronzed, arc 120°, divided on silver, two telescopes, and fittings complete, in mahogany case	5	0	0
Ditto, highly finished, arc 140°, three telescopes, extra fittings	8	0	0
Box Sextant, divided on silver, best make, with telescope, in leather case	4	4	0
Ditto, with supplementary arc, ditto, ditto	5	10	0
Optical Square, in case, 15*s.*; with adjustment, 22*s.*; double ...	2	2	0
Artificial Horizon, mercurial, superior make, iron bottle, etc.	4	0	0
Ditto, Captain George's, complete in sling leather case	5	5	0

STATION POINTERS and PLANE TABLES.

	£	s.	d.
Station Pointer, 12-in., divided to 30′, two verniers, in case	6	10	0
Ditto, 12-in., extending to 24-in. by lengthening arms, two clamp and tangent adjustments, divided on silver, in case	11	10	0
Plane Table, mahogany, on tripod stand, portable alidade, with trough compass and level, all packed in one case	7	0	0
Ditto, with telescope, as supplied to the Indian Government, with adjustment, £8 10*s.*; with vertical arc	10	0	0

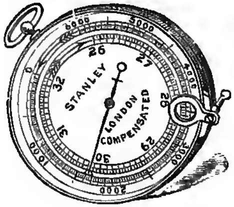


	£	*s.*	*d.*
Stanley's New Engineers' Level (Three Patents), centre cast in hard gun-metal with, and directly upon, the telescope body and object end. *The strongest and most compact level* yet made. Clamp and tangent adjustment. Tribrach stage. 14-in., £13 10*s.*; 12-in., £12; 10-in.	10	10	0
Stanley's New Model Level, very rigid, patent tribrach stage for stand or wall. Aluminium compass, prism reading 18-in.	18	10	0
Ditto, 16-in., £16 10*s.*; 14-in., £14 10*s.*; 12-in.	13	10	0
Fourteen-inch, ditto, without compass, £13 10*s.*; 12-in., ditto ...	12	0	0
With ball joint to set to any angle before screw adjustment, with compass, 14-in., £16 10*s.*; 12-in., £15 10*s.*; without compass, 14-in., £15 10*s.*; 12-in.	14	0	0
Any of the above with clamp and tangent adjustment, extra ...	1	0	0
Stanley's Patent Gradiometer, 14-in., specially designed for Drainage Work or Irrigation	20	0	0
Any of the above with framed stand, extra	1	0	0
Solid leather over-cases, with strap, for 18-in., 46*s.*; 16-in., 42*s.*; 14-in., 38*s.*; 12-in., 34*s.*; 10-in.	1	10	0
Capt. Abney's Pocket Level and Clinometer	1	18	0
Levelling Staves (Sopwith's), 14-ft., 45*s.*; 16-ft., 60*s*; 18-ft. ...	3	10	0

MAGNETIC COMPASSES, Etc.

Prismatic Compass, 2½-in., with shades and mirror, aluminium ring, in solid leather case, £3 5*s.*; 3-in., £3 15*s.*; 3½-in., £4; 4-in., £4 10*s.*; 4½-in., £5; 6-in.	6	10	0
Ditto, solid aluminium, 2½-in., £3 15*s.*; 3-in., £4 5*s.*; 3½-in., £4 10*s.*; 4-in., £5; 4½-in., £5 12*s.*; 6-in.	7	5	0
Stands fitted to the above, with ball and socket	1	4	0
,, ,, ,, Box Sextants	1	9	0

ANEROIDS, MOUNTAIN BAROMETERS, Etc., for taking Heights.



	£	s.	d.
Aneroid Barometer, 3-in. or 4½-in., with Stanley's improvements, constructed specially for altitude measurements. Of very refined workmanship, and has a new and exact system of reading. It may be relied upon for comparing depths and inclines of mines, altitudes of hills, etc., within a single yard with good observation, in case, £6; aluminium	6	10	0
Tourists' Pocket Aneroids, with altitudes ... ... 50*s.* to	4	0	0
Mountain Barometer, for altitudes, mercurial, complete, in leather case ... ... £7 and	10	10	0
Boiling-Point Apparatus, for taking heights, compact form ...	3	10	0

VARIOUS SURVEYING INSTRUMENTS.

Land Chains, light steel, 100 links, 13*s.*; 100 feet, 15*s.*; 20 metres, 13*s.*; 25 metres	0	16	0
Ditto, strong, 100 links, 16*s.*; 100 feet, 19*s.*; 20 metres	0	17	0
Tapes, Constantia, 100 links, 8*s.*; 50 feet, 7*s.*; 100 feet, 11*s.* 6*d.* 20 metres	0	18	0
Sounding Lines, with weight, 20 ft., 9*s.*; 30 ft., 14*s.*; 50 ft. ...	0	19	0
Sounding Chain, in case, 50 feet, £5; 100 feet	8	0	0
Cross Staff, French, 5*s.*; 7*s.* and 10*s.* Open with axis (best) ...	0	10	6
Offset Rods, 10-link, shod, and with hook, 5*s.* 6*d.*; jointed ...	0	7	6
Ranging Poles, 3 colours, per doz., 6ft., 30*s.*; 8ft., 36*s.*; 10 ft.	3	0	0
Passometer, to count steps for distance to 10,000, silver case ...	1	10	0
Field Books, large, in basil, with band, 1*s.* 8*d.* each; per doz.	0	19	0
Levelling Books, various patterns, 2*s.* each; per doz.	1	3	0
Tropical Umbrellas, portable, 12*s.*; with long stick	1	1	0
Ditto, 54-in. diam., holland	1	15	0
Sketching Cases (Sandhurst), 10*s.* 6*d.* Pea-Whistles, 1*s.* and	0	2	6

Cases of Drawing Instruments from £1 to £31 10*s.*

Eidographs and Pentagraphs, Protractors, Parallel Rules.

Drawing Scales of every description. Computing Scales & Planimeters.

Every article for the Surveyor or Explorer in stock.
Special Instruments made to order.

Complete Catalogue of Instruments Post Free.

W. F. STANLEY & CO., LTD.,
Instrument Makers to the Various Departments of His Majesty's Government,
4 & 5, GREAT TURNSTILE, HOLBORN, LONDON, W.C.

STATIONERS' HALL COURT, LONDON, E.C.

CROSBY LOCKWOOD & SON'S
Catalogue of
Scientific, Technical and Industrial Books.

	PAGE
MECHANICAL ENGINEERING .	1
CIVIL ENGINEERING	10
MARINE ENGINEERING, &c. .	17
MINING & METALLURGY . .	19
COLLIERY WORKING, &c. . .	21
ELECTRICITY	23
ARCHITECTURE & BUILDING .	28
SANITATION & WATER SUPPLY	28
CARPENTRY & TIMBER . .	29
DECORATIVE ARTS	31
NATURAL SCIENCE	33
CHEMICAL MANUFACTURES	34
INDUSTRIAL ARTS	38
COMMERCE, TABLES, &c. .	41
AGRICULTURE & GARDENING ·	43
AUCTIONEERING, VALUING, &c.	46
LAW & MISCELLANEOUS	47

MECHANICAL ENGINEERING, ETC.

THE MECHANICAL ENGINEER'S POCKET-BOOK.

Comprising Tables, Formulæ, Rules, and Data: A Handy Book of Reference for Daily Use in Engineering Practice. By D. KINNEAR CLARK, M. Inst. C.E., Fifth Edition, thoroughly Revised and Enlarged. By H. H. P. POWLES, A.M.I C.E., M.I.M.E. Small 8vo, 700 pp., bound in flexible Leather Cover, rounded corners. [*Just Published.* *Net* **6/0**

SUMMARY OF CONTENTS:—MATHEMATICAL TABLES.—MEASUREMENT OF SURFACES AND SOLIDS.—ENGLISH WEIGHTS AND MEASURES.—FRENCH METRIC WEIGHTS AND MEASURES.—FOREIGN WEIGHTS AND MEASURES.—MONEYS.—SPECIFIC GRAVITY, WEIGHT, AND VOLUME.—MANUFACTURED METALS.—STEEL PIPES.—BOLTS AND NUTS.—SUNDRY ARTICLES IN WROUGHT AND CAST IRON, COPPER, BRASS, LEAD, TIN, ZINC.—STRENGTH OF MATERIALS.—STRENGTH OF TIMBER.—STRENGTH OF CAST IRON.—STRENGTH OF WROUGHT IRON.—STRENGTH OF STEEL.—TENSILE STRENGTH OF COPPER, LEAD, &c.—RESISTANCE OF STONES AND OTHER BUILDING MATERIALS.—RIVETED JOINTS IN BOILER PLATES.—BOILER SHELLS.—WIRE ROPES AND HEMP ROPES.—CHAINS AND CHAIN CABLES.—FRAMING.—HARDNESS OF METALS, ALLOYS, AND STONES.—LABOUR OF ANIMALS.—MECHANICAL PRINCIPLES.—GRAVITY AND FALL OF BODIES.—ACCELERATING AND RETARDING FORCES.—MILL GEARING, SHAFTING, &c.—TRANSMISSION OF MOTIVE POWER.—HEAT.—COMBUSTION: FUELS.—WARMING, VENTILATION, COOKING STOVES.—STEAM.—STEAM ENGINES AND BOILERS.—RAILWAYS.—TRAMWAYS.—STEAM SHIPS.—PUMPING STEAM ENGINES AND PUMPS.—COAL GAS, GAS ENGINES, &c.—AIR IN MOTION.—COMPRESSED AIR.—HOT AIR ENGINES.—WATER POWER.—SPEED OF CUTTING TOOLS.—COLOURS.—ELECTRICAL ENGINEERING.

"Mr. Clark manifests what is an innate perception of what is likely to be useful in a pocket-book, and he is really unrivalled in the art of condensation. It is very difficult to hit upon any mechanical engineering subject concerning which this work supplies no information, and the excellent index at the end adds to its utility. In one word, it is an exceedingly handy and efficient tool, possessed of which the engineer will be saved many a wearisome calculation, or yet more wearisome hunt through various text-books and treatises, and, as such, we can heartily recommend it to our readers."—*The Engineer*,

"It would be found difficult to compress more matter within a similar compass, or produce a book of 700 pages which should be more compact or convenient for pocket reference. . . Will be appreciated by mechanical engineers of all classes."—*Practical Engineer*.

MR. HUTTON'S PRACTICAL HANDBOOKS.

THE WORKS' MANAGER'S HANDBOOK.

Comprising Modern Rules, Tables, and Data. For Engineers, Millwrights, and Boiler Makers; Tool Makers, Machinists, and Metal Workers; Iron and Brass Founders, &c. By W. S. HUTTON, Civil and Mechanical Engineer, Author of "The Practical Engineer's Handbook." Sixth Edition, carefully Revised, and Enlarged. In One handsome Volume, medium 8vo, strongly bound **15/0**

☞ *The Author having compiled Rules and Data for his own use in a great variety of modern engineering work, and having found his notes extremely useful, decided to publish them—revised to date—believing that a practical work, suited to the* DAILY REQUIREMENTS OF MODERN ENGINEERS, *would be favourably received.*

"Of this edition we may repeat the appreciative remarks we made upon the first and third. Since the appearance of the latter very considerable modifications have been made, although the total number of pages remains almost the same. It is a very useful collection of rules, tables, and workshop and drawing office data."—*The Engineer*, May 10, 1895.

"The author treats every subject from the point of view of one who has collected workshop notes for application in workshop practice, rather than from the theoretical or literary aspect. The volume contains a great deal of that kind of information which is gained only by practical experience, and is seldom written in books."—*The Engineer*, June 5, 1885.

"The volume is an exceedingly useful one, brimful with engineer's notes, memoranda, and rules, and well worthy of being on every mechanical engineer's bookshelf."—*Mechanical World.*

"The information is precisely that likely to be required in practice. . . . The work forms a desirable addition to the library not only of the works' manager, but of any one connected with general engineering."—*Mining Journal.*

"Brimful of useful information, stated in a concise form, Mr. Hutton's books have met a pressing want among engineers. The book must prove extremely useful to every practical man possessing a copy."—*Practical Engineer.*

THE PRACTICAL ENGINEER'S HANDBOOK.

Comprising a Treatise on Modern Engines and Boilers, Marine, Locomotive, and Stationary. And containing a large collection of Rules and Practical Data relating to Recent Practice in Designing and Constructing all kinds of Engines, Boilers, and other Engineering work. The whole constituting a comprehensive Key to the Board of Trade and other Examinations for Certificates of Competency in Modern Mechanical Engineering. By WALTER S. HUTTON, Civil and Mechanical Engineer, Author of "The Works' Manager's Handbook for Engineers," &c. With upwards of 420 Illustrations. Sixth Edition, Revised and Enlarged. Medium 8vo, nearly 560 pp., strongly bound. **18/0**

☞ *This Work is designed as a companion to the Author's* "WORKS' MANAGER'S HANDBOOK." *It possesses many new and original features, and contains, like its predecessor, a quantity of matter not originally intended for publication but collected by the Author for his own use in the construction of a great variety of* MODERN ENGINEERING WORK.

The information is given in a condensed and concise form, and is illustrated by upwards of 420 Engravings; and comprises a quantity of tabulated matter of great value to all engaged in designing, constructing, or estimating for ENGINES, BOILERS, *and* OTNER ENGINEERING WORK.

"We have kept it at hand for several weeks, referring to it as occasion arose, and we have not on a single occasion consulted its pages without finding the information of which we were in quest." —*Athenæum.*

"A thoroughly good practical handbook, which no engineer can go through without learning something that will be of service to him."—*Marine Engineer.*

"An excellent book of reference for engineers, and a valuable text-book for students of engineering."—*Scotsman.*

"This valuable manual embodies the results and experience of the leading authorities on mechanical engineering."—*Building News.*

"The author has collected together a surprising quantity of rules and practical data, and has shown much judgment in the selections he has made. . . . There is no doubt that this book is one of the most useful of its kind published, and will be a very popular compendium."—*Engineer.*

"A mass of information set down in simple language, and in such a form that it can be easily referred to at any time. The matter is uniformly good and well chosen, and is greatly elucidated by the illustrations. The book will find its way on to most engineers' shelves, where it will rank as one of the most useful books of reference."—*Practical Engineer.*

"Full of useful information, and should be found on the office shelf of all practical engineers —*English Mechanic.*

MR. HUTTON'S PRACTICAL HANDBOOKS—*continued.*

STEAM BOILER CONSTRUCTION.

A Practical Handbook for Engineers, Boiler-Makers, and Steam Users. Containing a large Collection of Rules and Data relating to Recent Practice in the Design, Construction, and Working of all Kinds of Stationary, Locomotive, and Marine Steam-Boilers. By WALTER S. HUTTON, Civil and Mechanical Engineer, Author of "The Works' Manager's Handbook," "The Practical Engineer's Handbook," &c. With upwards of 500 Illustrations. Fourth Edition, carefully Revised, and Enlarged. Medium 8vo, over 680 pages, cloth, strongly bound. [*Just Published.* **18/0**

☞ THIS WORK *is issued in continuation of the Series of Handbooks written by the Author, viz.:—*"THE WORKS' MANAGER'S HANDBOOK" *and* "THE PRACTICAL ENGINEER'S HANDBOOK," *which are so highly appreciated by engineers for the practical nature of their information; and is consequently written in the same style as those works.*

The Author believes that the concentration, in a convenient form for easy reference, of such a large amount of thoroughly practical information on Steam-Boilers, will be of considerable service to those for whom it is intended, and he trusts the book may be deemed worthy of as favourable a reception as has been accorded to its predecessors.

"One of the best, if not the best, books on boilers that has ever been published. The information is of the right kind, in a simple and accessible form. So far as generation is concerned, this is, undoubtedly, the standard book on steam practice."—*Electrical Review.*

"Every detail, both in boiler design and management, is clearly laid before the reader. The volume shows that boiler construction has been reduced to the condition of one of the most exact sciences; and such a book is of the utmost value to the *fin de siècle* Engineer and Works Manager." —*Marine Engineer.*

"There has long been room for a modern handbook on steam boilers; there is not that room now, because Mr. Hutton has filled it. It is a thoroughly practical book for those who are occupied in the construction, design, selection, or use of boilers."—*Engineer.*

"The book is of so important and comprehensive a character that it must find its way into the libraries of every one interested in boiler using or boiler manufacture if they wish to be thoroughly informed. We strongly recommend the book for the intrinsic value of its contents."—*Machinery Market.*

PRACTICAL MECHANICS' WORKSHOP COMPANION.

Comprising a great variety of the most useful Rules and Formulæ in Mechanical Science, with numerous Tables of Practical Data and Calculated Results for Facilitating Mechanical Operations. By WILLIAM TEMPLETON, Author of "The Engineer's Practical Assistant," &c., &c. Eighteenth Edition, Revised, Modernised, and considerably Enlarged by WALTER S. HUTTON, C.E., Author of "The Works' Manager's Handbook," "The Practical Engineer's Handbook," &c. Fcap. 8vo, nearly 500 pp., with 8 Plates and upwards of 250 Illustrative Diagrams, strongly bound for workshop or pocket wear and tear. **6/0**

"In its modernised form Hutton's 'Templeton' should have a wide sale, for it contains much valuable information which the mechanic will often find of use, and not a few tables and notes which he might look for in vain in other works. This modernised edition will be appreciated by all who have learned to value the original editions of 'Templeton.'"—*English Mechanic.*

"It has met with great success in the engineering workshop, as we can testify; and there are a great many men who, in a great measure, owe their rise in life to this little book."—*Building News.*

"This familiar text-book—well known to all mechanics and engineers—is of essential service to the every-day requirements of engineers, millwrights, and the various trades connected with engineering and building. The new modernised edition is worth its weight in gold."—*Building News.* (Second Notice.)

"This well-known and largely-used book contains information, brought up to date, of the sort so useful to the foreman and draughtsman. So much fresh information has been introduced as to constitute it practically a new book. It will be largely used in the office and workshop."—*Mechanical World.*

"The publishers wisely entrusted the task of revision of this popular, valuable, and useful book to Mr. Hutton, than whom a more competent man they could not have found."—*Iron.*

ENGINEER'S AND MILLWRIGHT'S ASSISTANT.

A Collection of Useful Tables, Rules, and Data. By WILLIAM TEMPLETON. Eighth Edition, with Additions. 18mo, cloth **2/6**

"Occupies a foremost place among books of this kind. A more suitable present to an apprentice to any of the mechanical trades could not possibly be made."—*Building News.*

"A deservedly popular work. It should be in the 'drawer' of every mechanic."—*English Mechanic.*

THE MECHANICAL ENGINEER'S REFERENCE BOOK.

For Machine and Boiler Construction. In Two Parts. Part I. General Engineering Data. Part II. Boiler Construction. With 51 Plates and numerous Illustrations. By Nelson Foley, M.I.N.A. Second Edition, Revised throughout and much Enlarged. Folio, half-bound. *Net* **£3 3s.**

Part I.—Measures.—Circumferences and Areas, &c., Squares, Cubes, Fourth Powers.—Square and Cube Roots.—Surface of Tubes.—Reciprocals.—Logarithms.—Mensuration.—Specific Gravities and Weights.—Work and Power.—Heat.—Combustion.—Expansion and Contraction.—Expansion of Gases.—Steam.—Static Forces.—Gravitation and Attraction.—Motion and Computation of Resulting Forces.—Accumulated Work.—Centre and Radius of Gyration.—Moment of Inertia.—Centre of Oscillation.—Electricity.—Strength of Materials.—Elasticity.—Test Sheets of Metals.—Friction.—Transmission of Power.—Flow of Liquids.—Flow of Gases.—Air Pumps, Surface Condensers, &c.—Speed of Steamships.—Propellers.—Cutting Tools.—Flanges.—Copper Sheets and Tubes.—Screws, Nuts, Bolt Heads, &c.—Various Recipes and Miscellaneous Matter.—With DIAGRAMS for Valve-Gear, Belting and Ropes, Discharge and Suction Pipes, Screw Propellers, and Copper Pipes.

Part II.—Treating of Power of Boilers.—Useful Ratios.—Notes on Construction.—Cylindrical Boiler Shells.—Circular Furnaces.—Flat Plates.—Stays.—Girders.—Screws.—Hydraulic Tests.—Riveting.—Boiler Setting, Chimneys, and Mountings.—Fuels, &c.—Examples of Boilers and Speeds of Steamships.—Nominal and Normal Horse Power.—With DIAGRAMS for all Boiler Calculations and Drawings of many Varieties of Boilers.

"Mr. Foley is well fitted to compile such a work. The diagrams are a great feature of the work. It may be stated that Mr. Foley has produced a volume which will undoubtedly fulfil the desire of the author and become indispensable to all mechanical engineers."—*Marine Engineer.*

"We have carefully examined this work, and pronounce it a most excellent reference book for the use of marine engineers."—*Journal of American Society of Naval Engineers.*

TEXT-BOOK ON THE STEAM ENGINE.

With a Supplement on Gas Engines and Part II. on Heat Engines. By T. M. Goodeve, M.A., Barrister-at-Law, Professor of Mechanics at the Royal College of Science, London; Author of "The Principles of Mechanics," "The Elements of Mechanism," &c. Fourteenth Edition. Crown 8vo, cloth . **6/0**

"Professor Goodeve has given us a treatise on the steam engine which will bear comparison with anything written by Huxley or Maxwell, and we can award it no higher praise."—*Engineer.*

"Mr. Goodeve's text-book is a work of which every young engineer should possess himself." —*Mining Journal.*

ON GAS ENGINES.

With Appendix describing a Recent Engine with Tube Igniter. By T. M. Goodeve, M.A. Crown 8vo, cloth **2/6**

"Like all Mr. Goodeve's writings, the present is no exception in point of general excellence. It is a valuable little volume."—*Mechanical World.*

GAS AND OIL ENGINE MANAGEMENT.

A Practical Guide for Users and Attendants, being Notes on Selection, Construction, and Management By M. Powis Bale, M.I M.E., A.M.I.C.E. Author of "Woodworking Machinery," &c. Crown 8vo, cloth.

[*Just Published. Net* **3/6**

THE GAS-ENGINE HANDBOOK.

A Manual of Useful Information for the Designer and the Engineer. By E. W. Roberts, M.E. With Forty Full-page Engravings. Small Fcap. 8vo, leather. *Net* **8/6**

A TREATISE ON STEAM BOILERS.

Their Strength, Construction, and Economical Working. By R. Wilson, C.E. Fifth Edition. 12mo, cloth **6/0**

"The best treatise that has ever been published on steam boilers."—*Engineer.*

THE MECHANICAL ENGINEER'S COMPANION

of Areas, Circumferences, Decimal Equivalents, in inches and feet, millimetres, squares, cubes, roots, &c.; Strength of Bolts, Weight of Iron, &c.; Weights, Measures, and other Data. Also Practical Rules for Engine Proportions. By R. Edwards, M.Inst.C.E. Fcap. 8vo, cloth. **3/6**

"A very useful little volume. It contains many tables, classified data and memoranda generally useful to engineers."—*Engineer.*

"What it professes to be, 'a handy office companion,' giving in a succinct form, a variety of information likely to be required by mechanical engineers in their everyday office work."—*Nature*

A HANDBOOK ON THE STEAM ENGINE.

With especial Reference to Small and Medium-sized Engines. For the Use of Engine Makers, Mechanical Draughtsmen, Engineering Students, and users of Steam Power. By Herman Haeder, C.E. Translated from the German with additions and alterations, by H. H. P. Powles, A.M.I.C.E., M.I.M.E. Third Edition, Revised. With nearly 1,100 Illustrations. Crown 8vo, cloth . . . . *Net* **7/6**

"A perfect encyclopædia of the steam engine and its details, and one which must take a permanent place in English drawing-offices and workshops."—*A Foreman Pattern-maker.*

"This is an excellent book, and should be in the hands of all who are interested in the construction and design of medium-sized stationary engines. . . . A careful study of its contents and the arrangement of the sections leads to the conclusion that there is probably no other book like it in this country. The volume aims at showing the results of practical experience, and it certainly may claim a complete achievement of this idea."—*Nature.*

"There can be no question as to its value. We cordially commend it to all concerned in the design and construction of the steam engine."—*Mechanical World.*

BOILER AND FACTORY CHIMNEYS.

Their Draught-Power and Stability. With a chapter on *Lightning Conductors.* By Robert Wilson, A.I.C.E., Author of "A Treatise on Steam Boilers," &c. Crown 8vo, cloth **3/6**

A valuable contribution to the literature of scientific building."—*The Builder.*

BOILER MAKER'S READY RECKONER & ASSISTANT.

With Examples of Practical Geometry and Templating, for the Use of Platers, Smiths, and Riveters. By John Courtney, Edited by D. K. Clark, M.I.C.E. Fourth Edition, 480 pp., with 140 Illustrations. Fcap. 8vo, half-bound **7/0**

"No workman or apprentice should be without this book."—*Iron Trade Circular.*

REFRIGERATION, COLD STORAGE, & ICE-MAKING:

A Practical Treatise on the Art and Science of Refrigeration. By A. J. Wallis-Tayler, A.M.Inst.C.E., Author of "Refrigerating and Ice-Making Machinery." 600 pp., with 360 Illustrations. Medium 8vo, cloth. *Net* **15/0**

"The author has to be congratulated on the completion and production of such an important work and it cannot fail to have a large body of readers, for it leaves out nothing that would in any way be of value to those interested in the subject."—*Steamship.*

"No one whose duty it is to handle the mammoth preserving installations of these latter days can afford to be without this valuable book."—*Glasgow Herald.*

THE POCKET BOOK OF REFRIGERATION AND ICE-MAKING.

Edited by A. J. Wallis-Tayler, A.M.Inst.C.E. Author of "Refrigerating and Ice-making Machinery," &c. Small Crown 8vo, cloth.

[*Just Published. Net* **3/6**

REFRIGERATING & ICE-MAKING MACHINERY.

A Descriptive Treatise for the Use of Persons Employing Refrigerating and Ice-Making Installations, and others. By A. J. Wallis-Tayler, A.-M. Inst. C.E. Third Edition, Enlarged. Crown 8vo, cloth . . **7/6**

"Practical, explicit, and profusely illustrated."—*Glasgow Herald.*

"We recommend the book, which gives the cost of various systems and illustrations showing details of parts of machinery and general arrangements of complete installations."—*Builder.*

"May be recommended as a useful description of the machinery, the processes, and of the facts, figures, and tabulated physics of refrigerating. It is one of the best compilations on the subject."—*Engineer.*

ENGINEERING ESTIMATES, COSTS, AND ACCOUNTS.

A Guide to Commercial Engineering. With numerous examples of Estimates and Costs of Millwright Work, Miscellaneous Productions, Steam Engines and Steam Boilers; and a Section on the Preparation of Costs Accounts. By A General Manager. Second Edition. 8vo, cloth. **12/0**

"This is an excellent and very useful book, covering subject-matter in constant requisition in every factory and workshop. . . . The book is invaluable, not only to the young engineer, but also to the estimate department of every works."—*Builder.*

"We accord the work unqualified praise. The information is given in a plain, straightforward manner, and bears throughout evidence of the intimate practical acquaintance of the author with every phase of commercial engineering."—*Mechanical World.*

HOISTING MACHINERY.

An Elementary Treatise on. Including the Elements of Crane Construction and Descriptions of the Various Types of Cranes in Use. By JOSEPH HORNER, A.M I.M.E., Author of "Pattern-Making," and other Works. Crown 8vo, with 215 Illustrations, including Folding Plates, cloth. [*Just Published.* *Net* **7/6**

AËRIAL OR WIRE-ROPE TRAMWAYS.

Their Construction and Management. By A. J. WALLIS-TAYLER, A.M.Inst.C.E. With 81 Illustrations. Crown 8vo, cloth **7/6**

"This is in its way an excellent volume. Without going into the minutiæ of the subject, it yet lays before its readers a very good exposition of the various systems of rope transmission in use, and gives as well not a little valuable information about their working, repair, and management. We can safely recommend it as a useful general treatise on the subject."—*The Engineer.*

MOTOR CARS OR POWER-CARRIAGES FOR COMMON ROADS.

By A. J. WALLIS-TAYLER, A. M. Inst. C.E., Author of "Modern Cycles," &c. 212 pp., with 76 Illustrations. Crown 8vo, cloth **4/6**

"The book is clearly expressed throughout, and is just the sort of work that an engineer thinking of turning his attention to motor-carriage work, would do well to read as a preliminary to starting operations."—*Engineering.*

PLATING AND BOILER MAKING.

A Practical Handbook for Workshop Operations. By JOSEPH G. HORNER, A.M.I.M.E. 380 pp. with 338 Illustrations. Crown 8vo, cloth . . **7/6**

"This work is characterised by that evidence of close acquaintance with workshop methods which will render the book exceedingly acceptable to the practical hand. We have no hesitation in commending the work as a serviceable and practical handbook on a subject which has not hitherto received much attention from those qualified to deal with it in a satisfactory manner."—*Mechanical World.*

PATTERN MAKING.

A Practical Treatise, embracing the Main Types of Engineering Construction, and including Gearing, Engine Work, Sheaves and Pulleys, Pipes and Columns, Screws, Machine Parts, Pumps and Cocks, the Moulding of Patterns in Loam and Greensand, estimating the weight of Castings &c. By JOSEPH G. HORNER, A.M.I.M.E. Third Edition, Enlarged. With 486 Illustrations. Crown 8vo, cloth. *Net* **7/6**

"A well-written technical guide, evidently written by a man who understands and has practised what he has written about. . . . We cordially recommend it to engineering students, young journeymen, and others desirous of being initiated into the mysteries of pattern-making."—*Builder.*

"An excellent *vade mecum* for the apprentice who desires to become master of his trade." —*English Mechanic.*

MECHANICAL ENGINEERING TERMS

(Lockwood's Dictionary of). Embracing those current in the Drawing Office, Pattern Shop, Foundry, Fitting, Turning, Smiths', and Boiler Shops, &c. Comprising upwards of 6,000 Definitions. Edited by J. G. HORNER, A.M.I.M.E. Third Edition, Revised, with Additions. Crown 8vo, cloth . . *Net* **7/6**

"Just the sort of handy dictionary required by the various trades engaged in mechanical engineering. The practical engineering pupil will find the book of great value in his studies, and every foreman engineer and mechanic should have a copy."—*Building News.*

TOOTHED GEARING.

A Practical Handbook for Offices and Workshops. By J. HORNER, A.M.I.M.E. With 184 Illustrations. Crown 8vo, cloth **6/0**

"We give the book our unqualified praise for its thoroughness of treatment, and recommend it to all interested as the most practical book on the subject yet written."—*Mechanical World.*

FIRES, FIRE-ENGINES, AND FIRE BRIGADES.

With a History of Fire-Engines, their Construction, Use, and Management; Foreign Fire Systems; Hints on Fire-Brigades, &c. By C. F. T. YOUNG, C.E. 8vo, cloth . . . . **£1 4s.**

"To such of our readers as are interested in the subject of fires and fire apparatus we can most heartily commend this book."—*Engineering.*

AËRIAL NAVIGATION.

A Practical Handbook on the Construction of Dirigible Balloons, Aërostats, Aëroplanes, and Aëremotors. By FREDERICK WALKER, C.E., Associate Member of the Aëronautic Institute. With 104 Illustrations. Large Crown 8vo, cloth . . . . *Net* **7/6**

STONE-WORKING MACHINERY.

A Manual dealing with the Rapid and Economical Conversion of Stone. With Hints on the Arrangement and Management of Stone Works. By M. POWIS BALE, M.I.M.E. Second Edition, enlarged. Crown 8vo, cloth . . **9/0**

"The book should be in the hands of every mason or student of stonework."—*Colliery Guardian.*

"A capital handbook for all who manipulate stone for building or ornamental purposes."—*Machinery Market.*

PUMPS AND PUMPING.

A Handbook for Pump Users. Being Notes on Selection, Construction, and Management. By M. POWIS BALE, M.I.M.E. Fourth Edition. Crown 8vo, cloth **3/6**

"The matter is set forth as concisely as possible. In fact, condensation rather than diffuseness has been the author's aim throughout; yet he does not seem to have omitted anything likely to be of use."—*Journal of Gas Lighting.*

"Thoroughly practical and clearly written."—*Glasgow Herald.*

MILLING MACHINES AND PROCESSES.

A Practical Treatise on Shaping Metals by Rotary Cutters. Including Information on Making and Grinding the Cutters. By PAUL N. HASLUCK, Author of "Lathe-Work." With upwards of 300 Engravings. Large crown 8vo, cloth **12/6**

"A new departure in engineering literature. . . . We can recommend this work to all interested in milling machines; it is what it professes to be—a practical treatise."—*Engineer.*

"A capital and reliable book which will no doubt be of considerable service both to those who are already acquainted with the process as well as to those who contemplate its adoption."—*Industries.*

LATHE-WORK.

A Practical Treatise on the Tools, Appliances, and Processes employed in the Art of Turning. By PAUL N. HASLUCK. Eighth Edition. Crown 8vo, cloth **5/0**

"Written by a man who knows not only how work ought to be done, but who also knows how to do it, and how to convey his knowledge to others. To all turners this book would be valuable."—*Engineering.*

"We can safely recommend the work to young engineers. To the amateur it will simply be invaluable. To the student it will convey a great deal of useful information."—*Engineer.*

SCREW-THREADS,

And Methods of Producing Them. With numerous Tables and complete Directions for using Screw-Cutting Lathes. By PAUL N. HASLUCK, Author of "Lathe-Work," &c. Sixth Edition. Waistcoat-pocket size . . **1/6**

"Full of useful information, hints and practical criticism. Taps, dies, and screwing tools generally are illustrated and their action described."—*Mechanical World.*

"It is a complete compendium of all the details of the screw-cutting lathe; in fact, a *multum in-parvo* on all the subjects it treats upon."—*Carpenter and Builder.*

TABLES AND MEMORANDA FOR ENGINEERS,

MECHANICS, ARCHITECTS, BUILDERS, &c.

Selected and Arranged by FRANCIS SMITH. Seventh Edition, Revised, including ELECTRICAL TABLES, FORMULÆ, and MEMORANDA. Waistcoat-pocket size, limp leather. [*Just Published.* **1/6**

"It would, perhaps, be as difficult to make a small pocket-book selection of notes and formulæ to suit ALL engineers as it would be to make a universal medicine; but Mr. Smith's waistcoat-pocket collection may be looked upon as a successful attempt."—*Engineer.*

"The best example we have ever seen of 270 pages of useful matter packed into the dimensions of a card-case."—*Building News.* "A veritable pocket treasury of knowledge."—*Iron.*

POCKET GLOSSARY OF TECHNICAL TERMS.

English-French, French-English; with Tables suitable for the Architectural, Engineering, Manufacturing, and Nautical Professions. By JOHN JAMES FLETCHER. Third Edition, 200 pp. Waistcoat-pocket size, limp leather **1/6**

"It is a very great advantage for readers and correspondents in France and England to have so large a number of the words relating to engineering and manufacturers collected in a liliputian volume. The little book will be useful both to students and travellers."—*Architect.*

"The glossary of terms is very complete, and many of the Tables are new and well arranged We cordially commend the book."—*Mechanical World.*

THE ENGINEER'S YEAR BOOK FOR 1904.

Comprising Formulæ, Rules, Tables, Data and Memoranda in Civil, Mechanical, Electrical, Marine and Mine Engineering. By H. R. Kempe, A.M. Inst. C.E., M.I.E.E., Principal Technical Officer, Engineer-in-Chief's Office, General Post Office, London, Author of "A Handbook of Electrical Testing," "The Electrical Engineer's Pocket-Book," &c. With 1,000 Illustrations, specially Engraved for the work. Crown 8vo, 900 pp., leather. [*Just Published.* **8/0**

"Kempe's Year Book really requires no commendation. Its sphere of usefulness is widely known, and it is used by engineers the world over."—*The Engineer.*

"The volume is distinctly in advance of most similar publications in this country."—*Engineering.*

"This valuable and well-designed book of reference meets the demands of all descriptions of engineers."—*Saturday Review.*

"Teems with up-to-date information in every branch of engineering and construction."—*Building News.*

"The needs of the engineering profession could hardly be supplied in a more admirable, complete and convenient form. To say that it more than sustains all comparisons is praise of the highest sort, and that may justly be said of it."—*Mining Journal.*

"There is certainly room for the newcomer, which supplies explanations and directions, as well as formulæ and tables. It deserves to become one of the most successful of the technical annuals."—*Architect.*

"Brings together with great skill all the technical information which an engineer has to use day by day. It is in every way admirably equipped, and is sure to prove successful."—*Scotsman.*

"The up-to-dateness of Mr. Kempe's compilation is a quality that will not be lost on the busy people for whom the work is intended."—*Glasgow Herald.*

THE PORTABLE ENGINE.

A Practical Manual on its Construction and Management. For the use of Owners and Users of Steam Engines generally. By William Dyson Wansbrough. Crown 8vo, cloth **3/6**

"This is a work of value to those who use steam machinery. . . . Should be read by every one who has a steam engine, on a farm or elsewhere."—*Mark Lane Express.*

IRON AND STEEL.

A Work for the Forge, Foundry, Factory, and Office. Containing ready, useful, and trustworthy Information for Ironmasters and their Stock-takers; Managers of Bar, Rail, Plate, and Sheet Rolling Mills; Iron and Metal Founders; Iron Ship and Bridge Builders; Mechanical, Mining, and Consulting Engineers; Architects, Contractors, Builders, &c. By Charles Hoare, Author of "The Slide Rule," &c. Ninth Edition. 32mo, leather . **6/0**

CONDENSED MECHANICS.

A Selection of Formulæ, Rules, Tables, and Data or the Use of Engineering Students, &c. By W. G. C. Hughes, A.M.I.C.E. Crown 8vo, cloth . **2/6**

"The book is well fitted for those who are preparing for examination and wish to refresh their knowledge by going through their formulæ again."—*Marine Engineer.*

THE SAFE USE OF STEAM.

Containing Rules for Unprofessional Steam Users. By an Engineer. Seventh Edition. Sewed . . . . **6d.**

"If steam-users would but learn this little book by heart, boiler explosions would become sensations by their rarity."—*English Mechanic.*

THE CARE AND MANAGEMENT OF STATIONARY ENGINES.

A Practical Handbook for Men-in-charge. By C. Hurst. Crown 8vo. *Net* **1/0**

THE LOCOMOTIVE ENGINE.

The Autobiography of an Old Locomotive Engine. By Robert Weatherburn, M.I.M.E. With Illustrations and Portraits of George and Robert Stephenson. Crown 8vo, cloth. *Net* **2/6**

THE LOCOMOTIVE ENGINE AND ITS DEVELOPMENT.

A Popular Treatise on the Gradual Improvements made in Railway Engines between 1803 and 1903. By Clement E. Stretton, C.E. Sixth Edition, Revised and Enlarged. Crown 8vo, cloth. [*Just Published. Net* **4/6**

"Students of railway history and all who are interested in the evolution of the modern locomotive will find much to attract and entertain in this volume."—*The Times.*

MODERN MACHINE SHOP TOOLS,

THEIR CONSTRUCTION, OPERATION, AND MANIPULATION.

Including both Hand and Machine Tools. An entirely New and Fully Illustrated Work, treating this Subject in a Concise and Comprehensive Manner. A Book of Practical Instruction in all Classes of Machine Shop Practice. Including Chapters on Filing, Fitting, and Scraping Surfaces; on Drills, Reamers, Taps, and Dies; the Lathe and its Tools; Planers, Shapers, and their Tools; Milling Machines and Cutters; Gear Cutters and Gear Cutting; Drilling Machines and Drill Work; Grinding Machines and their Work; Hardening and Tempering, Gearing, Belting, and Transmission Machinery; Useful Data and Tables. By William H. Van Dervoort, M.E. Illustrated by 673 Engravings of Latest Tools and Methods, all of which are fully described. Medium 8vo, cloth. [*Just Published. Net* **21/0**

LOCOMOTIVE ENGINE DRIVING.

A Practical Manual for Engineers in Charge of Locomotive Engines. By Michael Reynolds, formerly Locomotive Inspector, L. B. & S. C. R. Eleventh Edition. Including a Key to the Locomotive Engine. Crown 8vo, cloth **4/6**

"Mr. Reynolds has supplied a want, and has supplied it well. We can confidently recommend the book not only to the practical driver, but to everyone who takes an interest in the performance of locomotive engines."—*The Engineer.*

"Mr. Reynolds has opened a new chapter in the literature of the day. This admirable practical treatise, of the practical utility of which we have to speak in terms of warm commendation."—*Athenæum.*

THE MODEL LOCOMOTIVE ENGINEER,

Fireman, and Engine-Boy. Comprising a Historical Notice of the Pioneer Locomotive Engines and their Inventors. By Michael Reynolds. Second Edition, with Revised Appendix. Crown 8vo, cloth. **4/6**

"We should be glad to see this book in the possession of everyone in the kingdom who has ever laid, or is to lay, hands on a locomotive engine."—*Iron.*

CONTINUOUS RAILWAY BRAKES.

A Practical Treatise on the several Systems in Use in the United Kingdom: their Construction and Performance. By Michael Reynolds. 8vo, cloth **9/0**

"A popular explanation of the different brakes. It will be of great assistance in forming public opinion, and will be studied with benefit by those who take an interest in the brake."—*English Mechanic.*

STATIONARY ENGINE DRIVING.

A Practical Manual for Engineers in Charge of Stationary Engines. By Michael Reynolds. Sixth Edition. With Plates and Woodcuts. Crown 8vo, cloth **4/6**

"The author's advice on the various points treated is clear and practical."—*Engineering.*

"Our author leaves no stone unturned. He is determined that his readers shall not only know something about the stationary engine, but all about it."—*Engineer.*

ENGINE-DRIVING LIFE.

Stirring Adventures and Incidents in the Lives of Locomotive Engine-Drivers. By Michael Reynolds. Third Edition. Crown 8vo, cloth . **1/6**

"From first to last perfectly fascinating. Wilkie Collins's most thrilling conceptions are thrown into the shade by true incidents, endless in their variety, related in every page."—*North British Mail.*

THE ENGINEMAN'S POCKET COMPANION,

And Practical Educator for Enginemen, Boiler Attendants, and Mechanics. By Michael Reynolds. With 45 Illustrations and numerous Diagrams Fourth Edition, Revised. Royal 18mo, strongly bound for pocket wear **3/6**

"A most meritorious work, giving in a succinct and practical form all the information an engine-minder desirous of mastering the scientific principles of his daily calling would require."—*The Miller.*

CIVIL ENGINEERING, SURVEYING, ETC.

PIONEER IRRIGATION.

A Manual of Information or Farmers in the Colonies. By E. O. Mawson, Executive Engineer, Public Works Department, Bombay. With Numerous Plates and Diagrams. Medium 8vo, cloth.

[*Just ready. Price about* **7/6** *net.*

Summary of Contents:—Value of Irrigation, and Sources of Water Supply.—Dams and Weirs.—Canals.—Underground Water.—Methods of Irrigation.—Sewage Irrigation.—Imperial Automatic Sluice Gates.—The Cultivation of Irrigated Crops, Vegetables, and Fruit Trees.—Light Railways for Heavy Traffic.—Useful Memoranda, Tables, and Data.

TUNNELLING.

A Practical Treatise. By Charles Prelini, C.E. With additions by Charles S. Hill, C.E. With 150 Diagrams and Illustrations. Royal 8vo, cloth . . . . *Net* **16/0**

PRACTICAL TUNNELLING.

Explaining in detail Setting-out the Works, Shaft-sinking, and Heading-driving, Ranging the Lines and Levelling underground, Sub-Excavating, Timbering and the Construction of the Brickwork of Tunnels. By F. W. Simms, M. Inst. C.E. Fourth Edition, Revised and Further Extended, including the most recent (1895) Examples of Sub-aqueous and other Tunnels, by D. Kinnear Clark, M. Inst. C.E. With 34 Folding Plates. Imperial 8vo, cloth **£2 2s.**

"The present (1896) edition has been brought right up to date, and is thus rendered a work to which civil engineers generally should have ready access, and to which engineers who have construction work can hardly afford to be without, but which to the younger members of the profession is invaluable, as from its pages they can learn the state to which the science of tunnelling has attained."—*Railway News.*

THE WATER SUPPLY OF TOWNS AND THE CONSTRUCTION OF WATER-WORKS.

A Practical Treatise for the Use of Engineers and Students of Engineering. By W. K. Burton, A.M. Inst. C.E., Consulting Engineer to the Tokyo Water-works. Second Edition, Revised and Extended. With numerous Plates and Illustrations. Super-royal 8vo, buckram. [*Just Published.* **25/0**

I. Introductory.—II. Different Qualities of Water.—III. Quantity of Water to be Provided.—IV. On Ascertaining whether a Proposed Source of Supply is Sufficient.—V. On Estimating the Storage Capacity Required to be Provided.—VI. Classification of Water-works.—VII. Impounding Reservoirs.—VIII. Earthwork Dams.—IX. Masonry Dams.—X. The Purification of Water.—XI. Settling Reservoirs.—XII. Sand Filtration.—XIII. Purification of Water by Action of Iron, Softening of Water by Action of Lime, Natural Filtration.—XIV. Service or Clean Water Reservoirs—Water Towers—Stand Pipes.—XV. The Connection of Settling Reservoirs, Filter Beds and Service Reservoirs.—XVI. Pumping Machinery.—XVII. Flow of Water in Conduits—Pipes and Open Channels.—XVIII. Distribution Systems.—XIX. Special Provisions for the Extinction of Fire.—XX. Pipes for Water-works.—XXI. Prevention of Waste of Water.—XXII. Various Appliances used in Connection with Water-works.

Appendix I. By Prof. JOHN MILNE, F.R.S.—Considerations concerning the Probable Effects of Earthquakes on Water-works, and the Special Precautions to be taken in Earthquake Countries.

Appendix II. By JOHN DE RIJKE, C.E.—On Sand Dunes and Dune Sand as a Source of Water Supply.

"The chapter upon filtration of water is very complete, and the details of construction well illustrated. . . . The work should be specially valuable to civil engineers engaged in work in Japan, but the interest is by no means confined to that locality."—*Engineer.*

"We congratulate the author upon the practical commonsense shown in the preparation of this work. . . . The plates and diagrams have evidently been prepared with great care, and cannot fail to be of great assistance to the student."—*Builder.*

RURAL WATER SUPPLY.

A Practical Handbook on the Supply of Water and Construction of Waterworks for small Country Districts. By Allan Greenwell, A.M.I.C.E., and W. T. Curry, A.M.I.C.E., F.G.S. With Illustrations. Second Edition, Revised. Crown 8vo, cloth **5/0**

"We conscientiously recommend it as a very useful book for those concerned in obtaining water for small districts, giving a great deal of practical information in a small compass."—*Builder.*

"The volume contains valuable information upon all matters connected with water supply. . . It is full of details on points which are continually before water-works engineers."—*Nature.*

THE WATER SUPPLY OF CITIES AND TOWNS.

By William Humber, A. M. Inst. C.E., and M. Inst. M.E., Author of "Cast and Wrought Iron Bridge Construction," &c., &c. Illustrated with 50 Double Plates, 1 Single Plate, Coloured Frontispiece, and upwards of 250 Woodcuts, and containing 400 pp. of Text. Imp. 4to, elegantly and substantially half-bound in morocco . . . . *Net* **£6 6s.**

List of Contents. I. Historical Sketch of some of the Means that have been adopted for the Supply of Water to Cities and Towns.—II. Water and the Foreign Matter usually associated with it.—III. Rainfall and Evaporation.—IV. Springs and the Water-bearing Formations of various Districts.—V. Measurement and Estimation of the Flow of Water.—VI. On the Selection of the Source of Supply.—VII. Wells.—VIII. Reservoirs.—IX. The Purification of Water.—X. Pumps.—XI. Pumping Machinery.—XII. Conduits.—XIII. Distribution of Water.—XIV. Meters, Service Pipes, and House Fittings.—XV. The Law and Economy of Water-works.—XVI. Constant and Intermittent Supply.—XVII. Description of Plates.—Appendices, giving Tables of Rates of Supply, Velocities, &c., &c., together with Specifications of several Works illustrated, among which will be found: Aberdeen, Bideford, Canterbury, Dundee, Halifax, Lambeth, Rotherham, Dublin, and others.

"The most systematic and valuable work upon water supply hitherto produced in English, or in any other language. Mr. Humber's work is characterised almost throughout by an exhaustiveness much more distinctive of French and German than of English technical treatises." —*Engineer.*

THE PROGRESS OF ENGINEERING (1863-6).

By Wm. Humber, A.M.Inst.C.E. Complete in Four Vols. Containing 148 Double Plates, with Portraits and Copious Descriptive Letterpress. Impl. 4to, half-morocco. Price, complete, **£12 12s.**; or each Volume sold separately at **£3 3s.** per Volume. *Descriptive List of Contents on application.*

HYDRAULIC POWER ENGINEERING.

A Practical Manual on the Concentration and Transmission of Power by Hydraulic Machinery. By G. Croydon Marks, A.M. Inst. C.E. With nearly 200 Illustrations. 8vo, cloth. *Net* **9/0**

Summary of Contents. Principles of Hydraulics.—The Flow of Water.—Hydraulic Pressures, Material.—Test Load Packings for Sliding Surfaces.—Pipe Joints.—Controlling Valves.—Platform Lifts.—Workshop and Foundry Cranes.—Warehouse and Dock Cranes.—Hydraulic Accumulators.—Presses for Baling and other Purposes.—Sheet Metal Working and Forging Machinery.—Hydraulic Rivetters.—Hand, Power, and Steam Pumps.—Turbines.—Impulse Turbines.—Reaction Turbines.—Design of Turbines in Detail.—Water Wheels.—Hydraulic Engines.—Recent Achievements.—Pressure of Water.—Action of Pumps, &c.

"We have nothing but praise for this thoroughly valuable work. The author has succeeded in rendering his subject interesting as well as instructive."—*Practical Engineer.*

"Can be unhesitatingly recommended as a useful and up-to-date manual on hydraulic transmission and utilisation of power."—*Mechanical World*

HYDRAULIC TABLES, CO-EFFICIENTS, & FORMULÆ.

For Finding the Discharge of Water from Orifices, Notches, Weirs, Pipes, and Rivers. With New Formulæ, Tables, and General Information on Rain-fall, Catchment-Basins, Drainage, Sewerage, Water Supply for Towns and Mill Power. By John Neville, C.E., M.R.I.A. Third Edition, revised, with additions. Numerous Illustrations. Crown 8vo, cloth . . . **14/0**

"It is, of all English books on the subject, the one nearest to completeness."—*Architect.*

HYDRAULIC MANUAL.

Consisting of Working Tables and Explanatory Text. Intended as a Guide in Hydraulic Calculations and Field Operations. By Lowis D'A. Jackson, Author of "Aid to Survey Practice," "Modern Metrology," &c. Fourth Edition, Enlarged. Large crown 8vo, cloth **16/0**

"The author has constructed a manual which may be accepted as a trustworthy guide to this branch of the engineer's profession."—*Engineering.*

WATER ENGINEERING.

A Practical Treatise on the Measurement, Storage, Conveyance, and Utilisation of Water for the Supply of Towns, for Mill Power, and for other Purposes. By Charles Slagg, A.M.Inst.C.E. Second Edition. Crown 8vo, cloth. **7/6**

"As a small practical treatise on the water supply of towns, and on some applications of water-power, the work is in many respects excellent."—*Engineering.*

THE RECLAMATION OF LAND FROM TIDAL WATERS.

A Handbook for Engineers, Landed Proprietors, and others interested in Works of Reclamation. By ALEX. BEAZELEY, M.Inst. C.E. 8vo, cloth. *Net* **10/6**

"The book shows in a concise way what has to be done in reclaiming land from the sea, and the best way of doing it. The work contains a great deal of practical and useful information which cannot fail to be of service to engineers entrusted with the enclosure of salt marshes, and to landowners intending to reclaim land from the sea."—*The Engineer.*

"The author has carried out his task efficiently and well, and his book contains a large amount of information of great service to engineers and others interested in works of reclamation."—*Nature.*

MASONRY DAMS FROM INCEPTION TO COMPLETION.

Including numerous Formulæ, Forms of Specification and Tender, Pocket Diagram of Forces, &c. For the use of Civil and Mining Engineers. By C. F. COURTNEY, M. Inst. C.E. 8vo, cloth **9/0**

"The volume contains a good deal of valuable data. Many useful suggestions will be found in the remarks on site and position, location of dam, foundations and construction."—*Building News.*

RIVER BARS.

The Causes of their Formation, and their Treatment by "Induced Tidal Scour"; with a Description of the Successful Reduction by this Method of the Bar at Dublin. By I. J. MANN, Assist. Eng. to the Dublin Port and Docks Board. Royal 8vo, cloth **7/6**

"We recommend all interested in harbour works—and, indeed, those concerned in the improvements of rivers generally—to read Mr. Mann's interesting work."—*Engineer.*

TRAMWAYS: THEIR CONSTRUCTION AND WORKING.

Embracing a Comprehensive History of the System; with an exhaustive Analysis of the Various Modes of Traction, including Horse Power, Steam, Cable Traction, Electric Traction, &c.; a Description of the Varieties of Rolling Stock; and ample Details of Cost and Working Expenses. New Edition, Thoroughly Revised, and Including the Progress recently made in Tramway Construction, &c., &c. By D. KINNEAR CLARK, M. Inst. C.E. With 400 Illustrations. 8vo, 780 pp., buckram. **28/0**

"The new volume is one which will rank, among tramway engineers and those interested in tramway working, with the Author's world-famed book on railway machinery."—*The Engineer.*

SURVEYING AS PRACTISED BY CIVIL ENGINEERS AND SURVEYORS.

Including the Setting-out of Works for Construction and Surveys Abroad, with many Examples taken from Actual Practice. A Handbook for use in the Field and the Office, intended also as a Text-book for Students. By JOHN WHITELAW, Jun., A.M. Inst. C.E., Author of "Points and Crossings." With about 260 Illustrations. Demy 8vo, cloth . . . . *Net* **10/6**

"This work is written with admirable lucidity, and will certainly be found of distinct value both to students and to those engaged in actual practice."—*The Builder.*

PRACTICAL SURVEYING.

A Text-Book for Students preparing for Examinations or for Survey-work in the Colonies. By GEORGE W. USILL, A.M.I.C.E. With 4 Lithographic Plates and upwards of 330 Illustrations. Seventh Edition. Including Tables of Natural Sines, Tangents, Secants, &c. Crown 8vo, **7/6** cloth; or, on THIN PAPER, leather, gilt edges, rounded corners, for pocket use . . . **12/6**

"The best forms of instruments are described as to their construction, uses and modes of employment, and there are innumerable hints on work and equipment such as the author, in his experience as surveyor, draughtsman and teacher, has found necessary, and which the student in his inexperience will find most serviceable."—*Engineer.*

"The first book which should be put in the hands of a pupil of Civil Engineering."—*Architect.*

AID TO SURVEY PRACTICE.

For Reference in Surveying, Levelling, and Setting-out; and in Route Surveys of Travellers by Land and Sea. With Tables, Illustrations, and Records. By LOWIS D'A. JACKSON, A.M.I.C.E. Second Edition, Enlarged. 8vo, cloth **12/6**

"Mr. Jackson has produced a valuable *vade-mecum* for the surveyor. We can recommend this book as containing an admirable supplement to the teaching of the accomplished surveyor."—*Athenæum.*

"The author brings to his work a fortunate union of theory and practical experience which, aided by a clear and lucid style of writing, renders the book a very useful one."—*Builder.*

SURVEYING WITH THE TACHEOMETER.

A practical Manual for the use of Civil and Military Engineers and Surveyors. Including two series of Tables specially computed for the Reduction of Readings in Sexagesimal and in Centesimal Degrees. By NEIL KENNEDY, M. Inst. C.E. With Diagrams and Plates. Demy 8vo, cloth. *Net* **10/6**

"The work is very clearly written, and should remove all difficulties in the way of any surveyor desirous of making use of this useful and rapid instrument."—*Nature.*

ENGINEER'S & MINING SURVEYOR'S FIELD BOOK.

Consisting of a Series of Tables, with Rules, Explanations of Systems, and use of Theodolite for Traverse Surveying and plotting the work with minute accuracy by means of Straight Edge and Set Square only; Levelling with the Theodolite, Setting-out Curves with and without the Theodolite, Earthwork Tables, &c. By W. DAVIS HASKOLL, C.E. With numerous Woodcuts. Fourth Edition, Enlarged. Crown 8vo, cloth **12/0**

"The book is very handy; the separate tables of sines and tangents to every minute will make it useful for many other purposes, the genuine traverse tables existing all the same."—*Athenæum.*

LAND AND MARINE SURVEYING.

In Reference to the Preparation of Plans for Roads and Railways; Canals, Rivers, Towns' Water Supplies; Docks and Harbours. With Description and Use of Surveying Instruments. By W. DAVIS HASKOLL, C.E. Second Edition, Revised, with Additions. Large crown 8vo, cloth . . . **9/0**

"This book must prove of great value to the student. We have no hesitation in recommending it, feeling assured that it will more than repay a careful study."—*Mechanical World.*

"A most useful book for the student. We can strongly recommend it as a carefully-written and valuable text-book. It enjoys a well-deserved repute among surveyors."—*Builder.*

PRINCIPLES AND PRACTICE OF LEVELLING.

Showing its Application to Purposes of Railway and Civil Engineering in the Construction of Roads; with Mr. TELFORD'S Rules for the same. By FREDERICK W. SIMMS, M. Inst. C.E. Eighth Edition, with LAW'S Practical Examples for Setting-out Railway Curves, and TRAUTWINE'S Field Practice of Laying-out Circular Curves. With 7 Plates and numerous Woodcuts. 8vo **8/6**

"The text-book on levelling in most of our engineering schools and colleges."—*Engineer.*

"The publishers have rendered a substantial service to the profession, especially to the younger members, by bringing out the present edition of Mr. Simms's useful work."—*Engineering.*

AN OUTLINE OF THE METHOD OF CONDUCTING A TRIGONOMETRICAL SURVEY.

For the Formation of Geographical and Topographical Maps and Plans, Military Reconnaissance, LEVELLING, &c., with Useful Problems, Formulæ, and Tables. By Lieut.-General FROME, R.E. Fourth Edition, Revised and partly Re-written by Major-General Sir CHARLES WARREN, G.C.M.G., R.E. With 19 Plates and 115 Woodcuts, royal 8vo, cloth **16/0**

"No words of praise from us can strengthen the position so well and so steadily maintained by this work. Sir Charles Warren has revised the entire work, and made such additions as were necessary to bring every portion of the contents up to the present date."—*Broad Arrow.*

TABLES OF TANGENTIAL ANGLES AND MULTIPLES FOR SETTING-OUT CURVES.

From 5 to 200 Radius. By A. BEAZELEY, M. Inst. C.E. 6th Edition, Revised. With an Appendix on the use of the Tables for Measuring up Curves. Printed on 50 Cards, and sold in a cloth box, waistcoat-pocket size. **3/6**

"Each table is printed on a small card, which, placed on the theodolite, leaves the hands free to manipulate the instrument—no small advantage as regards the rapidity of work."—*Engineer.*

"Very handy: a man may know that all his day's work must fall on two of these cards, which he puts into his own card-case, and leaves the rest behind."—*Athenæum.*

HANDY GENERAL EARTH-WORK TABLES.

Giving the Contents in Cubic Yards of Centre and Slopes of Cuttings and Embankments from 3 inches to 80 feet in Depth or Height, for use with either 66 feet Chain or 100 feet Chain. By J. H. WATSON BUCK, M. Inst. C.E. On a Sheet mounted in cloth case **3/6**

EARTHWORK TABLES.

Showing the Contents in Cubic Yards of Embankments, Cuttings, &c., Heights or Depths up to an average of 80 feet. By JOSEPH BROADBENT, C.E., and FRANCIS CAMPIN, C.E. Crown 8vo, cloth **5/0**

"The way in which accuracy is attained, by a simple division of each cross section into three elements, two in which are constant and one variable, is ingenious."—*Athenæum.*

A MANUAL ON EARTHWORK.

By ALEX. J. GRAHAM, C.E. With numerous Diagrams. Second Edition. 18mo, cloth **2/6**

THE CONSTRUCTION OF LARGE TUNNEL SHAFTS.

A Practical and Theoretical Essay. By J. H. WATSON BUCK, M. Inst. C.E., Resident Engineer, L. and N. W. R. With Folding Plates, 8vo, cloth **12/0**

"Many of the methods given are of extreme practical value to the mason, and the observations on the form of arch, the rules for ordering the stone, and the construction of the templates, will be found of considerable use. We commend the book to the engineering profession."—*Building News.*

"Will be regarded by civil engineers as of the utmost value, and calculated to save much time and obviate many mistakes."—*Colliery Guardian.*

CAST & WROUGHT IRON BRIDGE CONSTRUCTION

(A Complete and Practical Treatise on), including Iron Foundations. In Three Parts.—Theoretical, Practical, and Descriptive. By WILLIAM HUMBER, A. M. Inst. C.E., and M. Inst. M.E. Third Edition, revised and much improved, with 115 Double Plates (20 of which now first appear in this edition), and numerous Additions to the Text. In 2 vols., imp. 4to, half-bound in morocco . . . . **£6 16s. 6d.**

"A very valuable contribution to the standard literature of civil engineering. In addition to elevations, plans, and sections, large scale details are given, which very much enhance the instructive worth of those illustrations."—*Civil Engineer and Architect's Journal.*

"Mr. Humber's stately volumes, lately issued—in which the most important bridges erected during the last five years, under the direction of the late Mr. Brunel, Sir W. Cubitt, Mr. Hawkshaw, Mr. Page, Mr. Fowler, Mr. Hemans, and others among our most eminent engineers, are drawn and specified in great detail."—*Engineer.*

ESSAY ON OBLIQUE BRIDGES

(Practical and Theoretical). With 13 large Plates. By the late GEORGE WATSON BUCK, M.I.C.E. Fourth Edition, revised by his Son, J. H. WATSON BUCK, M.I.C.E.; and with the addition of Description to Diagrams for Facilitating the Construction of Oblique Bridges, by W. H. BARLOW, M.I.C.E. Royal 8vo, cloth **12/0**

"The standard text-book for all engineers regarding skew arches is Mr. Buck's treatise, and it would be impossible to consult a better."—*Engineer.*

"Mr. Buck's treatise is recognised as a standard text-book, and his treatment has divested the subject of many of the intricacies supposed to belong to it. As a guide to the engineer and architect, on a confessedly difficult subject, Mr. Buck's work is unsurpassed."—*Building News.*

THE CONSTRUCTION OF OBLIQUE ARCHES

(A Practical Treatise on). By JOHN HART. Third Edition, with Plates. Imperial 8vo, cloth **8/0**

GRAPHIC AND ANALYTIC STATICS.

In their Practical Application to the Treatment of Stresses in Roofs Solid Girders, Lattice, Bowstring, and Suspension Bridges, Braced Iron Arches and Piers, and other Frameworks. By R. HUDSON GRAHAM, C.E. Containing Diagrams and Plates to Scale. With numerous Examples, many taken from existing Structures. Specially arranged for Class-work in Colleges and Universities. Second Edition, Revised and Enlarged. 8vo, cloth . **16/0**

"Mr. Graham's book will find a place wherever graphic and analytic statics are used or studied."—*Engineer.*

"The work is excellent from a practica point of view, and has evidently been prepared with much care. The directions for working are ample, and are illustrated by an abundance of well-selected examples. It is an excellent text-book for the practical draughtsman."—*Athenæum.*

WEIGHTS OF WROUGHT IRON & STEEL GIRDERS.

A Graphic Table for Facilitating the Computation of the Weights of Wrought Iron and Steel Girders, &c., for Parliamentary and other Estimates. By J. H. WATSON BUCK, M. Inst. C.E. On a Sheet **2/6**

GEOMETRY FOR TECHNICAL STUDENTS.

An Introduction to Pure and Applied Geometry and the Mensuration of Surfaces and Solids, including Problems in Plane Geometry useful in Drawing. By E. H. SPRAGUE, A.M.I.C.E. Crown 8vo, cloth. [*Just Published. Net* **1/0**

PRACTICAL GEOMETRY.

For the Architect, Engineer, and Mechanic. Giving Rules for the Delineation and Application of various Geometrical Lines, Figures, and Curves. By E. W. TARN, M.A., Architect. 8vo, cloth **9/0**

"No book with the same objects in view has ever been published in which the clearness of the rules laid down and the illustrative diagrams have been so satisfactory."—*Scotsman.*

THE GEOMETRY OF COMPASSES.

Or, Problems Resolved by the mere Description of Circles and the Use of Coloured Diagrams and Symbols. By OLIVER BYRNE. Coloured Plates. Crown 8vo, cloth **3/6**

EXPERIMENTS ON THE FLEXURE OF BEAMS

Resulting in the Discovery of New Laws of Failure by Buckling. By ALBERT E. GUY. Medium 8vo, cloth. [*Just Published. Net* **9/0**

HANDY BOOK FOR THE CALCULATION OF STRAINS

In Girders and Similar Structures and their Strength. Consisting of Formulæ and Corresponding Diagrams, with numerous details for Practical Application, &c. By WILLIAM HUMBER, A. M. Inst. C.E., &c. Fifth Edition. Crown 8vo, with nearly 100 Woodcuts and 3 Plates, cloth **7/6**

"The formulæ are neatly expressed, and the diagrams good."—*Athenæum.*

"We heartily commend this really *handy* book to our engineer and architect readers."—*English Mechanic.*

TRUSSES OF WOOD AND IRON.

Practical Applications of Science in Determining the Stresses, Breaking Weights, Safe Loads, Scantlings, and Details of Construction. With Complete Working Drawings. By WILLIAM GRIFFITHS, Surveyor. 8vo, cloth **4/6**

"This handy little book enters so minutely into every detail connected with the construction of roof trusses that no student need be ignorant of these matters."—*Practical Engineer.*

THE STRAINS ON STRUCTURES OF IRONWORK.

With Practical Remarks on Iron Construction. By F. W. SHEILDS, M.I.C.E. 8vo, cloth **5/0**

A TREATISE ON THE STRENGTH OF MATERIALS.

With Rules for Application in Architecture, the Construction of Suspension Bridges, Railways, &c. By PETER BARLOW, F.R.S. A new Edition, revised by his Sons, P. W. BARLOW, F.R.S., and W. H. BARLOW, F.R.S.; to which are added, Experiments by HODGKINSON, FAIRBAIRN, and KIRKALDY; and Formulæ for calculating Girders, &c. Edited by WM. HUMBER, A.M.I.C.E. 8vo, 400 pp., with 19 Plates and numerous Woodcuts, cloth **18/0**

"Valuable alike to the student, tyro, and the experienced practitioner, it will always rank in future as it has hitherto done, as the standard treatise on that particular subject."—*Engineer.*

SAFE RAILWAY WORKING.

A Treatise on Railway Accidents, their Cause and Prevention; with a Description of Modern Appliances and Systems. By CLEMENT E. STRETTON, C.E. With Illustrations and Coloured Plates. Third Edition, Enlarged. Crown 8vo, cloth **3/6**

"A book for the engineer, the directors, the managers; and, in short, all who wish for information on railway matters will find a perfect encyclopædia in 'Safe Railway Working.'"—*Railway Review.*

EXPANSION OF STRUCTURES BY HEAT.

By JOHN KEILY, C.E., late of the Indian Public Works Department. Crown 8vo, cloth **3/6**

"The aim the author has set before him, viz., to show the effects of heat upon metallic and other structures, is a laudable one, for this is a branch of physics upon which the engineer or architect can find but little reliable and comprehensive data in books."—*Builder.*

PUBLICATIONS OF THE ENGINEERING STANDARDS COMMITTEE.

The Engineering Standards Committee is the outcome of a Committee appointed by the Institution of Civil Engineers at the instance of Sir John Wolfe Barry, K.C.B., to inquire into the advisability of Standardising Rolled Iron and Steel Sections.

The Committee is supported by the Institution of Civil Engineers, the Institution of Mechanical Engineers, the Institution of Naval Architects, the Iron and Steel Institute, and the Institution of Electrical Engineers; and the value and importance of its labours has been emphatically recognised by his Majesty's Government, who have made a liberal grant from the Public Funds by way of contribution to the financial resources of the Committee.

The subjects already dealt with, or under consideration by the Committee, include not only Rolled Iron and Steel Sections, but Tests for Iron and Steel Material used in the Construction of Ships and their Machinery, Bridges and General Building Construction, Railway Rolling Stock Underframes, Component Parts of Locomotives, Railway and Tramway Rails, Electrical Plant, Insulating Materials, Screw Threads and Limit Gauges, Pipe Flanges, Cement, &c.

Reports already Published :—

1. BRITISH STANDARD SECTIONS.

List 1. Equal Angles.—List 2. Unequal Angles.—List 3. Bulb Angles. List 4. Bulb Tees.—List 5. Bulb Plates.—List 7. Channels.—List 8. Beams. F'cap. folio, sewed. [*Just Published. Net* **1/0**

2. BRITISH STANDARD TRAMWAY RAILS AND FISH PLATES: STANDARD SECTIONS AND SPECIFICATION.

F'cap. folio, sewed. [*Just Published. Net* **21/0**

3. REPORT ON THE INFLUENCE OF GAUGE LENGTH AND SECTION OF TEST BAR ON THE PERCENTAGE OF ELONGATION.

By Professor W. C. Unwin, F.R.S. F'cap. folio, sewed. [*Just Published. Net* **2/6**

4. PROPERTIES OF STANDARD BEAMS.

Demy 8vo, sewed. [*Just Published. Net* **1/0**

MARINE ENGINEERING, SHIPBUILDING, NAVIGATION, ETC.

THE NAVAL ARCHITECT'S AND SHIPBUILDER'S

POCKET-BOOK of Formulæ, Rules, and Tables, and Marine Engineer's and Surveyor's Handy Book of Reference. By CLEMENT MACKROW, M.I.N.A. Eighth Edition, Carefully Revised and Enlarged. Fcap., leather. *Net* **12/6**

SUMMARY OF CONTENTS:—SIGNS AND SYMBOLS, DECIMAL FRACTIONS.—TRIGONOMETRY.—PRACTICAL GEOMETRY.—MENSURATION.—CENTRES AND MOMENTS OF FIGURES.—MOMENTS OF INERTIA AND RADII OF GYRATION.—ALGEBRAICAL EXPRESSIONS FOR SIMPSON'S RULES.—MECHANICAL PRINCIPLES.—CENTRE OF GRAVITY.—LAWS OF MOTION.—DISPLACEMENT, CENTRE OF BUOYANCY.—CENTRE OF GRAVITY OF SHIP'S HULL.—STABILITY CURVES AND METACENTRES.—SEA AND SHALLOW-WATER WAVES.—ROLLING OF SHIPS.—PROPULSION AND RESISTANCE OF VESSELS.—SPEED TRIALS.—SAILING CENTRE OF EFFORT.—DISTANCES DOWN RIVERS, COAST LINES.—STEERING AND RUDDERS OF VESSELS.—LAUNCHING CALCULATIONS AND VELOCITIES.—WEIGHT OF MATERIAL AND GEAR.—GUN PARTICULARS AND WEIGHT.—STANDARD GAUGES.—RIVETED JOINTS AND RIVETING.—STRENGTH AND TESTS OF MATERIALS.—BINDING AND SHEARING STRESSES, &c.—STRENGTH OF SHAFTING, PILLARS, WHEELS, &c.—HYDRAULIC DATA, &c.—CONIC SECTIONS, CATENARIAN CURVES.—MECHANICAL POWERS, WORK.—BOARD OF TRADE REGULATIONS FOR BOILERS AND ENGINES.—BOARD OF TRADE REGULATIONS FOR SHIPS.—LLOYD'S RULES FOR BOILERS.—LLOYD'S WEIGHT OF CHAINS.—LLOYD'S SCANTLINGS FOR SHIPS.—DATA OF ENGINES AND VESSELS.—SHIPS' FITTINGS AND TESTS.—SEASONING PRESERVING TIMBER.—MEASUREMENT OF TIMBER.—ALLOYS, PAINTS, VARNISHES.—DATA FOR STOWAGE.—ADMIRALTY TRANSPORT REGULATIONS.—RULES FOR HORSE-POWER, SCREW PROPELLERS, &c.—PERCENTAGES FOR BUTT STRAPS, &c.—PARTICULARS OF YACHTS.—MASTING AND RIGGING VESSELS.—DISTANCES OF FOREIGN PORTS.—TONNAGE TABLES.—VOCABULARY OF FRENCH AND ENGLISH TERMS.—ENGLISH WEIGHTS AND MEASURES.—FOREIGN WEIGHTS AND MEASURES.—DECIMAL EQUIVALENTS.—FOREIGN MONEY.—DISCOUNT AND WAGES TABLES.—USEFUL NUMBERS AND READY RECKONERS.—TABLES OF CIRCULAR MEASURES.—TABLES OF AREAS OF AND CIRCUMFERENCES OF CIRCLES.—TABLES OF AREAS OF SEGMENTS OF CIRCLES.—TABLES OF SQUARES AND CUBES AND ROOTS OF NUMBERS.—TABLES OF LOGARITHMS OF NUMBERS.—TABLES OF HYPERBOLIC LOGARITHMS.—TABLES OF NATURAL SINES, TANGENTS, &c.—TABLES OF LOGARITHMIC SINES, TANGENTS, &c.

"In these days of advanced knowledge a work like this is of the greatest value. It contains a vast amount of information. We unhesitatingly say that it is the most valuable compilation for its specific purpose that has ever been printed. No naval architect, engineer, surveyor, seaman, wood or iron shipbuilder, can afford to be without this work."—*Nautical Magazine.*

"Should be used by all who are engaged in the construction or design of vessels. . . . Will be found to contain the most useful tables and formulæ required by shipbuilders, carefully collected from the best authorities, and put together in a popular and simple form. The book is one of exceptional merit."—*Engineer.*

"The professional shipbuilder has now, in a convenient and accessible form, reliable data for solving many of the numerous problems that present themselves in the course of his work."—*Iron.*

"There is no doubt that a pocket-book of this description must be a necessity in the shipbuilding trade. . . The volume contains a mass of useful information clearly expressed and presented in a handy form."—*Marine Engineer.*

WANNAN'S MARINE ENGINEER'S GUIDE

To Board of Trade Examinations for Certificates of Competency. Containing all Latest Questions to Date, with Simple, Clear, and Correct Solutions; 302 Elementary Questions with Illustrated Answers, and Verbal Questions and Answers; complete Set of Drawings with Statements completed. By A. C. WANNAN, C.E., Consulting Engineer, and E. W. I. WANNAN, M.I.M.E., Certificated First Class Marine Engineer. With numerous Engravings. Third Edition, Enlarged. 500 pages. Large crown 8vo, cloth . . *Net* **10/6**

"The book is clearly and plainly written and avoids unnecessary explanations and formulas, and we consider it a valuable book for students of marine engineering."—*Nautical Magazine.*

WANNAN'S MARINE ENGINEER'S POCKET-BOOK.

Containing Latest Board of Trade Rules and Data for Marine Engineers. By A. C. WANNAN. Third Edition, Revised, Enlarged, and Brought up to Date. Square 18mo. with thumb Index, leather. [*Just Published.* **5/0**

"There is a great deal of useful information in this little pocket-book. It is of the rule-of thumb order, and is, on that account, well adapted to the uses of the sea-going engineer."—*Engineer*

THE SHIPBUILDING INDUSTRY OF GERMANY.

Compiled and Edited by G. LEHMANN-FELSKOWSKI. With Coloured Prints Art Supplements, and numerous Illustrations throughout the text. Super royal 4to cloth. [*Just Published. Net* **10/6**

SEA TERMS, PHRASES, AND WORDS

(Technical Dictionary of) used in the English and French Languages (English-French, French-English). For the Use of Seamen, Engineers, Pilots, Shipbuilders, Shipowners, and Ship-brokers. Compiled by W. PIRRIE, late of the African Steamship Company. Fcap. 8vo, cloth limp **5/0**

"This volume will be highly appreciated by seamen, engineers, pilots, shipbuilders and shipowners. It will be found wonderfully accurate and complete."—*Scotsman.*

"A very useful dictionary, which has long been wanted by French and English engineers, masters, officers and others."—*Shipping World.*

ELECTRIC SHIP-LIGHTING.

A Handbook on the Practical Fitting and Running of Ships' Electrical Plant, for the Use of Shipowners and Builders, Marine Electricians and Sea-going Engineers in Charge. By J. W. URQUHART, Author of "Electric Light," "Dynamo Construction," &c. Second Edition, Revised and Extended. With numerous Illustrations. Crown 8vo, cloth **7/6**

MARINE ENGINEER'S POCKET-BOOK.

Consisting of useful Tables and Formulæ. By FRANK PROCTOR, A.I.N.A. Third Edition. Royal 32mo, leather. **4/0**

"We recommend it to our readers as going far to supply a long-felt want."—*Naval Science.*

"A most useful companion to all marine engineers."—*United Service Gazette.*

ELEMENTARY MARINE ENGINEERING.

A Manual for Young Marine Engineers and Apprentices. In the Form of Questions and Answers on Metals, Alloys, Strength of Materials, Construction and Management of Marine Engines and Boilers, Geometry, &c. With an Appendix of Useful Tables. By J. S. BREWER. Crown 8vo, cloth **1/6**

"Contains much valuable information for the class for whom it is intended, especially in the chapters on the management of boilers and engines."—*Nautical Magazine.*

MARINE ENGINES AND STEAM VESSELS.

A Treatise on. By ROBERT MURRAY, C.E. Eighth Edition, thoroughly Revised, with considerable Additions by the Author and by GEORGE CARLISLE, C.E., Senior Surveyor to the Board of Trade at Liverpool. Crown 8vo, cloth **4/6**

PRACTICAL NAVIGATION.

Consisting of THE SAILOR'S SEA-BOOK, by JAMES GREENWOOD and W. H. ROSSER; together with the requisite Mathematical and Nautical Tables for the Working of the Problems, by HENRY LAW, C.E., and Professor J. R. YOUNG. Illustrated. 12mo, strongly half-bound **7/0**

THE ART AND SCIENCE OF SAILMAKING.

By SAMUEL B. SADLER, Practical Sailmaker, late in the employment of Messrs. Ratsey and Lapthorne, of Cowes and Gosport. With Plates and other Illustrations. Small 4to, cloth **12/6**

"This extremely practical work gives a complete education in all the branches of the manufacture, cutting out, roping, seaming, and goring. It is copiously illustrated, and will form a first-rate text-book and guide."—*Portsmouth Times.*

CHAIN CABLES AND CHAINS.

Comprising Sizes and Curves of Links, Studs, &c., Iron for Cables and Chains, Chain Cable and Chain Making, Forming and Welding Links, Strength of Cables and Chains, Certificates for Cables, Marking Cables, Prices of Chain Cables and Chains, Historical Notes, Acts of Parliament, Statutory Tests, Charges for Testing, List of Manufacturers of Cables, &c., &c. By THOMAS W. TRAILL, F.E.R.N., M.Inst.C.E., Engineer-Surveyor-in-Chief, Board of Trade, Inspector of Chain Cable and Anchor Proving Establishments, and General Superintendent Lloyd's Committee on Proving Establishments. With numerous Tables, Illustrations, and Lithographic Drawings. Folio, cloth, bevelled boards. . . . . **£2 2s.**

"It contains a vast amount of valuable information. Nothing seems to be wanting to make it a complete and standard work of reference on the subject."—*Nautical Magazine.*

MINING, METALLURGY, AND COLLIERY WORKING.

THE OIL FIELDS OF RUSSIA AND THE RUSSIAN PETROLEUM INDUSTRY.

A Practical Handbook on the Exploration, Exploitation, and Management of Russian Oil Properties, including Notes on the Origin of Petroleum in Russia, a Description of the Theory and Practice of Liquid Fuel, and a Translation of the Rules and Regulations concerning Russian Oil Properties. By A. Beeby Thompson, A.M.I.M.E., late Chief Engineer and Manager of the European Petroleum Company's Russian Oil Properties. About 500 pp. With numerous Illustrations and Photographic Plates, and a Map of the Balakhany-Saboontchy-Romany Oil Field. Super-royal 8vo, cloth. [*Just Published. Net* **£3 3s.**

MACHINERY FOR METALLIFEROUS MINES.

A Practical Treatise for Mining Engineers, Metallurgists, and Managers of Mines. By E. Henry Davies, M.E., F.G.S. 600 pp. With Folding Plates and other Illustrations. Medium 8vo, cloth . . . . . *Net* **25/0**

"Deals exhaustively with the many and complex details which go to make up the sum total of machinery and other requirements for the successful working of metalliferous mines, and as a book of ready reference is of the highest value to mine managers and directors."—*Mining Journal.*

THE DEEP LEVEL MINES OF THE RAND,

And their Future Development, considered from the Commercial Point of View. By G. A. Denny (of Johannesburg), M.N.E.I.M.E., Consulting Engineer to the General Mining and Finance Corporation, Ltd., of London, Berlin, Paris, and Johannesburg. Fully Illustrated with Diagrams and Folding Plates. Royal 8vo, buckram . . . . *Net* **25/0**

"Mr. Denny by confining himself to the consideration of the future of the deep-level mines of the Rand breaks new ground, and by dealing with the subject rather from a commercial standpoint than from a scientific one, appeals to a wide circle of readers. The book cannot fail to prove of very great value to investors in South African mines."—*Mining Journal.*

PROSPECTING FOR GOLD.

A Handbook of Information and Hints for Prospectors based on Personal Experience. By Daniel J. Rankin, F.R.S.G.S., M.R.A.S., formerly Manager of the Central African Company, and Leader of African Gold Prospecting Expeditions. With Illustrations specially Drawn and Engraved for the Work F'cap. 8vo, leather . . . . *Net* **7/6**

"This well-compiled book contains a collection of the richest gems of useful knowledge for the prospector's benefit. A special table is given to accelerate the spotting at a glance of minerals associated with gold."—*Mining Journal.*

THE METALLURGY OF GOLD.

A Practical Treatise on the Metallurgical Treatment of Gold-bearing Ores. Including the Assaying, Melting, and Refining of Gold. By M. Eissler, M. Inst. M.M. Fifth Edition, Enlarged. With over 300 Illustrations and numerous Folding Plates. Medium 8vo, cloth . . . . . *Net* **21/0**

"This book thoroughly deserves its title of a 'Practical Treatise.' The whole process of gold mining, from the breaking of the quartz to the assay of the bullion, is described in clear and orderly narrative and with much, but not too much, fulness of detail."—*Saturday Review.*

THE CYANIDE PROCESS OF GOLD EXTRACTION.

And its Practical Application on the Witwatersrand Gold Fields and elsewhere. By M. Eissler, M. Inst. M.M. With Diagrams and Working Drawings. Third Edition, Revised and Enlarged. 8vo, cloth . . . . . *Net* **7/6**

"This book is just what was needed to acquaint mining men with the actual working of a process which is not only the most popular, but is, as a general rule, the most successful for the extraction of gold from tailings."—*Mining Journal.*

DIAMOND DRILLING FOR GOLD & OTHER MINERALS.

A Practical Handbook on the Use of Modern Diamond Core Drills in Prospecting and Exploiting Mineral-Bearing Properties, including Particulars of the Costs of Apparatus and Working. By G. A. Denny, M.N.E. Inst. M.E., M. Inst. M.M. Medium 8vo, 168 pp., with Illustrative Diagrams . **12/6**

"There is certainly scope for a work on diamond drilling, and Mr. Denny deserves grateful recognition for supplying a decided want."—*Mining Journal.*

GOLD ASSAYING.

A Practical Handbook for Assayers, Bankers, Chemists, Bullion Smelters, Goldsmiths, Mining and Metallurgical Engineers, Prospectors, Students, and others. By H. JOSHUA PHILLIPS, F.I.C., F.C.S., A.I.C.E., Author of "Engineering Chemistry," etc. Large Crown 8vo, cloth.

[Just ready, price about **7/6** *net.*

FIELD TESTING FOR GOLD AND SILVER.

A Practical Manual for Prospectors and Miners. By W. H. MERRITT, M.N.E. Inst. M.E., A.R.S.M., &c. With Photographic Plates and other Illustrations. Fcap. 8vo, leather . . . . *Net* **5/0**

"As an instructor of prospectors' classes Mr. Merritt has the advantage of knowing exactly the information likely to be most valuable to the miner in the field. The contents cover all the details of sampling and testing gold and silver ores. A useful addition to a prospector's kit."—*Mining Journal.*

THE PROSPECTOR'S HANDBOOK.

A Guide for the Prospector and Traveller in search of Metal-Bearing or other Valuable Minerals. By J. W. ANDERSON, M.A. (Camb.), F.R.G.S. Ninth Edition. Small crown 8vo, **3/6** cloth; or, leather **4/6**

"Will supply a much-felt want, especially among Colonists, in whose way are so often thrown many mineralogical specimens the value of which it is difficult to determine."—*Engineer.*

"How to find commercial minerals, and how to identify them when they are found, are the leading points to which attention is directed. The author has managed to pack as much practical detail into his pages as would supply material for a book three times its size."—*Mining Journal.*

THE METALLURGY OF SILVER.

A Practical Treatise on the Amalgamation, Roasting, and Lixiviation of Silver Ores. Including the Assaying, Melting, and Refining of Silver Bullion. By M. EISSLER, M. Inst. M.M. Third Edition. Crown 8vo, cloth . **10/6**

"A practical treatise, and a technical work which we are convinced will supply a long-felt want amongst practical men, and at the same time be of value to students and others indirectly connected with the industries."—*Mining Journal.*

THE HYDRO-METALLURGY OF COPPER.

Being an Account of Processes Adopted in the Hydro-Metallurgical Treatment of Cupriferous Ores, Including the Manufacture of Copper Vitriol, with Chapters on the Sources of Supply of Copper and the Roasting of Copper Ores. By M. EISSLER, M. Inst. M.M. 8vo, cloth . . . . . *Net* **12/9**

"In this volume the various processes for the extraction of copper by wet methods are fully detailed. Costs are given when available, and a great deal of useful information about the copper industry of the world is presented in an interesting and attractive manner."—*Mining Journal.*

THE METALLURGY OF ARGENTIFEROUS LEAD.

A Practical Treatise on the Smelting of Silver-Lead Ores and the Refining of Lead Bullion. Including Reports on various Smelting Establishments and Descriptions of Modern Smelting Furnaces and Plants in Europe and America. By M. EISSLER, M. Inst. M.M., Author of "The Metallurgy of Gold," &c. Crown 8vo, 400 pp., with 183 Illustrations, cloth **12/6**

"The numerous metallurgical processes, which are fully and extensively treated of, embrace all the stages experienced in the passage of the lead from the various natural states to its issue from the refinery as an article of commerce."—*Practical Engineer.*

METALLIFEROUS MINERALS AND MINING.

By D. C. DAVIES, F.G.S. Sixth Edition, thoroughly Revised and much Enlarged by his Son, E. HENRY DAVIES, M.E., F.G.S. 600 pp., with 173 Illustrations. Large crown 8vo, cloth . . . . . . *Net* **12/6**

"Neither the practical miner nor the general reader, interested in mines, can have a better book for his companion and his guide."—*Mining Journal.*

EARTHY AND OTHER MINERALS AND MINING.

By D. C. DAVIES, F.G.S., Author of "Metalliferous Minerals," &c. Third Edition, Revised and Enlarged by his Son, E. HENRY DAVIES, M.E., F.G.S. With about 100 Illustrations. Crown 8vo, cloth **12/6**

"We do not remember to have met with any English work on mining matters that contains the same amount of information packed in equally convenient form."—*Academy.*

BRITISH MINING.

A Treatise on the History, Discovery, Practical Development, and Future Prospects of Metalliferous Mines in the United Kingdom. By ROBERT HUNT, F.R.S., late Keeper of Mining Records. Upwards of 950 pp., with 230 Illustrations. Second Edition, Revised. Super-royal 8vo, cloth **£2 2s.**

POCKET-BOOK FOR MINERS AND METALLURGISTS.

Comprising Rules, Formulæ, Tables, and Notes for Use in Field and Office Work. By F. Danvers Power, F.G.S., M.E. Second Edition, Corrected. Fcap. 8vo, leather **9/0**

"This excellent book is an admirable example of its kind, and ought to find a large sale amongst English-speaking prospectors and mining engineers."—*Engineering.*

THE MINER'S HANDBOOK.

A Handy Book of Reference on the subjects of Mineral Deposits, Mining Operations, Ore Dressing, &c. For the Use of Students and others interested in Mining Matters. Compiled by John Milne, F.R.S., Professor of Mining in the Imperial University of Japan. Third Edition. Fcap. 8vo, leather **7/6**

"Professor Milne's handbook is sure to be received with favour by all connected with mining, and will be extremely popular among students."—*Athenæum.*

IRON ORES of GREAT BRITAIN and IRELAND.

Their Mode of Occurrence, Age and Origin, and the Methods of Searching for and Working Them. With a Notice of some of the Iron Ores of Spain. By J. D. Kendall, F.G.S., Mining Engineer. Crown 8vo, cloth . . **16/0**

MINE DRAINAGE.

A Complete Practical Treatise on Direct-Acting Underground Steam Pumping Machinery. By Stephen Michell. Second Edition, Re-written and Enlarged. With 250 Illustrations. Royal 8vo, cloth . *Net* **25/0**

HORIZONTAL PUMPING ENGINES.—ROTARY AND NON-ROTARY HORIZONTAL ENGINES.—SIMPLE AND COMPOUND STEAM PUMPS.—VERTICAL PUMPING ENGINES.—ROTARY AND NON-ROTARY VERTICAL ENGINES.—SIMPLE AND COMPOUND STEAM PUMPS.—TRIPLE-EXPANSION STEAM PUMPS.—PULSATING STEAM PUMPS.—PUMP VALVES.—SINKING PUMPS, &c., &c.

"This volume contains an immense amount of important and interesting new matter. The book should undoubtedly prove of great use to all who wish for information on the subject."—*The Engineer.*

ELECTRICITY AS APPLIED TO MINING.

By Arnold Lupton, M.Inst.C.E., M.I.M.E., M.I.E.E., late Professor of Coal Mining at the Yorkshire College, Victoria University, Mining Engineer and Colliery Manager; G. D. Aspinall Parr, M.I.E.E., A.M.I.M.E., Associate of the Central Technical College, City and Guilds of London, Head of the Electrical Engineering Department, Yorkshire College, Victoria University; and Herbert Perkin, M.I.M.E., Certified Colliery Manager, Assistant Lecturer in the Mining Department of the Yorkshire College, Victoria University. With about 170 Illustrations. Medium 8vo, cloth. *Net* **9/0**

(For Summary of Contents, see page 23.)

THE COLLIERY MANAGER'S HANDBOOK.

A Comprehensive Treatise on the Laying-out and Working of Collieries, Designed as a Book of Reference for Colliery Managers, and for the Use of Coal Mining Students preparing for First-class Certificates. By Caleb Pamely, Mining Engineer and Surveyor; Member of the North of England Institute of Mining and Mechanical Engineers; and Member of the South Wales Institute of Mining Engineers. With 700 Plans, Diagrams, and other Illustrations. Fourth Edition, Revised and Enlarged. 964 pp. Medium 8vo, cloth **£1 5s.**

GEOLOGY.—SEARCH FOR COAL.—MINERAL LEASES AND OTHER HOLDINGS.—SHAFT SINKING.—FITTING UP THE SHAFT AND SURFACE ARRANGEMENTS.—STEAM BOILERS AND THEIR FITTINGS.—TIMBERING AND WALLING.—NARROW WORK AND METHODS OF WORKING.—UNDERGROUND CONVEYANCE.—DRAINAGE.—THE GASES MET WITH IN MINES; VENTILATION.—ON THE FRICTION OF AIR IN MINES.—THE PRIESTMAN OIL ENGINE; PETROLEUM AND NATURAL GAS.—SURVEYING AND PLANNING.—SAFETY LAMPS AND FIREDAMP DETECTORS.—SUNDRY AND INCIDENTAL OPERATIONS AND APPLIANCES.—COLLIERY EXPLOSIONS.—MISCELLANEOUS QUESTIONS AND ANSWERS.—*Appendix:* SUMMARY OF REPORT OF H.M. COMMISSIONERS ON ACCIDENTS IN MINES.

"Mr. Pamely's work is eminently suited to the purpose for which it is intended, being clear, interesting, exhaustive, rich in detail, and up to date, giving descriptions of the latest machines in every department. A mining engineer could scarcely go wrong who followed this work."—*Colliery Guardian.*

"Mr. Pamely has not only given us a comprehensive reference book of a very high order suitable to the requirements of mining engineers and colliery managers, but has also provided mining students with a class-book that is as interesting as it is instructive."—*Colliery Manager.*

"This is the most complete 'all-round' work on coal-mining published in the English language. . . . No library of coal-mining books is complete without it."—*Colliery Engineer* (Scranton, Pa., U.S.A.).

COLLIERY WORKING AND MANAGEMENT.

Comprising the Duties of a Colliery Manager, the Oversight and Arrangement of Labour and Wages, and the different Systems of Working Coal Seams. By H. F. Bulman and R. A. S. Redmayne. 350 pp., with 28 Plates and other Illustrations, including Underground Photographs. Medium 8vo, cloth. **15/0**

"This is, indeed, an admirable Handbook for Colliery Managers, in fact it is an indispensable adjunct to a Colliery Manager's education, as well as being a most useful and interesting work on the subject for all who in any way have to do with coal mining. The underground photographs are an attractive feature of the work, being very lifelike and necessarily true representations of the scenes they depict."—*Colliery Guardian.*

"Mr. Bulman and Mr. Redmayne, who are both experienced Colliery Managers of great literary ability, are to be congratulated on having supplied an authoritative work dealing with a side of the subject of coal mining which has hitherto received but scant treatment. The authors elucidate their text by 119 woodcuts and 28 plates, most of the latter being admirable reproductions of photographs taken underground with the aid of the magnesium flash-light. These illustrations are excellent."—*Nature.*

COAL AND COAL MINING.

By the late Sir Warington W. Smyth, M.A., F.R.S., Chief Inspector of the Mines of the Crown and of the Duchy of Cornwall. Eighth Edition, Revised and Extended by T. Forster Brown, Mining and Civil Engineer, Chief Inspector of the Mines of the Crown and of the Duchy of Cornwall. Crown 8vo, cloth. **3/6**

"As an outline is given of every known coal-field in this and other countries, as well as of the principal methods of working, the book will doubtless interest a very large number of readers."—*Mining Journal.*

NOTES AND FORMULÆ FOR MINING STUDENTS.

By John Herman Merivale, M.A., Late Professor of Mining in the Durham College of Science, Newcastle-upon-Tyne. Fourth Edition, Revised and Enlarged. By H. F. Bulman, A.M.Inst.C.E. Small crown 8vo, cloth. **2/6**

"The author has done his work in a creditable manner, and has produced a book that will be of service to students and those who are practically engaged in mining operations."—*Engineer.*

INFLAMMABLE GAS AND VAPOUR IN THE AIR

(The Detection and Measurement of). By Frank Clowes, D.Sc., Lond., F.I.C. With a Chapter on The Detection and Measurement of Petroleum Vapour by Boverton Redwood, F.R.S.E., Consulting Adviser to the Corporation of London under the Petroleum Acts. Crown 8vo, cloth. *Net* **5/0**

"Professor Clowes has given us a volume on a subject of much industrial importance . . . Those interested in these matters may be recommended to study this book, which is easy of comprehension and contains many good things."—*The Engineer.*

COAL & IRON INDUSTRIES of the UNITED KINGDOM.

Comprising a Description of the Coal Fields, and of the Principal Seams of Coal, with Returns of their Produce and its Distribution, and Analyses of Special Varieties. Also, an Account of the Occurrence of Iron Ores in Veins or Seams; Analyses of each Variety; and a History of the Rise and Progress of Pig Iron Manufacture. By Richard Meade. 8vo, cloth . . **£1 8s.**

"Of this book we may unreservedly say that it is the best of its class which we have ever met. . . . A book of reference which no one engaged in the iron or coal trades should omit from his library."—*Iron and Coal Trades Review.*

ASBESTOS AND ASBESTIC.

Their Properties, Occurrence, and Use. By Robert H. Jones, F.S.A., Mineralogist, Hon. Mem. Asbestos Club, Black Lake, Canada. With Ten Collotype Plates and other Illustrations. Demy 8vo, cloth. . **16/0**

"An interesting and invaluable work."—*Colliery Guardian.*

GRANITES AND OUR GRANITE INDUSTRIES.

By George F. Harris, F.G.S. With Illustrations. Crown 8vo, cloth **2/6**

TRAVERSE TABLES.

For use in Mine Surveying. By William Lintern, C.E. With two plates. Small crown 8vo, cloth . . . . *Net* **3/0**

ELECTRICITY, ELECTRICAL ENGINEERING, ETC.

THE ELEMENTS OF ELECTRICAL ENGINEERING.

A First Year's Course for Students. By TYSON SEWELL, A.I.E.E., Assistant Lecturer and Demonstrator in Electrical Engineering at the Polytechnic, Regent Street, London. Second Edition, Revised, with Additional Chapters on Alternating Current Working, and Appendix of Questions and Answers. 450 pages, with 274 Illustrations. Demy 8vo, cloth. [*Just Published. Net* **7/6**

OHM'S LAW.—UNITS EMPLOYED IN ELECTRICAL ENGINEERING.—SERIES AND PARALLEL CIRCUITS; CURRENT DENSITY AND POTENTIAL DROP IN THE CIRCUIT.—THE HEATING EFFECT OF THE ELECTRIC CURRENT.—THE MAGNETIC EFFECT OF AN ELECTRIC CURRENT.—THE MAGNETISATION OF IRON.—ELECTRO-CHEMISTRY; PRIMARY BATTERIES.—ACCUMULATORS.—INDICATING INSTRUMENTS; AMMETERS, VOLTMETERS, OHMMETERS.—ELECTRICITY SUPPLY METERS.—MEASURING INSTRUMENTS, AND THE MEASUREMENT OF ELECTRICAL RESISTANCE.—MEASUREMENT OF POTENTIAL DIFFERENCE, CAPACITY, CURRENT STRENGTH, AND PERMEABILITY.—ARC LAMPS.—INCANDESCENT LAMPS; MANUFACTURE AND INSTALLATION; PHOTOMETRY.—THE CONTINUOUS CURRENT DYNAMO.—DIRECT CURRENT MOTORS.—ALTERNATING CURRENTS.—TRANSFORMERS, ALTERNATORS, SYNCHRONOUS MOTORS.—POLYPHASE WORKING.—APPENDIX OF QUESTIONS AND ANSWERS.

"An excellent treatise for students of the elementary facts connected with electrical engineering."—*The Electrician.*

"One of the best books for those commencing the study of electrical engineering. Everything is explained in simple language which even a beginner cannot fail to understand."—*Engineer.*

"One welcomes this book, which is sound in its treatment, and admirably calculated to give students the knowledge and information they most require."—*Nature.*

CONDUCTORS FOR ELECTRICAL DISTRIBUTION.

Their Materials and Manufacture, The Calculation of Circuits, Pole-Line Construction, Underground Working, and other Uses. By F. A. C. PERRINE, A.M., D.Sc.; formerly Professor of Electrical Engineering, Leland Stanford, Jr., University; M.Amer.I.E.E. 8vo, cloth. [*Just Published. Net* **20/-**

CONDUCTOR MATERIALS—ALLOYED CONDUCTORS—MANUFACTURE OF WIRE—WIRE-FINISHING—WIRE INSULATION—CABLES—CALCULATION OF CIRCUITS—KELVIN'S LAW OF ECONOMY IN CONDUCTORS—MULTIPLE ARC DISTRIBUTION—ALTERNATING CURRENT CALCULATION—OVERHEAD LINES—POLE LINE—LINE INSULATORS—UNDERGROUND CONDUCTORS.

WIRELESS TELEGRAPHY;

Its Origins, Development, Inventions, and Apparatus. By CHARLES HENRY SEWALL. With 85 Diagrams and Illustrations. Demy 8vo, cloth. [*Just Published. Net* **10/6**

ARMATURE WINDINGS OF DIRECT CURRENT DYNAMOS.

Extension and Application of a General Winding Rule. By E. ARNOLD, Engineer. Assistant Professor in Electrotechnics and Machine Design at the Riga Polytechnic School. Translated from the Original German by FRANCIS B. DE GRESS, M.E., Chief of Testing Department, Crocker-Wheeler Company. With 146 Illustrations. Medium 8vo, cloth . . *Net* **12/-**

ELECTRICITY AS APPLIED TO MINING.

By ARNOLD LUPTON, M.Inst.C.E., M.I.M.E., M.I.E.E., late Professor of Coal Mining at the Yorkshire College, Victoria University, Mining Engineer and Colliery Manager; G. D. ASPINALL PARR, M.I.E.E., A.M.I.M.E., Associate of the Central Technical College, City and Guilds of London, Head of the Electrical Engineering Department, Yorkshire College, Victoria University; and HERBERT PERKIN, M.I.M.E. Certificated Colliery Manager, Assistant Lecturer in the Mining Department of the Yorkshire College, Victoria University. With about 170 Illustrations. Medium 8vo, cloth. *Net* **9/-**

INTRODUCTORY.—DYNAMIC ELECTRICITY.—DRIVING OF THE DYNAMO.—THE STEAM TURBINE.—DISTRIBUTION OF ELECTRICAL ENERGY.—STARTING AND STOPPING ELECTRICAL GENERATORS AND MOTORS.—ELECTRIC CABLES.—CENTRAL ELECTRICAL PLANTS.—ELECTRICITY APPLIED TO PUMPING AND HAULING.—ELECTRICITY APPLIED TO COAL-CUTTING.—TYPICAL ELECTRIC PLANTS RECENTLY ERECTED.—ELECTRIC LIGHTING BY ARC AND GLOW LAMPS—MISCELLANEOUS APPLICATIONS OF ELECTRICITY—ELECTRICITY AS COMPARED WITH OTHER MODES OF TRANSMITTING POWER.—DANGERS OF ELECTRICITY.

DYNAMO, MOTOR AND SWITCHBOARD CIRCUITS FOR ELECTRICAL ENGINEERS.

A Practical Handbook dealing with Direct, Alternating and Polyphase Currents. By WILLIAM R. BOWKER, C.E., M.E., E.E., Lecturer on Physics and Electrical Engineering at the Municipal Technical School, Bury. 8vo, cloth. [*Just ready, price about* **6/0** *net*

DYNAMO ELECTRIC MACHINERY: its CONSTRUCTION, DESIGN, and OPERATION.

By SAMUEL SHELDON, A M., Ph.D , Professor of Physics and Electrical Engineering at the Polytechnic Institute of Brooklyn, assisted by HOBART MASON, B.S.

In two volumes, sold separately, as follows:—

Vol. I.—DIRECT CURRENT MACHINES. Third Edition, Revised. Large crown 8vo. 280 pages, with 200 Illustrations . . . *Net* **12/0**

Vol. II.—ALTERNATING CURRENT MACHINES. Large crown 8vo. 260 pages, with 184 Illustrations . . . . *Net* **12/0**

Designed as Text-books for use in Technical Educational Institutions, and by Engineers whose work includes the handling of Direct and Alternating Current Machines respectively, and for Students proficient in mathematics.

ELECTRICAL AND MAGNETIC CALCULATIONS.

For the Use of Electrical Engineers and Artisans, Teachers, Students, and all others interested in the Theory and Application of Electricity and Magnetism. By A. A. ATKINSON, Professor of Electricity in Ohio University. Crown 8vo, cloth . . . . *Net* **9/0**

"To teachers and those who already possess a fair knowledge of their subject we can recommend this book as being useful to consult when requiring data or formulæ which it is neither convenient nor necessary to retain by memory."—*The Electrician.*

SUBMARINE TELEGRAPHS.

Their History, Construction, and Working. Founded in part on WÜNSCHENDORFF'S "Traité de Télégraphie Sous-Marine," and Compiled from Authoritative and Exclusive Sources. By CHARLES BRIGHT, F.R.S.E., A.M.Inst.C.E., M.I.E.E. 780 pp., fully Illustrated, including Maps and Folding Plates. Royal 8vo, cloth . . . . *Net* **£3 3s.**

"There are few, if any, persons more fitted to write a treatise on submarine telegraphy than Mr. Charles Bright. He has done his work admirably, and has written in a way which will appeal as much to the layman as to the engineer. This admirable volume must, for many years to come, hold the position of the English classic on submarine telegraphy."—*Engineer.*

"This book is full of information. It makes a book of reference which should be in every engineer's library."—*Nature.*

"Mr. Bright's interestingly written and admirably illustrated book will meet with a welcome reception from cable men."—*Electrician.*

"The author deals with his subject from all points of view—political and strategical as well as scientific. The work will be of interest, not only to men of science, but to the general public. We can strongly recommend it."—*Athenæum.*

THE ELECTRICAL ENGINEER'S POCKET-BOOK.

Consisting of Modern Rules, Formulæ, Tables, and Data. By H. R. KEMPE, M.I.E.E., A.M.Inst.C.E., Technical Officer Postal Telegraphs, Author of "A Handbook of Electrical Testing," &c. Second Edition, thoroughly Revised, with Additions. With numerous Illustrations. Royal 32mo, oblong, leather **5/0**

"It is the best book of its kind."—*Electrical Engineer.*
"The Electrical Engineer's Pocket-Book is a good one."—*Electrician.*
"Strongly recommended to those engaged in the electrical industries."—*Electrical Review.*

POWER TRANSMITTED BY ELECTRICITY.

And applied by the Electric Motor, including Electric Railway Construction. By P. ATKINSON, A.M., Ph.D. Third Edition, Fully Revised, and New Matter added. With 94 Illustrations. Crown 8vo, cloth . . *Net* **9/0**

DYNAMIC ELECTRICITY AND MAGNETISM.

By PHILIP ATKINSON, A.M., Ph.D., Author of "Elements of Static Electricity," &c. Crown 8vo, 417 pp., with 120 Illustrations, cloth . **10/6**

THE MANAGEMENT OF DYNAMOS.

A Handybook of Theory and Practice for the Use of Mechanics, Engineers, Students, and others in Charge of Dynamos. By G. W. LUMMIS-PATERSON. Third Edition, Revised. Crown 8vo, cloth **4/6**

"An example which deserves to be taken as a model by other authors. The subject is treated in a manner which any intelligent man who is fit to be entrusted with charge of an engine should be able to understand. It is a useful book to all who make, tend, or employ electric machinery." —*Architect.*

THE STANDARD ELECTRICAL DICTIONARY.

A Popular Encyclopædia of Words and Terms Used in the Practice of Electrical Engineering. Containing upwards of 3,000 definitions, By T. O'CONOR SLOANE, A.M., Ph.D. Third Edition, with Appendix. Crown 8vo, 690 pp., 390 Illustrations, cloth . . . . *Net* **7/6**

"The work has many attractive features in it, and is, beyond doubt, a well put together and useful publication. The amount of ground covered may be gathered from the fact that in the index about 5,000 references will be found."—*Electrical Review.*

ELECTRIC LIGHT FITTING.

A Handbook for Working Electrical Engineers, embodying Practical Notes on Installation Management. By J. W. URQUHART, Electrician, Author of "Electric Light," &c. With numerous Illustrations. Third Edition, Revised, with Additions. Crown 8vo, cloth **5/0**

"This volume deals with the mechanics of electric lighting, and is addressed to men who are already engaged in the work, or are training for it. The work traverses a great deal of ground, and may be read as a sequel to the author's useful work on 'Electric Light.'"—*Electrician.*

"The book is well worth the perusal of the workman, for whom it is written."—*Electrica Review.*

ELECTRIC LIGHT.

Its Production and Use, Embodying Plain Directions for the Treatment of Dynamo-Electric Machines, Batteries, Accumulators, and Electric Lamps. By J. W. URQUHART, C.E. Sixth Edition, Enlarged. Crown 8vo, cloth. **7/6**

"The whole ground of electric lighting is more or less covered and explained in a very clear and concise manner."—*Electrical Review.*

"A *vade-mecum* of the salient facts connected with the science of electric lighting."—*Electrician.*

DYNAMO CONSTRUCTION.

A Practical Handbook for the Use of Engineer-Constructors and Electricians-in-Charge. Embracing Framework Building, Field Magnet and Armature Winding and Grouping, Compounding, &c. By J. W. URQUHART. Second Edition, Enlarged, with 114 Illustrations. Crown 8vo, cloth . . **7/6**

"Mr. Urquhart's book is the first one which deals with these matters in such a way that the engineering student can understand them. The book is very readable, and the author leads his readers up to difficult subjects by reasonably simple tests."—*Engineering Review.*

ELECTRIC SHIP-LIGHTING.

A Handbook on the Practical Fitting and Running of Ships' Electrical Plant. For the Use of Shipowners and Builders, Marine Electricians, and Seagoing Engineers-in-Charge. By J. W. URQUHART, C.E. Second Edition, Revised and Extended. With 88 Illustrations, Crown 8vo, cloth **7/6**

"The subject of ship electric lighting is one of vast importance and Mr. Urquhart is to be highly complimented for placing such a valuable work at the service of marine electricians."—*The Steamship.*

ELECTRIC LIGHTING (ELEMENTARY PRINCIPLES OF).

By ALAN A. CAMPBELL SWINTON, M.Inst.C.E., M.I.E.E. Fifth Edition. With 16 Illustrations. Crown 8vo, cloth **1/6**

ELECTRIC LIGHT FOR COUNTRY HOUSES.

A Practical Handbook on the Erection and Running of Small Installations, with Particulars of the Cost of Plant and Working. By J. H. KNIGHT. Third Edition, Revised. Crown 8vo, wrapper **1/0**

HOW TO MAKE A DYNAMO.

A Practical Treatise for Amateurs. Containing Illustrations and Detailed Instructions for Constructing a Small Dynamo to Produce the Electric Light. By ALFRED CROFTS. Sixth Edition, Revised. Crown 8vo, cloth . **2/0**

THE STUDENT'S TEXT-BOOK OF ELECTRICITY.

By H. M. NOAD, F.R.S. 650 pp., with 470 Illustrations. Crown 8vo, cloth. **9/0**

ARCHITECTURE, BUILDING, ETC.

PRACTICAL BUILDING CONSTRUCTION.

A Handbook for Students Preparing for Examinations, and a Book of Reference for Persons Engaged in Building. By John Parnell Allen, Surveyor, Lecturer on Building Construction at the Durham College of Science, Newcastle-on-Tyne. Fourth Edition, Revised and Enlarged. Medium 8vo, 570 pp., with 1,000 Illustrations, cloth.
[*Just Published. Net* **7/6**

"The most complete exposition of building construction we have seen. It contains all that is necessary to prepare students for the various examinations in building construction."—*Building News.*

"The author depends nearly as much on his diagrams as on his type. The pages suggest the hand of a man of experience in building operations—and the volume must be a blessing to many teachers as well as to students."—*The Architect.*

"The work is sure to prove a formidable rival to great and small competitors alike, and bids fair to take a permanent place as a favourite student's text-book. The large number of illustrations deserve particular mention for the great merit they possess for purposes of reference in exactly corresponding to convenient scales."—*Journal of the Royal Institute of British Architects.*

PRACTICAL MASONRY.

A Guide to the Art of Stone Cutting. Comprising the Construction, Setting Out, and Working of Stairs, Circular Work, Arches, Niches, Domes, Pendentives, Vaults, Tracery Windows, &c.; to which are added Supplements relating to Masonry Estimating and Quantity Surveying, and to Building Stones, and a Glossary of Terms. For the Use of Students, Masons, and other Workmen. By William R. Purchase, Building Inspector to the Borough of Hove. Fourth Edition, Enlarged. Royal 8vo, 210 pp., with 52 Lithographic Plates, comprising over 400 Diagrams, cloth.
[*Just Published. Net* **7/6**

"Mr. Purchase's 'Practical Masonry' will undoubtedly be found useful to all interested in this important subject, whether theoretically or practically. Most of the examples given are from actual work carried out, the diagrams being carefully drawn. The book is a practical treatise on the subject, the author himself having commenced as an operative mason, and afterwards acted as foreman mason on many large and important buildings prior to the attainment of his present position. It should be found of general utility to architectural students and others, as well as to those to whom it is specially addressed."—*Journal of the Royal Institute of British Architects.*

MODERN PLUMBING, STEAM AND HOT WATER HEATING.

A New Practical Work for the Plumber, the Heating Engineer, the Architect, and the Builder. By J. J. Lawler, Author of "American Sanitary Plumbing," &c. With 284 Illustrations and Folding Plates. 4to, cloth . *Net* **21/-**

HEATING BY HOT WATER,

VENTILATION AND HOT WATER SUPPLY.

By Walter Jones, M.I.M.E. 340 pages, with 140 Illustrations. Royal 8vo, cloth. [*Just Published. Net* **6/0**

CONCRETE: ITS NATURE AND USES.

A Book for Architects, Builders, Contractors, and Clerks of Works. By George L. Sutcliffe, A.R.I.B.A. 350 pp., with Illustrations. Crown 8vo, cloth **7/6**

"The author treats a difficult subject in a lucid manner. The manual fills a long-felt gap. It is careful and exhaustive; equally useful as a student's guide and an architect's book of reference."—*Journal of the Royal Institute of British Architects.*

LOCKWOOD'S BUILDER'S PRICE BOOK for 1904.

A Comprehensive Handbook of the Latest Prices and Data for Builders, Architects, Engineers, and Contractors. Re-constructed, Re-written, and Greatly Enlarged. By Francis T. W. Miller. 800 closely-printed pages, crown 8vo, cloth **4/0**

"This book is a very useful one, and should find a place in every English office connected with the building and engineering professions."—*Industries.*

"An excellent book of reference."—*Architect.*

"In its new and revised form this Price Book is what a work of this kind should be—comprehensive, reliable, well arranged, legible, and well bound."—*British Architect.*

DECORATIVE PART OF CIVIL ARCHITECTURE.

By Sir William Chambers, F.R.S. With Portrait, Illustrations, Notes, and an Examination of Grecian Architecture, by Joseph Gwilt, F.S.A. Revised and Edited by W. H. Leeds. 66 Plates, 4to, cloth . . **21/0**

THE MECHANICS OF ARCHITECTURE.

A Treatise on Applied Mechanics, especially Adapted to the Use of Architects. By E. W. Tarn, M.A., Author of "The Science of Building," &c. Second Edition, Enlarged. Illustrated with 125 Diagrams. Crown 8vo, cloth **7/6**

"The book is a very useful and helpful manual of architectural mechanics."—*Builder.*

A HANDY BOOK OF VILLA ARCHITECTURE.

Being a Series of Designs for Villa Residences in various Styles. With Outline Specifications and Estimates. By C. Wickes, Architect, Author of "The Spires and Towers of England," &c. 61 Plates, 4to, half-morocco, gilt edges . . . . **£1 11s. 6d.**

"The whole of the designs bear evidence of their being the work of an artistic architect, and they will prove very valuable and suggestive."—*Building News.*

THE ARCHITECT'S GUIDE.

Being a Text-book of Useful Information for Architects, Engineers, Surveyors, Contractors, Clerks of Works, &c., &c. By F. Rogers. Crown 8vo, cloth. **3/6**

ARCHITECTURAL PERSPECTIVE.

The whole Course and Operations of the Draughtsman in Drawing a Large House in Linear Perspective. Illustrated by 43 Folding Plates. By F. O. Ferguson. Third Edition. 8vo, boards **3/6**

"It is the most intelligible of the treatises on this ill-treated subject that I have met with."—E. Ingress Bell, Esq., in the *R.I.B.A. Journal.*

PRACTICAL RULES ON DRAWING.

For the Operative Builder and Young Student in Architecture. By George Pyne. 14 Plates, 4to, boards **7/6**

MEASURING AND VALUING ARTIFICERS' WORK

(The Student's Guide to the Practice of). Containing Directions for taking Dimensions, Abstracting the same, and bringing the Quantities into Bill, with Tables of Constants for Valuation of Labour, and for the Calculation of Areas and Solidities. Originally edited by E. Dobson, Architect. With Additions by E. W. Tarn, M.A. Seventh Edition, Revised. With 8 Plates and 63 Woodcuts. Crown 8vo, cloth. **7/6**

"This edition will be found the most complete treatise on the principles of measuring and valuing artificers' work that has yet been published."—*Building News.*

TECHNICAL GUIDE, MEASURER, AND ESTIMATOR.

For Builders and Surveyors. Containing Technical Directions for Measuring Work in all the Building Trades, Complete Specifications for Houses, Roads, and Drains, and an Easy Method of Estimating the parts of a Building collectively. By A. C. Beaton. Ninth Edition. Waistcoat-pocket size, gilt edges **1/6**

"No builder, architect, surveyor, or valuer should be without his 'Beaton.'"—*Building News.*

SPECIFICATIONS FOR PRACTICAL ARCHITECTURE.

A Guide to the Architect, Engineer, Surveyor, and Builder. With an Essay on the Structure and Science of Modern Buildings. Upon the Basis of the Work by Alfred Bartholomew, thoroughly Revised, Corrected, and greatly added to by Frederick Rogers, Architect. Third Edition, Revised. 8vo, cloth **15/0**

"The work is too well known to need any recommendation from us. It is one of the books with which every young architect must be equipped."—*Architect.*

THE HOUSE-OWNER'S ESTIMATOR.

Or, What will it Cost to Build, Alter, or Repair? A Price Book for Unprofessional People as well as the Architectural Surveyor and Builder. By J. D. Simon. Edited by F. T. W. Miller, A.R.I.B.A. Fifth Edition. Carefully Revised. Crown 8vo, cloth. . . . . *Net* **3/6**

"In two years it will repay its cost a hundred times over."—*Field.*

SANITATION AND WATER SUPPLY.

THE HEALTH OFFICER'S POCKET-BOOK.

A Guide to Sanitary Practice and Law. For Medical Officers of Health, Sanitary Inspectors, Members of Sanitary Authorities, &c. By EDWARD F. WILLOUGHBY, M.D. (Lond.), &c. Second Edition, Revised and Enlarged. Fcap. 8vo, leather . . . . *Net* **10/6**

"It is a mine of condensed information of a pertinent and useful kind on the various subjects of which it treats. The different subjects are succinctly but fully and scientifically dealt with."—*The Lancet.*

"We recommend all those engaged in practical sanitary work to furnish themselves with a copy for reference."—*Sanitary Journal.*

THE BACTERIAL PURIFICATION OF SEWAGE:

Being a Practical Account of the Various Modern Biological Methods of Purifying Sewage. By SIDNEY BARWISE, M.D. (Lond.), D.P.H. (Camb.), etc. With 10 Page Plates and 2 Folding Diagrams. Royal 8vo, cloth. *Net* **6/0**

THE PURIFICATION OF SEWAGE.

Being a Brief Account of the Scientific Principles of Sewage Purification, and their Practical Application. By SIDNEY BARWISE, M.D. (Lond.), M.R.C.S., D.P.H. (Camb.), Fellow of the Sanitary Institute, Medical Officer of Health to the Derbyshire County Council. Crown 8vo, cloth **5/0**

WATER AND ITS PURIFICATION.

A Handbook for the Use of Local Authorities, Sanitary Officers, and others interested in Water Supply. By S. RIDEAL, D.Sc. Lond., F.I.C. Second Edition, Revised, with Additions, including numerous Illustrations and Tables. Large Crown 8vo, cloth . . . . *Net* **9/0**

RURAL WATER SUPPLY.

A Practical Handbook on the Supply of Water and Construction of Waterworks for Small Country Districts. By ALLAN GREENWELL, A.M.I.C.E., and W. T. CURRY, A.M.I.C.E. Revised Edition. Crown 8vo, cloth **5/0**

THE WATER SUPPLY OF CITIES AND TOWNS.

By WILLIAM HUMBER, A.M. Inst. C.E., and M.Inst. M.E. Imp. 4to, half-bound morocco. (See page 11.) . . . . *Net* **£6 6s.**

THE WATER SUPPLY OF TOWNS AND THE CONSTRUCTION OF WATER-WORKS.

By PROFESSOR W. K. BURTON, A.M. Inst. C.E. Second Edition, Revised and Extended. Royal 8vo, cloth. (See page 10.) . . . . . **£1 5s.**

WATER ENGINEERING.

A Practical Treatise on the Measurement, Storage, Conveyance, and Utilisation of Water for the Supply of Towns. By C. SLAGG, A.M. Inst. C.E. **7/6**

SANITARY WORK IN SMALL TOWNS AND VILLAGES.

By CHARLES SLAGG, A. M. Inst. C.E. Crown 8vo, cloth **3/0**

PLUMBING.

A Text-book to the Practice of the Art or Craft of the Plumber. By W. P. BUCHAN. Ninth Edition, Enlarged, with 500 Illustrations. Crown 8vo, **3/6**

VENTILATION.

A Text-book to the Practice of the Art of Ventilating Buildings. By W. P. BUCHAN, R.P. Crown 8vo, cloth **3/6**

CARPENTRY, TIMBER, ETC.

THE ELEMENTARY PRINCIPLES OF CARPENTRY.

A Treatise on the Pressure and Equilibrium of Timber Framing, the Resistance of Timber, and the Construction of Floors, Arches, Bridges, Roofs, Uniting Iron and Stone with Timber, &c. To which is added an Essay on the Nature and Properties of Timber, &c., with Descriptions of the kinds of Wood used in Building; also numerous Tables of the Scantlings of Timber for different purposes, the Specific Gravities of Materials, &c. By THOMAS TREDGOLD, C.E. With an Appendix of Specimens of Various Roofs of Iron and Stone, Illustrated. Seventh Edition, thoroughly Revised and considerably Enlarged by E. WYNDHAM TARN, M.A., Author of "The Science of Building," &c. With 61 Plates, Portrait of the Author, and several Woodcuts. In One large Vol., 4to, cloth . . . . **£1 5s.**

"Ought to be in every architect's and every builder's library."—*Builder.*

"A work whose monumental excellence must commend it wherever skilful carpentry is concerned. The author's principles are rather confirmed than impaired by time. The additional plates are of great intrinsic value."—*Building News.*

WOODWORKING MACHINERY.

Its Rise, Progress, and Construction. With Hints on the Management of Saw Mills and the Economical Conversion of Timber. Illustrated with Examples of Recent Designs by leading English, French, and American Engineers. By M. POWIS BALE, A.M.Inst.C.E., M.I.M.E. Second Edition, Revised, with large Additions, large crown 8vo, 440 pp., cloth **9/0**

"Mr. Bale is evidently an expert on the subject, and he has collected so much information that his book is all-sufficient for builders and others engaged in the conversion of timber."—*Architect.*

"The most comprehensive compendium of wood-working machinery we have seen. The author is a thorough master of his subject."—*Building News.*

SAW MILLS.

Their Arrangement and Management, and the Economical Conversion of Timber. By M. POWIS BALE, A.M.Inst.C.E. Second Edition, Revised. Crown 8vo, cloth. **10/6**

"The *administration* of a large sawing establishment is discussed, and the subject examined from a financial standpoint. Hence the size, shape, order, and disposition of saw mills and the like are gone into in detail, and the course of the timber is traced from its reception to its delivery in its converted state. We could not desire a more complete or practical treatise."—*Builder.*

THE CARPENTER'S GUIDE.

Or, Book of Lines for Carpenters; comprising all the Elementary Principles essential for acquiring a knowledge of Carpentry. Founded on the late PETER NICHOLSON'S standard work. A New Edition, Revised by ARTHUR ASHPITEL, F.S.A. Together with Practical Rules on Drawing, by GEORGE PYNE. With 74 Plates, 4to, cloth . . . . **£1 1s.**

A PRACTICAL TREATISE ON HANDRAILING.

Showing New and Simple Methods for Finding the Pitch of the Plank, Drawing the Moulds, Bevelling, Jointing-up, and Squaring the Wreath. By GEORGE COLLINGS. Revised and Enlarged, to which is added A TREATISE ON STAIR-BUILDING. Third Edition. With Plates and Diagrams. 12mo, cloth. **2/6**

"Will be found of practical utility in the execution of this difficult branch of joinery."—*Builder.*

"Almost every difficult phase of this somewhat intricate branch of joinery is elucidated by the aid of plates and explanatory letterpress."—*Furniture Gazette.*

CIRCULAR WORK IN CARPENTRY AND JOINERY.

A Practical Treatise on Circular Work of Single and Double Curvature. By GEORGE COLLINGS. With Diagrams. Fourth Edition, 12mo, cloth . **2/6**

"An excellent example of what a book of this kind should be. Cheap in price, clear in definition, and practical in the examples selected."—*Builder.*

THE CABINET-MAKER'S GUIDE TO THE ENTIRE CONSTRUCTION OF CABINET WORK.

By RICHARD BITMEAD. Illustrated with Plans, Sections and Working Drawings. Crown 8vo, cloth **2/6**

HANDRAILING COMPLETE IN EIGHT LESSONS.

On the Square-Cut System. By J. S. Goldthorp, Teacher of Geometry and Building Construction at the Halifax Mechanics' Institute. With Eight Plates and over 150 Practical Exercises. 4to, cloth **3/6**

"Likely to be of considerable value to joiners and others who take a pride in good work. The arrangement of the book is excellent. We heartily commend it to teachers and students."—*Timber Trades Journal.*

TIMBER MERCHANT'S and BUILDER'S COMPANION.

Containing New and Copious Tables of the Reduced Weight and Measurement of Deals and Battens, of all sizes, and other Useful Tables for the use of Timber Merchants and Builders. By William Dowsing. Fourth Edition, Revised and Corrected. Crown 8vo, cloth **3/0**

"We are glad to see a fourth edition of these admirable tables, which for correctness and simplicity of arrangement leave nothing to be desired."—*Timber Trades Journal.*

THE PRACTICAL TIMBER MERCHANT.

Being a Guide for the Use of Building Contractors, Surveyors, Builders, &c., comprising useful Tables for all purposes connected with the Timber Trade, Marks of Wood, Essay on the Strength of Timber, Remarks on the Growth of Timber, &c. By W. Richardson. Second Edition. Fcap. 8vo, cloth . **3/6**

"This handy manual contains much valuable information for the use of timber merchants, builders, foresters, and all others connected with the growth, sale, and manufacture of timber."—*Journal of Forestry.*

PACKING-CASE TABLES.

Showing the number of Superficial Feet in Boxes or Packing-Cases, from six inches square and upwards. By W. Richardson, Timber Broker. Third Edition. Oblong 4to, cloth **3/6**

"Invaluable labour-saving tables."—*Ironmonger.*
"Will save much labour and calculation."—*Grocer.*

GUIDE TO SUPERFICIAL MEASUREMENT.

Tables calculated from 1 to 200 inches in length by 1 to 108 inches in breadth. For the use of Architects, Surveyors, Engineers, Timber Merchants, Builders, &c. By James Hawkings. Fifth Edition. Fcap., cloth. **3/6**

"These tables will be found of great assistance to all who require to make calculations of superficial measurement."—*English Mechanic.*

PRACTICAL FORESTRY.

And its Bearing on the Improvement of Estates. By Charles E. Curtis, F.S.I., Professor of Forestry, Field Engineering, and General Estate Management, at the College of Agriculture, Downton. Second Edition, Revised. Crown 8vo, cloth **3/6**

Prefatory Remarks.—Objects of Planting.—Choice of a Forester.—Choice of Soil and Site.—Laying Out of Land for Plantations.—Preparation of the Ground for Planting.—Drainage.—Planting.—Distances and Distribution of Trees in Plantations.—Trees and Ground Game.—Attention after Planting.—Thinning of Plantations.—Pruning of Forest Trees.—Realization.—Methods of Sale.—Measurement of Timber.—Measurement and Valuation of Larch Plantation.—Fire Lines.—Cost of Planting.

"Mr. Curtis has in the course of a series of short pithy chapters afforded much information of a useful and practical character on the planting and subsequent treatment of trees."—*Illustrated Carpenter and Builder.*

THE ELEMENTS OF FORESTRY.

Designed to afford Information concerning the Planting and Care of Forest Trees for Ornament or Profit, with suggestions upon the Creation and Care of Woodlands. By F. B. Hough. Large crown 8vo, cloth **10/0**

TIMBER IMPORTER'S, TIMBER MERCHANT'S, AND BUILDER'S STANDARD GUIDE.

By Richard E. Grandy. Comprising:—An Analysis of Deal Standards, Home and Foreign, with Comparative Values and Tabular Arrangements for fixing Net Landed Cost on Baltic and North American Deals, including all intermediate Expenses, Freight, Insurance, &c.; together with copious Information for the Retailer and Builder. Third Edition, Revised. 12mo, cloth **2/0**

"Everything it pretends to be: built up gradually, it leads one from a forest to a treenail, and throws in, as a makeweight, a host of material concerning bricks, columns, cisterns, &c."—*English Mechanic.*

DECORATIVE ARTS, ETC.

SCHOOL OF PAINTING FOR THE IMITATION OF WOODS AND MARBLES.

As Taught and Practised by A. R. Van der Burg and P. Van der Burg, Directors of the Rotterdam Painting Institution. Royal folio, 18½ by 12½ in., Illustrated with 24 full-size Coloured Plates; also 12 plain Plates, comprising 154 Figures. Fourth Edition cloth. [*Just Published. Net* **£1 5s.**

List of Plates.

1. Various Tools Required for Wood Painting.—2, 3. Walnut; Preliminary Stages of Graining and Finished Specimen.—4. Tools Used for Marble Painting and Method of Manipulation.—5, 6. St. Remi Marble; Earlier Operations and Finished Specimen.—7. Methods of Sketching Different Grains, Knots, &c.—8, 9. Ash: Preliminary Stages and Finished Specimen.—10. Methods of Sketching Marble Grains.—11, 12. Breche Marble; Preliminary Stages of Working and Finished Specimen.—13. Maple; Methods of Producing the Different Grains.—14, 15. Bird's-Eye Maple; Preliminary Stages and Finished Specimen.—16. Methods of Sketching the Different Species of White Marble.—17, 18. White Marble; Preliminary Stages of Process and Finished Specimen.—19. Mahogany; Specimens of Various Grains and Methods of Manipulation.—20, 21. Mahogany; Earlier Stages and Finished Specimen.—22, 23, 24. Sienna Marble; Varieties of Grain, Preliminary Stages and Finished Specimen.—25, 26, 27. Juniper Wood; Methods of Producing Grain, &c.; Preliminary Stages and Finished Specimen.—28, 29, 30. Vert de Mer Marble; Varieties of Grain and Methods of Working, Unfinished and Finished Specimens.—31, 32, 33. Oak; Varieties of Grain, Tools Employed and Methods of Manipulation, Preliminary Stages and Finished Specimen.—34, 35, 36. Waulsort Marble; Varieties of Grain, Unfinished and Finished Specimens.

"Those who desire to attain skill in the art of painting woods and marbles will find advantage in consulting this book. . . . Some of the Working Men's Clubs should give their young men the opportunity to study it."—*Builder.*

"A comprehensive guide to the art. The explanations of the processes, the manipulation and management of the colours, and the beautifully executed plates will not be the least valuable to the student who aims at making his work a faithful transcript of nature."—*Building News.*

"Students and novices are fortunate who are able to become the possessors of so noble a work."—*The Architect.*

ELEMENTARY DECORATION.

A Guide to the Simpler Forms of Everyday Art. Together with PRACTICAL HOUSE DECORATION. By James W. Facey. With numerous Illustrations. In One Vol., strongly half-bound **5/0**

HOUSE PAINTING, GRAINING, MARBLING, AND SIGN WRITING.

A Practical Manual of. By Ellis A. Davidson. Eighth Edition. With Coloured Plates and Wood Engravings. Crown 8vo, cloth . . . **6/0**

"A mass of information of use to the amateur and of value to the practical man."—*English Mechanic.*

THE DECORATOR'S ASSISTANT.

A Modern Guide for Decorative Artists and Amateurs, Painters, Writers, Gilders, &c. Containing upwards of 600 Receipts, Rules, and Instructions; with a variety of Information for General Work connected with every Class of Interior and Exterior Decorations, &c. Eighth Edition. Cr. 8vo . **1/0**

"Full of receipts of value to decorators, painters, gilders, &c. The book contains the gist of larger treatises on colour and technical processes. It would be difficult to meet with a work so full of varied information on the painter's art."—*Building News.*

MARBLE DECORATION

And the Terminology of British and Foreign Marbles. A Handbook for Students. By George H. Blagrove, Author of "Shoring and its Application," &c. With 28 Illustrations. Crown 8vo, cloth **3/6**

"This most useful and much wanted handbook should be in the hands of every architect and builder."—*Building World.*

"A carefully and usefully written treatise; the work is essentially practical."—*Scotsman.*

DELAMOTTE'S WORKS ON ILLUMINATION AND ALPHABETS.

ORNAMENTAL ALPHABETS, ANCIENT & MEDIÆVAL.

From the Eighth Century, with Numerals; including Gothic, Church-Text, large and small, German, Italian, Arabesque, Initials for Illumination, Monograms, Crosses, &c., &c., for the use of Architectural and Engineering Draughtsmen, Missal Painters, Masons, Decorative Painters, Lithographers, Engravers, Carvers, &c. Collected and Engraved by F. DELAMOTTE, and printed in Colours. New and Cheaper Edition. Royal 8vo, oblong, ornamental boards **2/6**

"For those who insert enamelled sentences round gilded chalices, who blazon shop legends over shop-doors, who letter church walls with pithy sentences from the Decalogue, this book will be useful."—*Athenæum.*

MODERN ALPHABETS, PLAIN AND ORNAMENTAL.

Including German, Old English, Saxon, Italic, Perspective, Greek, Hebrew, Court Hand, Engrossing, Tuscan, Riband, Gothic, Rustic, and Arabesque; with several Original Designs, and an Analysis of the Roman and Old English Alphabets, large and small, and Numerals, for the use of Draughtsmen, Surveyors, Masons, Decorative Painters, Lithographers, Engravers, Carvers, &c. Collected and Engraved by F. DELAMOTTE, and printed in Colours. New and Cheaper Edition. Royal 8vo, oblong, ornamental boards . **2/6**

"There is comprised in it every possible shape into which the letters of the alphabet and numerals can be formed, and the talent which has been expended in the conception of the various plain and ornamental letters is wonderful."—*Standard.*

MEDIÆVAL ALPHABETS AND INITIALS.

By F. G. DELAMOTTE. Containing 21 Plates and Illuminated Title, printed in Gold and Colours. With an Introduction by J. WILLIS BROOKS. Fifth Edition. Small 4to, ornamental boards . . . . *Net* **5/0**

"A volume in which the letters of the alphabet come forth glorified in gilding and all the colours of the prism interwoven and intertwined and intermingled."—*Sun.*

A PRIMER OF THE ART OF ILLUMINATION.

For the Use of Beginners; with a Rudimentary Treatise on the Art, Practical Directions for its Exercise, and Examples taken from Illuminated MSS., printed in Gold and Colours. By F. DELAMOTTE. New and Cheaper Edition. Small 4to, ornamental boards **6/0**

"The examples of ancient MSS. recommended to the student, which, with much good sense, the author chooses from collections accessible to all, are selected with judgment and knowledge as well as taste."—*Athenæum.*

THE EMBROIDERER'S BOOK OF DESIGN.

Containing Initials, Emblems, Cyphers, Monograms, Ornamental Borders, Ecclesiastical Devices, Mediæval and Modern Alphabets, and National Emblems. Collected by F. DELAMOTTE, and printed in Colours. Oblong royal 8vo, ornamental wrapper . . . . *Net* **2/0**

"The book will be of great assistance to ladies and young children who are endowed with the art of plying the needle in this most ornamental and useful pretty work."—*East Anglian Times.*

WOOD-CARVING FOR AMATEURS.

With Hints on Design. By A LADY. With 10 Plates. New and Cheaper Edition. Crown 8vo, in emblematic wrapper **2/0**

"The handicraft of the wood-carver, so well as a book can impart it, may be learnt from 'A Lady's' publication."—*Athenæum.*

PAINTING POPULARLY EXPLAINED.

By THOMAS JOHN GULLICK, Painter, and JOHN TIMBS, F.S.A. Including Fresco, Oil, Mosaic, Water-Colour, Water-Glass, Tempera, Encaustic, Miniature, Painting on Ivory, Vellum, Pottery, Enamel, Glass, &c. Fifth Edition. Crown 8vo, cloth **5/0**

⁂ *Adopted as a Prize Book at South Kensington.*

"Much may be learned, even by those who fancy they do not require to be taught, from the careful perusal of this unpretending but comprehensive treatise."—*Art Journal.*

NATURAL SCIENCE, ETC.

THE VISIBLE UNIVERSE.

Chapters on the Origin and Construction of the Heavens. By J. E. GORE, F.R.A.S., Author of "Star Groups," &c. Illustrated by 6 Stellar Photographs and 12 Plates. Demy 8vo, cloth **16/0**

STAR GROUPS.

A Student's Guide to the Constellations. By J. ELLARD GORE, F.R.A.S., M.R.I.A., &c., Author of "The Visible Universe," "The Scenery of the Heavens," &c. With 30 Maps. Small 4to, cloth **5/0**

AN ASTRONOMICAL GLOSSARY.

Or, Dictionary of Terms used in Astronomy. With Tables of Data and Lists of Remarkable and Interesting Celestial Objects. By J. ELLARD GORE, F.R.A.S., Author of "The Visible Universe," &c. Small crown 8vo, cloth. **2/6**

THE MICROSCOPE.

Its Construction and Management. Including Technique, Photo-micrography, and the Past and Future of the Microscope. By Dr. HENRI VAN HEURCK. Re-Edited and Augmented from the Fourth French Edition, and Translated by WYNNE E. BAXTER, F.G.S. Imp. 8vo, cloth **18/0**

A MANUAL OF THE MOLLUSCA.

A Treatise on Recent and Fossil Shells. By S. P. WOODWARD, A.L.S., F.G.S. With an Appendix on RECENT AND FOSSIL CONCHOLOGICAL DISCOVERIES, by RALPH TATE, A.L.S., F.G.S. With 23 Plates and upwards of 300 Woodcuts. Reprint of Fourth Edition (1880). Crown 8vo, cloth **7/6**

THE TWIN RECORDS OF CREATION.

Or, Geology and Genesis, their Perfect Harmony and Wonderful Concord. By G. W. V. LE VAUX. 8vo, cloth **5/0**

LARDNER'S HANDBOOKS OF SCIENCE.

HANDBOOK OF MECHANICS.

Enlarged and re-written by B. LOEWY, F.R.A.S. Post 8vo, cloth . **6/0**

HANDBOOK OF HYDROSTATICS AND PNEUMATICS.

Revised and Enlarged by B. LOEWY, F.R.A.S. Post 8vo, cloth . **5/0**

HANDBOOK OF HEAT.

Edited and re-written by B. LOEWY, F.R.A.S. Post 8vo, cloth . **6/0**

HANDBOOK OF OPTICS.

New Edition. Edited by T. OLVER HARDING, B.A. Small 8vo, cloth **5/0**

ELECTRICITY, MAGNETISM, AND ACOUSTICS.

Edited by GEO. C. FOSTER, B.A. Small 8vo, cloth **5/0**

HANDBOOK OF ASTRONOMY.

Revised and Edited by EDWIN DUNKIN, F.R.A.S. 8vo, cloth . . . **9/6**

MUSEUM OF SCIENCE AND ART.

With upwards of 1,200 Engravings. In Six Double Volumes, **£1 1s.** Cloth, or half-morocco . . . . **£1 11s. 6d.**

NATURAL PHILOSOPHY FOR SCHOOLS . . **3/6**

ANIMAL PHYSIOLOGY FOR SCHOOLS . . **3/6**

THE ELECTRIC TELEGRAPH.

Revised by E. B. BRIGHT, F.R.A.S. Fcap. 8vo, cloth **2/6**

CHEMICAL MANUFACTURES, CHEMISTRY, ETC.

THE OIL FIELDS OF RUSSIA AND THE RUSSIAN PETROLEUM INDUSTRY.

A Practical Handbook on the Exploration, Exploitation, and Management of Russian Oil Properties, including Notes on the Origin of Petroleum in Russia, a Description of the Theory and Practice of Liquid Fuel, and a Translation of the Rules and Regulations concerning Russian Oil Properties. By A. Beeby Thompson, A.M.I.M.E., late Chief Engineer and Manager of the European Petroleum Company's Russian Oil Properties. About 500 pp., with numerous Illustrations and Photographic Plates, and a Map of the Balakhany-Saboontchy-Romany Oil Field. Super-royal 8vo, cloth.
[*Just Published. Net* **£3 3s.**

THE ANALYSIS OF OILS AND ALLIED SUBSTANCES.

By A. C. Wright, M.A.Oxon., B.Sc.Lond., formerly Assistant Lecturer in Chemistry at the Yorkshire College, Leeds, and Lecturer in Chemistry at the Hull Technical School. Demy 8vo, cloth. . . . . . . *Net* **9/0**

THE GAS ENGINEER'S POCKET-BOOK.

Comprising Tables, Notes and Memoranda relating to the Manufacture, Distribution and Use of Coal Gas and the Construction of Gas Works. By H. O'Connor, A.M.Inst.C.E. Second Edition, Revised. 470 pp., crown 8vo, fully Illustrated, leather **10/6**

"The book contains a vast amount of information. The author goes consecutively through the engineering details and practical methods involved in each of the different processes or parts of a gas-works. He has certainly succeeded in making a compilation of hard matters of fact absolutely interesting to read."—*Gas World.*

"The volume contains a great quantity of specialised information, compiled, we believe, from trustworthy sources, which should make it of considerable value to those for whom it is specifically produced."—*Engineer.*

LIGHTING BY ACETYLENE

Generators, Burners, and Electric Furnaces. By William E. Gibbs, M.E. With 66 Illustrations. Crown 8vo, cloth **7/6**

ENGINEERING CHEMISTRY.

A Practical Treatise for the Use of Analytical Chemists, Engineers, Iron Masters, Iron Founders, Students and others. Comprising Methods of Analysis and Valuation of the Principal Materials used in Engineering Work, with numerous Analyses, Examples and Suggestions. By H. Joshua Phillips F.I.C., F.C.S. Third Edition, Revised and Enlarged. Crown 8vo, 420 pp., with Plates and other Illustrations, cloth. . . . . . *Net* **10/6**

"In this work the author has rendered no small service to a numerous body of practical men. . . . The analytical methods may be pronounced most satisfactory, being as accurate as the despatch required of engineering chemists permits."—*Chemical News.*

"The analytical methods given are, as a whole, such as are likely to give rapid and trustworthy results in experienced hands. . . . There is much excellent descriptive matter in the work, the chapter on 'Oils and Lubrication' being specially noticeable in this respect."—*Engineer.*

NITRO-EXPLOSIVES.

A Practical Treatise concerning the Properties, Manufacture, and Analysis of Nitrated Substances, including the Fulminates, Smokeless Powders, and Celluloid. By P. Gerald Sanford, F.I.C., Consulting Chemist to the Cotton Powder Company, Limited, &c. With Illustrations. Crown 8vo, cloth. **9/0**

"One of the very few text-books in which can be found just what is wanted. Mr. Sanford goes steadily through the whole list of explosives commonly used, he names any given explosive, and tells us of what it is composed and how it is manufactured. The book is excellent."—*Engineer.*

A HANDBOOK ON MODERN EXPLOSIVES.

A Practical Treatise on the Manufacture and Use of Dynamite, Gun-Cotton, Nitro-Glycerine and other Explosive Compounds, including Collodion-Cotton. With Chapters on Explosives in Practical Application. By M. Eissler, M.E. Second Edition, Enlarged. Crown 8vo, cloth **12/6**

"A veritable mine of information on the subject of explosives employed for military, mining and blasting purposes."—*Army and Navy Gazette.*

A MANUAL OF THE ALKALI TRADE.

Including the Manufacture of Sulphuric Acid, Sulphate of Soda, and Bleaching Powder. By JOHN LOMAS, Alkali Manufacturer. With 232 Illustrations and Working Drawings, Second Edition, with Additions. Super-royal 8vo, cloth . . . . **£1 10s.**

"We find not merely a sound and luminous explanation of the chemical principles of the trade, but a notice of numerous matters which have a most important bearing on the successful conduct of alkali works, but which are generally overlooked by even experienced technological authors."—*Chemical Review.*

DANGEROUS GOODS.

Their Sources and Properties, Modes of Storage and Transport. With Notes and Comments on Accidents arising therefrom. A Guide for the Use of Government and Railway Officials, Steamship Owners, &c. By H. JOSHUA PHILLIPS, F.I.C., F.C.S. Crown 8vo, 374 pp., cloth **9/0**

"Merits a wide circulation, and an intelligent, appreciative study."—*Chemical News.*

THE BLOWPIPE IN CHEMISTRY, MINERALOGY, Etc.

Containing all known Methods of Anhydrous Analysis, many Working Examples, and Instructions for Making Apparatus. By Lieut.-Colonel W. A. ROSS, R.A., F.G.S. Second Edition, Enlarged. Crown 8vo, cloth . **5/0**

"The student who goes conscientiously through the course of experimentation here laid down will gain a better insight into inorganic chemistry and mineralogy than if he had 'got up' any of the best text-books of the day, and passed any number of examinations in their contents."—*Chemical News.*

THE MANUAL OF COLOURS AND DYE-WARES.

Their Properties, Applications, Valuations, Impurities and Sophistications. For the Use of Dyers, Printers, Drysalters, Brokers, &c. By J. W. SLATER. Second Edition, Revised and greatly Enlarged. Crown 8vo, cloth . **7/6**

"There is no other work which covers precisely the same ground. To students preparing for examinations in dyeing and printing it will prove exceedingly useful."—*Chemical News.*

A HANDY BOOK FOR BREWERS.

Being a Practical Guide to the Art of Brewing and Malting. Embracing the Conclusions of Modern Research which bear upon the Practice of Brewing. By HERBERT EDWARDS WRIGHT, M.A. Second Edition, Enlarged. Crown 8vo, 530 pp., cloth **12/6**

"May be consulted with advantage by the student who is preparing himself for examinational tests, while the scientific brewer will find in it a *résumé* of all the most important discoveries of modern times. The work is written throughout in a clear and concise manner, and the author takes great care to discriminate between vague theories and well-established facts."—*Brewers' Journal.*

"We have great pleasure in recommending this handy book, and have no hesitation in saying that it is one of the best—if not the best—which has yet been written on the subject of beer-brewing in this country; it should have a place on the shelves of every brewer's library."—*Brewers' Guardian.*

FUELS: SOLID, LIQUID, AND GASEOUS.

Their Analysis and Valuation. For the Use of Chemists and Engineers. By H. J. PHILLIPS, F.C.S., formerly Analytical and Consulting Chemist to the G.E. Rlwy. Fourth Edition. Crown 8vo, cloth **2/0**

"Ought to have its place in the laboratory of every metallurgical establishment and wherever fuel is used on a large scale."—*Chemical News.*

THE ARTISTS' MANUAL OF PIGMENTS.

Showing their Composition, Conditions of Permanency, Non-Permanency, and Adulterations, &c., with Tests of Purity. By H. C. STANDAGE. Third Edition. Crown 8vo, cloth **2/6**

"This work is indeed *multum-in-parvo*, and we can, with good conscience, recommend it to all who come in contact with pigments, whether as makers, dealers, or users."—*Chemical Review.*

A POCKET-BOOK OF MENSURATION AND GAUGING.

Containing Tables, Rules, and Memoranda for Revenue Officers, Brewers, Spirit Merchants, &c. By J. B. MANT, Inland Revenue. Second Edition, Revised. 18mo, leather **4/0**

"Should be in the hands of every practical brewer."—*Brewers' Journal.*

INDUSTRIAL ARTS, TRADES, AND MANUFACTURES.

TEA MACHINERY AND TEA FACTORIES.

A Descriptive Treatise on the Mechanical Appliances required in the Cultivation of the Tea Plant and the Preparation of Tea for the Market. By A. J. WALLIS-TAYLER, A. M. Inst. C.E. Medium 8vo, 468 pp. With 218 Illustrations . . . . *Net* **25/0**

SUMMARY OF CONTENTS.

MECHANICAL CULTIVATION OR TILLAGE OF THE SOIL.—PLUCKING OR GATHERING THE LEAF.—TEA FACTORIES.—THE DRESSING, MANUFACTURE, OR PREPARATION OF TEA BY MECHANICAL MEANS.—ARTIFICIAL WITHERING OF THE LEAF.—MACHINES FOR ROLLING OR CURLING THE LEAF.—FERMENTING PROCESS.—MACHINES FOR THE AUTOMATIC DRYING OR FIRING OF THE LEAF.—MACHINES FOR NON-AUTOMATIC DRYING OR FIRING OF THE LEAF.—DRYING OR FIRING MACHINES.—BREAKING OR CUTTING, AND SORTING MACHINES.—PACKING THE TEA.—MEANS OF TRANSPORT ON TEA PLANTATIONS.—MISCELLANEOUS MACHINERY AND APPARATUS.—FINAL TREATMENT OF THE TEA.—TABLES AND MEMORANDA.

"The subject of tea machinery is now one of the first interest to a large class of people, to whom we strongly commend the volume."—*Chamber of Commerce Journal.*

"When tea planting was first introduced into the British possessions little, if any, machinery was employed, but now its use is almost universal. This volume contains a very full account of the machinery necessary for the proper outfit of a factory, and also a description of the processes best carried out by this machinery."—*Journal Society of Arts.*

FLOUR MANUFACTURE.

A Treatise on Milling Science and Practice. By FRIEDRICH KICK, Imperial Regierungsrath, Professor of Mechanical Technology in the Imperial German Polytechnic Institute, Prague. Translated from the Second Enlarged and Revised Edition with Supplement. By H. H. P. POWLES, Assoc. Memb. Institution of Civil Engineers. Nearly 400 pp. Illustrated with 28 Folding Plates, and 167 Woodcuts. Royal 8vo, cloth . . . . . . **£1 5s.**

"This invaluable work is, and will remain, the standard authority on the science of milling. . . . The miller who has read and digested this work will have laid the foundation, so to speak, of a successful career; he will have acquired a number of general principles which he can proceed to apply. In this handsome volume we at last have the accepted text-book of modern milling in good, sound English, which has little, if any, trace of the German idiom."—*The Miller.*

"The appearance of this celebrated work in English is very opportune, and British millers will, we are sure, not be slow in availing themselves of its pages."—*Millers' Gazette.*

COTTON MANUFACTURE.

A Manual of Practical Instruction of the Processes of Opening, Carding, Combing, Drawing, Doubling and Spinning of Cotton, the Methods of Dyeing, &c. For the Use of Operatives, Overlookers, and Manufacturers. By JOHN LISTER, Technical Instructor, Pendleton. 8vo, cloth . . **7/6**

"This invaluable volume is a distinct advance in the literature of cotton manufacture."—*Machinery.*

"It is thoroughly reliable, fulfilling nearly all the requirements desired."—*Glasgow Herald.*

MODERN CYCLES.

A Practical Handbook on their Construction and Repair. By A. J. WALLIS-TAYLER, A. M. Inst. C. E., Author of "Refrigerating Machinery," &c. With upwards of 300 Illustrations. Crown 8vo, cloth **10/6**

"The large trade that is done in the component parts of bicycles has placed in the way of men mechanically inclined extraordinary facilities for building bicycles for their own use. . . . The book will prove a valuable guide for all those who aspire to the manufacture or repair of their own machines."—*The Field.*

"A most comprehensive and up-to-date treatise."—*The Cycle.*

"A very useful book, which is quite entitled to rank as a standard work for students of cycle construction."—*Wheeling.*

MOTOR CARS OR POWER CARRIAGES FOR COMMON ROADS.

By A. J. WALLIS-TAYLER, Assoc. Memb. Inst. C.E., Author of "Modern Cycles," &c. 212 pp., with 76 Illustrations. Crown 8vo, cloth . . **4/6**

"The book is clearly expressed throughout, and is just the sort of work that an engineer, thinking of turning his attention to motor-carriage work, would do well to read as a preliminary to starting operations."—*Engineering*

PRACTICAL TANNING.

A Handbook of Modern Processess, Receipts, and Suggestions for the Treatment of Hides, Skins, and Pelts of every Description. By L. A. Flemming, American Tanner. 472 pages. 8vo, cloth. [*Just Published. Net* **25/0**

THE ART OF LEATHER MANUFACTURE.

Being a Practical Handbook, in which the Operations of Tanning, Currying, and Leather Dressing are fully Described, and the Principles of Tanning Explained, and many Recent Processes Introduced; as also Methods for the Estimation of Tannin, and a Description of the Arts of Glue Boiling, Gut Dressing, &c. By Alexander Watt. Fourth Edition. Crown 8vo cloth. **9/0**

"A sound, comprehensive treatise on tanning and its accessories. The book is an eminently valuable production, which redounds to the credit of both author and publishers."—*Chemical Review.*

THE ART OF SOAP-MAKING.

A Practical Handbook of the Manufacture of Hard and Soft Soaps, Toilet Soaps, &c. Including many New Processes, and a Chapter on the Recovery of Glycerine from Waste Leys. By Alexander Watt. Sixth Edition, including an Appendix on Modern Candlemaking. Crown 8vo, cloth . **7/6**

"The work will prove very useful, not merely to the technological student, but to the practical soap boiler who wishes to understand the theory of his art."—*Chemical News.*

"A thoroughly practical treatise. We congratulate the author on the success of his endeavour to fill a void in English technical literature."—*Nature.*

PRACTICAL PAPER-MAKING.

A Manual for Paper-Makers and Owners and Managers of Paper-Mills. With Tables, Calculations, &c. By G. Clapperton, Paper-Maker. With Illustrations of Fibres from Micro-Photographs. Crown 8vo, cloth . . **5/0**

"The author caters for the requirements of responsible mill hands, apprentices, &c., whilst his manual will be found of great service to students of technology, as well as to veteran paper-makers and mill owners. The illustrations form an excellent feature."—*The World's Paper Trade Review.*

THE ART OF PAPER-MAKING.

A Practical Handbook of the Manufacture of Paper from Rags, Esparto, Straw, and other Fibrous Materials. Including the Manufacture of Pulp from Wood Fibre, with a Description of the Machinery and Appliances used. To which are added Details of Processes for Recovering Soda from Waste Liquors. By Alexander Watt. With Illustrations. Crown 8vo, cloth . . **7/6**

"It may be regarded as the standard work on the subject. The book is full of valuable information. The 'Art of Paper-Making' is in every respect a model of a text-book, either for a technical class, or for the private student."—*Paper and Printing Trades Journal.*

A TREATISE ON PAPER.

For Printers and Stationers. With an Outline of Paper Manufacture; Complete Tables of Sizes, and Specimens of Different Kinds of Paper. By Richard Parkinson, late of the Manchester Technical School. Demy 8vo, cloth **3/6**

CEMENTS, PASTES, GLUES, AND GUMS.

A Practical Guide to the Manufacture and Application of the various Agglutinants required in the Building, Metal-Working, Wood-Working, and Leather-Working Trades, and for Workshop and Office Use. With upwards of 900 Recipes. By H. C. Standage. Third Edition. Crown 8vo, cloth . **2/0**

"We have pleasure in speaking favourably of this volume. So far as we have had experience, which is not inconsiderable, this manual is trustworthy."—*Athenæum.*

THE CABINET-MAKER'S GUIDE

TO THE ENTIRE CONSTRUCTION OF CABINET WORK.

Including Veneering, Marquetrie, Buhlwork, Mosaic, Inlaying, &c. By Richard Bitmead. Illustrated with Plans, Sections, and Working Drawings. Small crown 8vo, cloth **2/6**

FRENCH POLISHING AND ENAMELLING.

A Practical Work of Instruction. Including Numerous Recipes for making Polishes, Varnishes, Glaze-Lacquers, Revivers, &c. By Richard Bitmead, Author of "The Cabinet-Maker's Guide." Small crown 8vo, cloth . **1/6**

WATCH REPAIRING, CLEANING, AND ADJUSTING.

A Practical Handbook dealing with the Materials and Tools Used, and the Methods of Repairing, Cleaning, Altering, and Adjusting all kinds of English and Foreign Watches, Repeaters, Chronographs and Marine Chronometers. By F. J. GARRARD, Springer and Adjuster of Marine Chronometers and Deck Watches for the Admiralty. With over 200 Illustrations. Crown 8vo, cloth. [*Just Published. Net* **4/6**

MODERN HOROLOGY, IN THEORY AND PRACTICE.

Translated from the French of CLAUDIUS SAUNIER, ex-Director of the School of Horology at Macon, by JULIEN TRIPPLIN, F.R.A.S., Besançon Watch Manufacturer, and EDWARD RIGG, M.A., Assayer in the Royal Mint. With Seventy-eight Woodcuts and Twenty-two Coloured Copper Plates. Second Edition. Super-royal 8vo, **£2 2s.** cloth; half-calf . . . **£2 10s.**

"There is no horological work in the English language at all to be compared to this production of M. Saunier's for clearness and completeness. It is alike good as a guide for the student and as a reference for the experienced horologist and skilled workman."—*Horological Journal.*

"The latest, the most complete, and the most reliable of those literary productions to which continental watchmakers are indebted for the mechanical superiority over their English brethren—in fact, the Book of Books is M. Saunier's 'Treatise.'"—*Watchmaker, Jeweller, and Silversmith.*

THE WATCH ADJUSTER'S MANUAL.

A Practical Guide for the Watch and Chronometer Adjuster in Making, Springing, Timing and Adjusting for Isochronism, Positions and Temperatures. By C. E. FRITTS. 370 pp., with Illustrations, 8vo, cloth . . . **16/0**

THE WATCHMAKER'S HANDBOOK.

Intended as a Workshop Companion for those engaged in Watchmaking and the Allied Mechanical Arts. Translated from the French of CLAUDIUS SAUNIER, and enlarged by JULIEN TRIPPLIN, F.R.A.S., and EDWARD RIGG, M.A., Assayer in the Royal Mint. Third Edition. Cr. 8vo, cloth. . **9/0**

"Each part is truly a treatise in itself. The arrangement is good and the language is clear and concise. It is an admirable guide for the young watchmaker."—*Engineering.*

HISTORY OF WATCHES & OTHER TIMEKEEPERS.

By JAMES F. KENDAL, M.B.H. Inst. **1/6** boards; or cloth, gilt . **2/6**

"The best which has yet appeared on this subject in the English language."—*Industries.*

"Open the book where you may, there is interesting matter in it concerning the ingenious devices of the ancient or modern horologer."—*Saturday Review.*

ELECTRO-PLATING & ELECTRO-REFINING OF METALS.

Being a new edition of ALEXANDER WATT'S "ELECTRO-DEPOSITION." Revised and Largely Rewritten by ARNOLD PHILIP, B.Sc., A.I.E.E., Principal Assistant to the Admiralty Chemist. Large Crown 8vo, cloth. . *Net* **12/6**

"Altogether the work can be highly recommended to every electro-plater, and is of undoubted interest to every electro-metallurgist."—*Electrical Review.*

"Eminently a book for the practical worker in electro-deposition. It contains practical descriptions of methods, processes and materials, as actually pursued and used in the workshop."—*Engineer.*

ELECTRO-METALLURGY.

Practically Treated. By ALEXANDER WATT. Tenth Edition, including the most recent Processes. 12mo, cloth **3/6**

"From this book both amateur and artisan may learn everything necessary for the successful prosecution of electroplating."—*Iron.*

JEWELLER'S ASSISTANT IN WORKING IN GOLD.

A Practical Treatise for Masters and Workmen, Compiled from the Experience of Thirty Years' Workshop Practice. By GEORGE E. GEE. Crown 8vo. **7/6**

"This manual of technical education is apparently destined to be a valuable auxiliary to a handicraft which is certainly capable of great improvement."—*The Times.*

ELECTROPLATING.

A Practical Handbook on the Deposition of Copper, Silver, Nickel, Gold, Aluminium, Brass, Platinum, &c., &c. By J. W. URQUHART, C.E. Fourth Edition, Revised. Crown 8vo, cloth **5/0**

"An excellent practical manual."—*Engineering*

"An excellent work, giving the newest information."—*Horological Journal.*

ELECTROTYPING.

The Reproduction and Multiplication of Printing Surfaces and Works of Art by the Electro-Deposition of Metals. By J. W. URQUHART, C.E. Crown 8vo, cloth **5/0**

"The book is thoroughly practical; the reader is, therefore, conducted through the leading laws of electricity, then through the metals used by electrotypers, the apparatus, and the depositing processes, up to the final preparation of the work."—*Art Journal.*

GOLDSMITH'S HANDBOOK.

By GEORGE E. GEE, Jeweller, &c. Fifth Edition. 12mo, cloth . . **3/0**

"A good, sound educator."—*Horological Journal.*

SILVERSMITH'S HANDBOOK.

By GEORGE E. GEE, Jeweller, &c. Third Edition, with numerous Illustrations. 12mo, cloth **3/0**

"The chief merit of the work is its practical character. . . . The workers in the trade will speedily discover its merits when they sit down to study it."—*English Mechanic.*

*** *The above two works together, strongly half-bound, price* 7s.

SHEET METAL WORKER'S INSTRUCTOR.

Comprising a Selection of Geometrical Problems and Practical Rules for Describing the Various Patterns Required by Zinc, Sheet-Iron, Copper, and Tin-Plate Workers. By REUBEN HENRY WARN, Practical Tin-Plate Worker. New Edition, Revised and greatly Enlarged by JOSEPH G. HORNER, A.M.I.M.E. Crown 8vo, 254 pp., with 430 Illustrations, cloth . . **7/6**

SAVOURIES AND SWEETS

Suitable for Luncheons and Dinners. By Miss M. L. ALLEN (Mrs. A. MACAIRE), Author of "Breakfast Dishes," &c. Twenty-ninth Edition. F'cap 8vo, sewed **1/0**

BREAKFAST DISHES

For Every Morning of Three Months. By Miss ALLEN (Mrs A. MACAIRE), Author of "Savouries and Sweets," &c. Twenty-second Edition. F'cap 8vo, sewed **1/0**

BREAD & BISCUIT BAKER'S & SUGAR-BOILER'S ASSISTANT.

Including a large variety of Modern Recipes. With Remarks on the Art of Bread-making. By ROBERT WELLS. Third Edition. Crown 8vo . . **1/0**

"A large number of wrinkles for the ordinary cook, as well as the baker."—*Saturday Review.*

PASTRYCOOK & CONFECTIONER'S GUIDE.

For Hotels, Restaurants, and the Trade in general, adapted also for Family Use. By R. WELLS, Author of "The Bread and Biscuit Baker" . . **1/0**

"We cannot speak too highly of this really excellent work. In these days of keen competition our readers cannot do better than purchase this book."—*Bakers' Times.*

ORNAMENTAL CONFECTIONERY.

A Guide for Bakers, Confectioners and Pastrycooks; including a variety of Modern Recipes, and Remarks on Decorative and Coloured Work. With 129 Original Designs. By ROBERT WELLS. Crown 8vo, cloth **5/0**

"A valuable work, practical, and should be in the hands of every baker and confectioner. The illustrative designs are worth treble the amount charged for the work."—*Bakers' Times.*

MODERN FLOUR CONFECTIONER.

Containing a large Collection of Recipes for Cheap Cakes, Biscuits, &c. With remarks on the Ingredients Used in their Manufacture. By R. WELLS. **1/0**

"The work is of a decidedly practical character, and in every recipe regard is had to economical working."—*North British Daily Mail.*

RUBBER HAND STAMPS

And the Manipulation of Rubber. A Practical Treatise on the Manufacture of Indiarubber Hand Stamps, Small Articles of Indiarubber, The Hektograph, Special Inks, Cements, and Allied Subjects. By T. O'CONOR SLOANE, A.M., Ph.D. With numerous Illustrations. Square 8vo, cloth. **5/0**

HANDYBOOKS FOR HANDICRAFTS.

BY PAUL N. HASLUCK.

Editor of "Work" (New Series), Author of "Lathe Work," "Milling Machines," &c.
Crown 8vo, 144 pp., price 1s. each.

☞ *These* HANDYBOOKS *have been written to supply information for* WORKMEN, STUDENTS, *and* AMATEURS *in the several Handicrafts, on the actual* PRACTICE *of the* WORKSHOP, *and are intended to convey in plain language* TECHNICAL KNOWLEDGE *of the several* CRAFTS. *In describing the processes employed, and the manipulation of material, workshop terms are used; workshop practice is fully explained; and the text is freely illustrated with drawings of modern tools, appliances, and processes.*

METAL TURNER'S HANDYBOOK.

A Practical Manual for Workers at the Foot-Lathe. With 100 Illustrations. **1/0**

"The book will be of service alike to the amateur and the artisan turner. It displays thorough knowledge of the subject."—*Scotsman.*

WOOD TURNER'S HANDYBOOK.

A Practical Manual for Workers at the Lathe. With over 100 Illustrations. **1/0**

"We recommend the book to young turners and amateurs. A multitude of workmen have hitherto sought in vain for a manual of this special industry."—*Mechanical World.*

WATCH JOBBER'S HANDYBOOK.

A Practical Manual on Cleaning, Repairing, and Adjusting. With upwards of 100 Illustrations **1/0**

"We strongly advise all young persons connected with the watch trade to acquire and study this inexpensive work."—*Clerkenwell Chronicle.*

PATTERN MAKER'S HANDYBOOK.

A Practical Manual on the Construction of Patterns for Founders. With upwards of 100 Illustrations **1/0**

"A most valuable, if not indispensable manual for the pattern maker."—*Knowledge.*

MECHANIC'S WORKSHOP HANDYBOOK.

A Practical Manual on Mechanical Manipulation, embracing Information on various Handicraft Processes. With Useful Notes and Miscellaneous Memoranda. Comprising about 200 Subjects **1/0**

"A very clever and useful book, which should be found in every workshop; and it should certainly find a place in all technical schools."—*Saturday Review.*

MODEL ENGINEER'S HANDYBOOK.

A Practical Manual on the Construction of Model Steam Engines. With upwards of 100 Illustrations. **1/0**

"Mr. Hasluck has produced a very good little book."—*Builder.*

CLOCK JOBBER'S HANDYBOOK.

A Practical Manual on Cleaning, Repairing, and Adjusting. With upwards of 100 Illustrations **1/0**

"It is of inestimable service to those commencing the trade."—*Coventry Standard.*

CABINET WORKER'S HANDYBOOK.

A Practical Manual on the Tools, Materials, Appliances, and Processes employed in Cabinet Work. With upwards of 100 Illustrations . . **1/0**

"Mr. Hasluck's thorough-going little Handybook is amongst the most practical guides we have seen for beginners in cabinet-work."—*Saturday Review.*

WOODWORKER'S HANDYBOOK.

Embracing Information on the Tools, Materials, Appliances and Processes Employed in Woodworking. With 104 Illustrations. **1/0**

"Written by a man who knows, not only how work ought to be done, but how to do it, and how to convey his knowledge to others."—*Engineering.*

"Mr. Hasluck writes admirably, and gives complete instructions."—*Engineer.*

"Mr. Hasluck combines the experience of a practical teacher with the manipulative skill and scientific knowledge of processes of the trained mechanician, and the manuals are marvels of what can be produced at a popular price."—*Schoolmaster.*

"Helpful to workmen of all ages and degrees of experience."—*Daily Chronicle.*

"Concise, clear, and practical."—*Saturday Review.*

COMMERCE, COUNTING-HOUSE WORK, TABLES, ETC.

LESSONS IN COMMERCE.

By Professor R. GAMBARO, of the Royal High Commercial School at Genoa. Edited and Revised by JAMES GAULT, Professor of Commerce and Commercial Law in King's College, London. Fourth Edition. Crown 8vo, cloth . **3/6**

"The publishers of this work have rendered considerable service to the cause of commercial education by the opportune production of this volume. . . . The work is peculiarly acceptable to English readers and an admirable addition to existing class books. In a phrase, we think the work attains its object in furnishing a brief account of those laws and customs of British trade with which the commercial man interested therein should be familiar."—*Chamber of Commerce Journal.*

"An invaluable guide in the hands of those who are preparing for a commercial career, and, in fact, the information it contains on matters of business should be impressed on every one."—*Counting House.*

THE FOREIGN COMMERCIAL CORRESPONDENT.

Being Aids to Commercial Correspondence in Five Languages—English, French, German, Italian, and Spanish. By CONRAD E. BAKER. Third Edition, Carefully Revised Throughout. Crown 8vo, cloth . . . **4/6**

"Whoever wishes to correspond in all the languages mentioned by Mr. Baker cannot do better than study this work, the materials of which are excellent and conveniently arranged. They consist not of entire specimen letters, but—what are far more useful—short passages, sentences, or phrases expressing the same general idea in various forms."—*Athenæum.*

"A careful examination has convinced us that it is unusually complete, well arranged and reliable. The book is a thoroughly good one."—*Schoolmaster.*

FACTORY ACCOUNTS: their PRINCIPLES & PRACTICE.

A Handbook for Accountants and Manufacturers, with Appendices on the Nomenclature of Machine Details; the Income Tax Acts; the Rating of Factories; Fire and Boiler Insurance; the Factory and Workshop Acts, &c., including also a Glossary of Terms and a large number of Specimen Rulings. By EMILE GARCKE and J. M. FELLS. Fifth Edition, Revised and Enlarged. Demy 8vo, cloth **7/6**

"A very interesting description of the requirements of Factory Accounts. . . . The principle of assimilating the Factory Accounts to the general commercial books is one which we thoroughly agree with."—*Accountants' Journal.*

"Characterised by extreme thoroughness. There are few owners of factories who would not derive great benefit from the perusal of this most admirable work."—*Local Government Chronicle.*

MODERN METROLOGY.

A Manual of the Metrical Units and Systems of the present Century. With an Appendix containing a proposed English System. By LOWIS D. A. JACKSON, A. M. Inst. C. E., Author of "Aid to Survey Practice," &c. Large crown 8vo, cloth **12/6**

"We recommend the work to all interested in the practical reform of our weights and measures."—*Nature.*

A SERIES OF METRIC TABLES.

In which the British Standard Measures and Weights are compared with those of the Metric System at present in Use on the Continent. By C. H. DOWLING, C.E. 8vo, cloth **10/6**

"Mr. Dowling's Tables are well put together as a ready reckoner for the conversion of one system into the other."—*Athenæum.*

IRON AND METAL TRADES' COMPANION.

For Expeditiously Ascertaining the Value of any Goods bought or sold by Weight, from 1s. per cwt. to 112s. per cwt., and from one farthing per pound to one shilling per pound. By THOMAS DOWNIE. Strongly bound in leather, 396 pp. **9/0**

"A most useful set of tables, nothing like them before existed."—*Building News.*

"Although specially adapted to the iron and metal trades, the tables will be found useful in every other business in which merchandise is bought and sold by weight."—*Railway News.*

NUMBER, WEIGHT, AND FRACTIONAL CALCULATOR.

Containing upwards of 250,000 Separate Calculations, showing at a Glance the Value at 422 Different Rates, ranging from $\frac{1}{128}$th of a Penny to 20s. each, or per cwt., and £20 per ton, of any number of articles consecutively, from 1 to 470. Any number of cwts., qrs., and lbs., from 1 cwt. to 470 cwts. Any number of tons, cwts., qrs., and lbs., from 1 to 1,000 tons. By WILLIAM CHADWICK, Public Accountant. Third Edition, Revised and Improved. 8vo, strongly bound **18/0**

"It is as easy of reference for any answer or any number of answers as a dictionary. For making up accounts or estimates the book must prove invaluable to all who have any considerable quantity of calculations involving price and measure in any combination to do."—*Engineer.*

"The most perfect work of the kind yet prepared."—*Glasgow Herald.*

THE WEIGHT CALCULATOR.

Being a Series of Tables upon a New and Comprehensive Plan, exhibiting at one Reference the exact Value of any Weight from 1 lb. to 15 tons, at 300 Progressive Rates, from 1*d.* to 168*s.* per cwt., and containing 186,000 Direct Answers, which, with their Combinations, consisting of a single addition (mostly to be performed at sight), will afford an aggregate of 10,266,000 Answers; the whole being calculated and designed to ensure correctness and promote despatch. By HENRY HARBEN, Accountant. Sixth Edition, carefully Corrected. Royal 8vo, strongly half-bound. [*Just Published.* **£1 5s.**

"A practical and useful work of reference for men of business generally."—*Ironmonger.*

"Of priceless value to business men. It is a necessary book in all mercantile offices."—*Sheffield Independent.*

THE DISCOUNT GUIDE.

Comprising several Series of Tables for the Use of Merchants, Manufacturers, Ironmongers, and Others, by which may be ascertained the Exact Profit arising from any mode of using Discounts, either in the Purchase or Sale of Goods, and the method of either Altering a Rate of Discount, or Advancing a Price, so as to produce, by one operation, a sum that will realise any required Profit after allowing one or more Discounts: to which are added Tables of Profit or Advance from 1¼ to 90 per cent., Tables of Discount from 1¼ to 98¾ per cent., and Tables of Commission, &c., from ⅛ to 10 per cent. By HENRY HARBEN, Accountant. New Edition, Corrected. Demy 8vo, half-bound . **£1 5s.**

"A book such as this can only be appreciated by business men, to whom the saving of time means saving of money. The work must prove of great value to merchants, manufacturers, and general traders."—*British Trade Journal.*

TABLES OF WAGES.

At 54, 52, 50 and 48 Hours per Week. Showing the Amounts of Wages from One quarter of an hour to Sixty-four hours, in each case at Rates of Wages advancing by One Shilling from 4s. to 55s. per week. By THOS. GARBUTT, Accountant. Square crown 8vo, half-bound **6/0**

IRON-PLATE WEIGHT TABLES.

For Iron Shipbuilders, Engineers, and Iron Merchants. Containing the Calculated Weights of upwards of 150,000 different sizes of Iron Plates from 1 foot by 6 in. by ¼ in. to 10 feet by 5 feet by 1 in. Worked out on the Basis of 40 lbs. to the square foot of Iron of 1 inch in thickness. By H. BURLINSON and W. H. SIMPSON. 4to, half-bound . . . . **£1 5s.**

AGRICULTURE, FARMING, GARDENING, ETC.

THE COMPLETE GRAZIER AND FARMER'S AND CATTLE BREEDER'S ASSISTANT.

A Compendium of Husbandry. Originally Written by WILLIAM YOUATT. Fourteenth Edition, entirely Re-written, considerably Enlarged, and brought up to Present Requirements, by WILLIAM FREAM, LL.D., Assistant Commissioner, Royal Commission on Agriculture, Author of "The Elements of Agriculture," &c. Royal 8vo, 1,100 pp., 450 Illustrations, handsomely bound. **£1 11s. 6d.**

BOOK I. ON THE VARIETIES, BREEDING, REARING, FATTENING AND MANAGEMENT OF CATTLE.
BOOK II. ON THE ECONOMY AND MANAGEMENT OF THE DAIRY.
BOOK III. ON THE BREEDING, REARING, AND MANAGEMENT OF HORSES.
BOOK IV. ON THE BREEDING, REARING, AND FATTENING OF SHEEP.
BOOK V. ON THE BREEDING, REARING, AND FATTENING OF SWINE.
BOOK VI. ON THE DISEASES OF LIVE STOCK.
BOOK VII. ON THE BREEDING, REARING, AND MANAGEMENT OF POULTRY.
BOOK VIII. ON FARM OFFICES AND IMPLEMENTS OF HUSBANDRY.
BOOK IX. ON THE CULTURE AND MANAGEMENT OF GRASS LANDS.
BOOK X. ON THE CULTIVATION AND APPLICATION OF GRASSES, PULSE AND ROOTS.
BOOK XI. ON MANURES AND THEIR APPLICATION TO GRASS LAND AND CROPS.
BOOK XII. MONTHLY CALENDARS OF FARMWORK.

"Dr. Fream is to be congratulated on the successful attempt he has made to give us a work which will at once become the standard classic of the farm practice of the country. We believe that it will be found that it has no compeer among the many works at present in existence. . . . The illustrations are admirable, while the frontispiece, which represents the well-known bull, New Year's Gift, bred by the Queen, is a work of art."—*The Times.*

"The book must be recognised as occupying the proud position of the most exhaustive work of reference in the English language on the subject with which it deals."—*Athenæum.*

"The most comprehensive guide to modern farm practice that exists in the English language to-day. . . . The book is one that ought to be on every farm and in the library of every land owner."—*Mark Lane Express.*

"In point of exhaustiveness and accuracy the work will certainly hold a pre-eminent and unique position among books dealing with scientific agricultural practice. It is, in fact, an agricultural library of itself."—*North British Agriculturist.*

FARM LIVE STOCK OF GREAT BRITAIN.

By ROBERT WALLACE, F.L.S., F.R.S.E., &c., Professor of Agriculture and Rural Economy in the University of Edinburgh. Third Edition, thoroughly Revised and considerably Enlarged. With over 120 Phototypes of Prize Stock. Demy 8vo, 384 pp., with 79 Plates and Maps, cloth. . . . **12/6**

"A really complete work on the history, breeds, and management of the farm stock of Great Britain, and one which is likely to find its way to the shelves of every country gentleman's library."—*The Times.*

"The 'Farm Live Stock of Great Britain' is a production to be proud of, and its issue not the least of the services which its author has rendered to agricultural science."—*Scottish Farmer.*

NOTE-BOOK OF AGRICULTURAL FACTS & FIGURES FOR FARMERS AND FARM STUDENTS.

By PRIMROSE McCONNELL, B.Sc., Fellow of the Highland and Agricultural Society, Author of "Elements of Farming." Sixth Edition, Re-written, Revised, and greatly Enlarged. Fcap. 8vo, 480 pp., leather, gilt edges . . **6/0**

CONTENTS.—SURVEYING AND LEVELLING.—WEIGHTS AND MEASURES.—MACHINERY AND BUILDINGS. — LABOUR. — OPERATIONS. — DRAINING. — EMBANKING. — GEOLOGICAL MEMORANDA. — SOILS. — MANURES. — CROPPING. — CROPS.—ROTATIONS. — WEEDS. — FEEDING.—DAIRYING.—LIVE STOCK.—HORSES.—CATTLE. — SHEEP.—PIGS.—POULTRY.—FORESTRY.—HORTICULTURE.—MISCELLANEOUS.

"No farmer, and certainly no agricultural student, ought to be without this *multum-in-parvo* manual of all subjects connected with the farm."—*North British Agriculturist.*

"This little pocket-book contains a large amount of useful information upon all kinds of agricultural subjects. Something of the kind has long been wanted."—*Mark Lane Express.*

"The amount of information it contains is most surprising; the arrangement of the matter is so methodical—although so compressed—as to be intelligible to everyone who takes a glance through its pages. They teem with information."—*Farm and Home.*

THE ELEMENTS OF AGRICULTURAL GEOLOGY.

A Scientific Aid to Practical Farming. By PRIMROSE McCONNELL. Author of "Note-Book of Agricultural Facts and Figures," &c. Royal 8vo, cloth. *Net* **21/0**

"On every page the work bears the impress of a masterly knowledge of the subject dealt with, and we have nothing but unstinted praise to offer."—*Field.*

BRITISH DAIRYING.

A Handy Volume on the Work of the Dairy-Farm. For the Use of Technical Instruction Classes, Students in Agricultural Colleges and the Working Dairy-Farmer. By Prof. J. P. SHELDON. With Illustrations. Second Edition, Revised. Crown 8vo, cloth **2/6**

"Confidently recommended as a useful text-book on dairy farming."—*Agricultural Gazette.*

"Probably the best half-crown manual on dairy work that has yet been produced."—*North British Agriculturist.*

"It is the soundest little work we have yet seen on the subject."—*The Times.*

MILK, CHEESE, AND BUTTER.

A Practical Handbook on their Properties and the Processes of their Production. Including a Chapter on Cream and the Methods of its Separation from Milk. By JOHN OLIVER, late Principal of the Western Dairy Institute, Berkeley. With Coloured Plates and 200 Illustrations. Crown 8vo, cloth. **7/6**

"An exhaustive and masterly production. It may be cordially recommended to all students and practitioners of dairy science."—*North British Agriculturist.*

"We recommend this very comprehensive and carefully-written book to dairy-farmers and students of dairying. It is a distinct acquisition to the library of the agriculturist."—*Agricultural Gazette.*

SYSTEMATIC SMALL FARMING.

Or, The Lessons of My Farm. Being an Introduction to Modern Farm Practice for Small Farmers. By R. SCOTT BURN, Author of "Outlines of Modern Farming," &c. Crown 8vo, cloth. **6/0**

"This is the completest book of its class we have seen, and one which every amateur farmer will read with pleasure, and accept as a guide."—*Field.*

OUTLINES OF MODERN FARMING.

By R. SCOTT BURN. Soils, Manures, and Crops—Farming and Farming Economy—Cattle, Sheep, and Horses—Management of Dairy, Pigs, and Poultry—Utilisation of Town-Sewage, Irrigation, &c. Sixth Edition. In One Vol., 1,250 pp., half-bound, profusely Illustrated **12/0**

FARM ENGINEERING, The COMPLETE TEXT-BOOK of.

Comprising Draining and Embanking; Irrigation and Water Supply; Farm Roads, Fences and Gates; Farm Buildings; Barn Implements and Machines; Field Implements and Machines; Agricultural Surveying, &c. By Professor JOHN SCOTT. In One Vol., 1,150 pp., half-bound, with over 600 Illustrations. **12/0**

"Written with great care, as well as with knowledge and ability. The author has done his work well; we have found him a very trustworthy guide wherever we have tested his statements. The volume will be of great value to agricultural students."—*Mark Lane Express.*

THE FIELDS OF GREAT BRITAIN.

A Text-Book of Agriculture. Adapted to the Syllabus of the Science and Art Department. For Elementary and Advanced Students. By HUGH CLEMENTS (Board of Trade). Second Edition, Revised, with Additions. 18mo, cloth **2/6**

"It is a long time since we have seen a book which has pleased us more, or which contains such a vast and useful fund of knowledge."—*Educational Times.*

TABLES and MEMORANDA for FARMERS, GRAZIERS, AGRICULTURAL STUDENTS, SURVEYORS, LAND AGENTS, AUCTIONEERS, &c.

With a New System of Farm Book-keeping. By SIDNEY FRANCIS. Fifth Edition. 272 pp., waistcoat-pocket size, limp leather **1/6**

"Weighing less than 1 oz., and occupying no more space than a match-box, it contains a mass of facts and calculations which has never before, in such handy form, been obtainable. Every operation on the farm is dealt with. The work may be taken as thoroughly accurate, the whole of the tables having been revised by Dr. Fream. We cordially recommend it."—*Bell's Weekly Messenger.*

THE ROTHAMSTED EXPERIMENTS AND THEIR PRACTICAL LESSONS FOR FARMERS.

Part I. STOCK. Part II. CROPS. By C. J. R. TIPPER. Crown 8vo, cloth. **3/6**

"We have no doubt that the book will be welcomed by a large class of farmers and others interested in agriculture."—*Standard.*

FERTILISERS AND FEEDING STUFFS.

Their Properties and Uses. A Handbook for the Practical Farmer. By BERNARD DYER, D.Sc.(Lond.). With the Text of the Fertilisers and Feeding Stuffs Act of 1893, The Regulations and Forms of the Board of Agriculture, and Notes on the Act by A. J. DAVID, B.A., LL.M. Fourth Edition, Revised. Crown 8vo, cloth. [*Just Published.* **1/0**

"This little book is precisely what it professes to be—'A Handbook for the Practical Farmer.' Dr. Dyer has done farmers good service in placing at their disposal so much useful information in so intelligible a form."—*The Times.*

BEES FOR PLEASURE AND PROFIT.

A Guide to the Manipulation of Bees, the Production of Honey, and the General Management of the Apiary. By G. GORDON SAMSON. With numerous Illustrations. Crown 8vo, wrapper **1/0**

BOOK-KEEPING for FARMERS and ESTATE OWNERS.

A Practical Treatise, presenting, in Three Plans, a System adapted for all Classes of Farms. By JOHNSON M. WOODMAN, Chartered Accountant. Fourth Edition. Crown 8vo, cloth. [*Just Published.* **2/6**

"The volume is a capital study of a most important subject."—*Agricultural Gazette.*

WOODMAN'S YEARLY FARM ACCOUNT BOOK.

Giving Weekly Labour Account and Diary, and showing the Income and Expenditure under each Department of Crops, Live Stock, Dairy, &c., &c. With Valuation, Profit and Loss Account, and Balance Sheet at the End of the Year. By JOHNSON M. WOODMAN, Chartered Accountant. Second Edition. Folio, half-bound . . . . *Net* **7/6**

"Contains every requisite for keeping farm accounts readily and accurately."—*Agriculture.*

THE FORCING GARDEN.

Or, How to Grow Early Fruits, Flowers and Vegetables. With Plans and Estimates for Building Glasshouses, Pits and Frames. With Illustrations. By SAMUEL WOOD. Crown 8vo, cloth **3/6**

"A good book, containing a great deal of valuable teaching."—*Gardeners' Magazine.*

A PLAIN GUIDE TO GOOD GARDENING.

Or, How to Grow Vegetables, Fruits, and Flowers. By S. WOOD. Fourth Edition, with considerable Additions, and numerous Illustrations. Crown 8vo, cloth **3/6**

"A very good book, and one to be highly recommended as a practical guide. The practical directions are excellent."—*Athenæum.*

MULTUM-IN-PARVO GARDENING.

Or, How to Make One Acre of Land produce £620 a year, by the Cultivation of Fruits and Vegetables; also, How to Grow Flowers in Three Glass Houses, so as to realise £176 per annum clear Profit. By SAMUEL WOOD, Author of "Good Gardening," &c. Sixth Edition, Crown 8vo, sewed . . . **1/0**

THE LADIES' MULTUM-IN-PARVO FLOWER GARDEN.

And Amateur's Complete Guide. By S. WOOD. Crown 8vo, cloth . **3/6**

POTATOES: HOW TO GROW AND SHOW THEM.

A Practical Guide to the Cultivation and General Treatment of the Potato. By J. PINK. Crown 8vo **2/0**

MARKET AND KITCHEN GARDENING.

By C. W. SHAW, late Editor of "Gardening Illustrated." Crown 8vo, cloth. **3/6**

AUCTIONEERING, VALUING, LAND SURVEYING, ESTATE AGENCY, ETC.

INWOOD'S TABLES FOR PURCHASING ESTATES AND FOR THE VALUATION OF PROPERTIES,

Including Advowsons, Assurance Policies, Copyholds, Deferred Annuities, Freeholds, Ground Rents, Immediate Annuities, Leaseholds, Life Interests, Mortgages, Perpetuities, Renewals of Leases, Reversions, Sinking Funds, &c., &c. 27th Edition, Revised and Extended by WILLIAM SCHOOLING, F.R.A.S., with Logarithms of Natural Numbers and THOMAN'S Logarithmic Interest and Annuity Tables. 360 pp., Demy 8vo, cloth. [*Just Published. Net* **8/0**

"Those interested in the purchase and sale of estates, and in the adjustment of compensation cases, as well as in transactions in annuities, life insurances, &c., will find the present edition of eminent service."—*Engineering.*

"This valuable book has been considerably enlarged and improved by the labours of Mr. Schooling, and is now very complete indeed."—*Economist.*

"Altogether this edition will prove of extreme value to many classes of professional men in saving them many long and tedious calculations."—*Investors' Review.*

THE APPRAISER, AUCTIONEER, BROKER, HOUSE AND ESTATE AGENT AND VALUER'S POCKET ASSISTANT.

For the Valuation for Purchase, Sale, or Renewal of Leases, Annuities, and Reversions, and of Property generally; with Prices for Inventories, &c. By JOHN WHEELER, Valuer, &c. Sixth Edition, Re-written and greatly Extended by C. NORRIS. Royal 32mo, cloth **5/0**

"A neat and concise book of reference, containing an admirable and clearly-arranged list of prices for inventories, and a very practical guide to determine the value of furniture, &c."—*Standard.*

"Contains a large quantity of varied and useful information as to the valuation for purchase, sale, or renewal of leases, annuities and reversions, and of property generally, with prices for inventories, and a guide to determine the value of interior fittings and other effects."—*Builder.*

AUCTIONEERS: THEIR DUTIES AND LIABILITIES.

A Manual of Instruction and Counsel for the Young Auctioneer. By ROBERT SQUIBBS, Auctioneer. Second Edition, Revised. Demy 8vo, cloth . **12/6**

"The work is one of general excellent character, and gives much information in a compendious and satisfactory form."—*Builder.*

"May be recommended as giving a great deal of information on the law relating to auctioneers, in a very readable form."—*Law Journal.*

THE AGRICULTURAL VALUER'S ASSISTANT.

A Practical Handbook on the Valuation of Landed Estates; including Example of a Detailed Report on Management and Realisation; Forms of Valuations of Tenant Right; Lists of Local Agricultural Customs; Scales of Compensation under the Agricultural Holdings Act, and a Brief Treatise on Compensation under the Lands Clauses Acts, &c. By TOM BRIGHT, Agricultural Valuer. Author of "The Agricultural Surveyor and Estate Agent's Handbook." Fourth Edition, Revised, with Appendix containing a Digest of the Agricultural Holdings Acts, 1883 and 1900. Crown 8vo, cloth . *Net* **6/0**

"Full of tables and examples in connection with the valuation of tenant-right, estates, labour, contents and weights of timber, and farm produce of all kinds."—*Agricultural Gazette.*

"An eminently practical handbook, full of practical tables and data of undoubted interest and value to surveyors and auctioneers in preparing valuations of all kinds."—*Farmer.*

POLE PLANTATIONS AND UNDERWOODS.

A Practical Handbook on Estimating the Cost of Forming, Renovating, Improving, and Grubbing Plantations and Underwoods, their Valuation for Purposes of Transfer, Rental, Sale or Assessment. By TOM BRIGHT. Crown 8vo, cloth **3/6**

"To valuers, foresters and agents it will be a welcome aid."—*North British Agriculturist.*

"Well calculated to assist the valuer in the discharge of his duries, and of undoubted interest and use both to surveyors and auctioneers in preparing valuations of all kinds."—*Kent Herald.*

AGRICULTURAL SURVEYOR AND ESTATE AGENT'S HANDBOOK.

Of Practical Rules, Formulæ, Tables, and Data. A Comprehensive Manual for the Use of Surveyors, Agents, Landowners, and others interested in the Equipment, the Management, or the Valuation of Landed Estates. By TOM BRIGHT, Agricultural Surveyor and Valuer, Author of "The Agricultural Valuer's Assistant," &c. With Illustrations. Fcap. 8vo, Leather. *Net* **7/6**

"An exceedingly useful book, the contents of which are admirably chosen. The classes for whom the work is intended will find it convenient to have this comprehensive handbook accessible for reference."—*Live Stock Journal.*

"It is a singularly compact and well informed compendium of the facts and figures likely to be required in estate work, and is certain to prove of much service to those to whom it is addressed."—*Scotsman.*

THE LAND VALUER'S BEST ASSISTANT.

Being Tables on a very much Improved Plan, for Calculating the Value of Estates. With Tables for reducing Scotch, Irish, and Provincial Customary Acres to Statute Measure, &c. By R. HUDSON, C.E. New Edition. Royal 32mo, leather, elastic band **4/0**

"Of incalculable value to the country gentleman and professional man."—*Farmers' Journal.*

THE LAND IMPROVER'S POCKET-BOOK.

Comprising Formulæ, Tables, and Memoranda required in any Computation relating to the Permanent Improvement of Landed Property. By JOHN EWART, Surveyor. Second Edition, Revised. Royal 32mo, oblong, leather . **4/0**

"A compendious and handy little volume."—*Spectator.*

THE LAND VALUER'S COMPLETE POCKET-BOOK.

Being the above Two Works bound together. Leather **7/6**

HANDBOOK OF HOUSE PROPERTY.

A Popular and Practical Guide to the Purchase, Tenancy, and Compulsory Sale of Houses and Land, including Dilapidations and Fixtures: with Examples of all kinds of Valuations, Information on Building and on the right use of Decorative Art. By E. L. TARBUCK, Architect and Surveyor. Sixth Edition. 12mo, cloth **5/0**

"The advice is thoroughly practical."—*Law Journal.*

"For all who have dealings with house property, this is an indispensable guide."—*Decoration.*

"Carefully brought up to date, and much improved by the addition of a division on Fine Art. A well-written and thoughtful work."—*Land Agents' Record.*

LAW AND MISCELLANEOUS.

MODERN JOURNALISM.

A Handbook of Instruction and Counsel for the Young Journalist. By JOHN B. MACKIE, Fellow of the Institute of Journalists. Crown 8vo, cloth . **2/0**

"This invaluable guide to journalism is a work which all aspirants to a journalistic career will read with advantage."—*Journalist.*

HANDBOOK FOR SOLICITORS AND ENGINEERS

Engaged in Promoting Private Acts of Parliament and Provisional Orders for the Authorisation of Railways, Tramways, Gas and Water Works, &c. By L. L. MACASSEY, of the Middle Temple, Barrister-at-Law, M.I.C.E. 8vo, cloth . . . . **£1 5s.**

PATENTS for INVENTIONS, HOW to PROCURE THEM.

Compiled for the Use of Inventors, Patentees and others. By G. G. M. HARDINGHAM, Assoc. Mem. Inst. C.E., &c. Demy 8vo, cloth . . **1/6**

CONCILIATION & ARBITRATION in LABOUR DISPUTES.

A Historical Sketch and Brief Statement of the Present Position of the Question at Home and Abroad. By J. S. JEANS. Crown 8vo, 200 pp., cloth **2/6**

EVERY MAN'S OWN LAWYER.

A Handy-Book of the Principles of Law and Equity. With a Concise Dictionary of Legal Terms. By A BARRISTER. Forty-first Edition, carefully Revised, and comprising New Acts of Parliament, including the *Motor Car Act*, 1903; *Employment of Children Act*, 1903; *Pistols Act*, 1903; *Poor Prisoners' Defence Act*, 1903; *Education Acts of* 1902 *and* 1903; *Housing of the Working Classes Act*, 1903, &c. *Judicial Decisions* pronounced during the year have also been duly noted. Crown 8vo, 800 pp., strongly bound in cloth. *Just Published.* **6/8**

*** *This Standard Work of Reference forms* A COMPLETE EPITOME OF THE LAWS OF ENGLAND, *comprising (amongst other matter)*:

THE RIGHTS AND WRONGS OF INDIVIDUALS

LANDLORD AND TENANT
VENDORS AND PURCHASERS
LEASES AND MORTGAGES
JOINT-STOCK COMPANIES
MASTERS, SERVANTS AND WORKMEN
CONTRACTS AND AGREEMENTS
MONEY LENDERS, SURETISHIP
PARTNERSHIP, SHIPPING LAW
SALE AND PURCHASE OF GOODS
CHEQUES, BILLS AND NOTES
BILLS OF SALE, BANKRUPTCY
LIFE, FIRE, AND MARINE INSURANCE
LIBEL AND SLANDER
CRIMINAL LAW
PARLIAMENTARY ELECTIONS
COUNTY COUNCILS
DISTRICT AND PARISH COUNCILS
BOROUGH CORPORATIONS
TRUSTEES AND EXECUTORS
CLERGY AND CHURCHWARDENS
COPYRIGHT, PATENTS, TRADE MARKS
HUSBAND AND WIFE, DIVORCE
INFANCY, CUSTODY OF CHILDREN
PUBLIC HEALTH AND NUISANCES
INNKEEPERS AND SPORTING
TAXES AND DEATH DUTIES

FORMS OF WILLS, AGREEMENTS, NOTICES, &C.

☞ *The object of this work is to enable those who consult it to help themselves to the law; and thereby to dispense, as far as possible, with professional assistance and advice. There are many wrongs and grievances which persons submit to from time to time through not knowing how or where to apply for redress; and many persons have as great a dread of a lawyer's office as of a lion's den. With this book at hand it is believed that many a* SIX-AND-EIGHTPENCE *may be saved; many a wrong redressed; many a right reclaimed; many a law suit avoided; and many an evil abated. The work has established itself as the standard legal adviser of all classes, and has also made a reputation for itself as a useful book of reference for lawyers residing at a distance from law libraries, who are glad to have at hand a work embodying recent decisions and enactments*

*** OPINIONS OF THE PRESS.

"The amount of information given in the volume is simply wonderful. The continued popularity of the work shows that it fulfils a useful purpose."—*Law Journal.*

"As a book of reference this volume is without a rival."—*Pall Mall Gazette.*

"No Englishman ought to be without this book."—*Engineer.*

"Ought to be in every business establishment and in all libraries."—*Sheffield Post.*

"The 'Concise Dictionary' adds considerably to its value."—*Westminster Gazette.*

"It is a complete code of English Law written in plain language, which all can understand. . . . Should be in the hands of every business man, and all who wish to abolish lawyers' bills."—*Weekly Times.*

"A useful and concise epitome of the law, compiled with considerable care."—*Law Magazine.*

"A complete digest of the most useful facts which constitute English law."—*Globe.*

"Admirably done, admirably arranged, and admirably cheap."—*Leeds Mercury.*

"A concise, cheap, and complete epitome of the English law. So plainly written that he who runs may read, and he who reads may understand."—*Figaro.*

"A dictionary of legal facts well put together. The book is a very useful one."—*Spectator.*

LABOUR CONTRACTS.

A Popular Handbook on the Law of Contracts for Works and Services. By DAVID GIBBONS. Fourth Edition, with Appendix of Statutes by T. F. UTTLEY, Solicitor. Fcap. 8vo, cloth **3/6**

BRADBURY, AGNEW, & CO. LD., PRINTERS, LONDON AND TONBRIDGE.

WEALE'S SERIES
OF
SCIENTIFIC AND TECHNICAL WORKS.

> "It is not too much to say that no books have ever proved more popular with or more useful to young engineers and others than the excellent treatises comprised in WEALE'S SERIES."—**Engineer.**

A New Classified List.

PAGE
CIVIL ENGINEERING AND SURVEYING 2
MINING AND METALLURGY 3
MECHANICAL ENGINEERING . . 4
NAVIGATION, SHIPBUILDING, ETC. . 5

PAGE
ARCHITECTURE AND BUILDING . 6
INDUSTRIAL AND USEFUL ARTS . . 8
AGRICULTURE, GARDENING, ETC. . 10
MATHEMATICS, ARITHMETIC, ETC. . 12

BOOKS OF REFERENCE AND MISCELLANEOUS VOLUMES . . 14

Capio Lumen

CROSBY LOCKWOOD AND SON,
7, STATIONERS' HALL COURT, LONDON, E.C.
1904.

CIVIL ENGINEERING & SURVEYING.

Civil Engineering.

By HENRY LAW, M.Inst.C.E. Including a Treatise on HYDRAULIC ENGINEERING by G. R. BURNELL, M.I.C.E. Seventh Edition, revised, with LARGE ADDITIONS by D. K. CLARK, M.I.C.E. **6/6**

Pioneer Engineering:

A Treatise on the Engineering Operations connected with the Settlement of Waste Lands in New Countries. By EDWARD DOBSON, M.INST.C.E. With numerous Plates. Second Edition **4/6**

Iron Bridges of Moderate Span:

Their Construction and Erection. By HAMILTON W. PENDRED. With 40 Illustrations **2/0**

Iron and Steel Bridges and Viaducts.

A Practical Treatise upon their Construction for the use of Engineers, Draughtsmen, and Students. By FRANCIS CAMPIN, C.E. With numerous Illustrations **3/6**

Constructional Iron and Steel Work,

As applied to Public, Private, and Domestic Buildings. By FRANCIS CAMPIN, C.E. **3/6**

Tubular and other Iron Girder Bridges.

Describing the Britannia and Conway Tubular Bridges. By G. DRYSDALE DEMPSEY, C.E. Fourth Edition **2/0**

Materials and Construction:

A Theoretical and Practical Treatise on the Strains, Designing, and Erection of Works of Construction. By FRANCIS CAMPIN, C.E. . . **3/0**

Sanitary Work in the Smaller Towns and in Villages.

By CHARLES SLAGG, Assoc. M.Inst.C.E. Third Edition . . **3/0**

Construction of Roads and Streets.

By H. LAW, C.E., and D. K. CLARK, C.E. Sixth Edition, revised, with Additional Chapters by A. J. WALLIS-TAYLER, A.M. Inst. C.E. . **6/0**

Gas Works (The Construction of),

And the Manufacture and Distribution of Coal Gas. By S. HUGHES, C.E. Re-written by WILLIAM RICHARDS, C.E. Eighth Edition . . **5/6**

Water Works

For the Supply of Cities and Towns. With a Description of the Principal Geological Formations of England as influencing Supplies of Water. By SAMUEL HUGHES, F.G.S., C.E. Enlarged Edition **4/0**

The Power of Water,

As applied to drive Flour Mills, and to give motion to Turbines and other Hydrostatic Engines. By JOSEPH GLYNN, F.R.S. New Edition . **2/0**

Wells and Well-Sinking.

By JOHN GEO. SWINDELL, A.R.I.B.A., and G. R. BURNELL, C.E. Revised Edition. With a New Appendix on the Qualities of Water. Illustrated **2/0**

The Drainage of Lands, Towns, and Buildings.

By G. D. DEMPSEY, C.E. Revised, with large Additions on Recent Practice, by D. K. CLARK, M.I.C.E. Third Edition **4/6**

The Blasting and Quarrying of Stone,

For Building and other Purposes. With Remarks on the Blowing up of Bridges. By Gen. Sir J. BURGOYNE, K.C.B. **1/6**

Foundations and Concrete Works.

With Practical Remarks on Footings, Planking, Sand, Concrete, Béton, Pile-driving, Caissons, and Cofferdams. By E. DOBSON, M.R.I.B.A. Ninth Edition . . . **1/6**

Pneumatics,

Including Acoustics and the Phenomena of Wind Currents, for the Use of Beginners. By CHARLES TOMLINSON, F.R.S. Fourth Edition . 1/6

Land and Engineering Surveying.

For Students and Practical Use. By T. BAKER, C.E. Nineteenth Edition, Revised and Extended by F. E. DIXON, A.M. Inst. C.E., Professional Associate of the Institution of Surveyors. With numerous Illustrations and two Lithographic Plates 2/0

Mensuration and Measuring.

For Students and Practical Use. With the Mensuration and Levelling of Land for the purposes of Modern Engineering. By T. BAKER, C.E. New Edition by E. NUGENT, C.E. 1/6

MINING AND METALLURGY.

Mining Calculations,

For the use of Students Preparing for the Examinations for Colliery Managers' Certificates, comprising numerous Rules and Examples in Arithmetic, Algebra, and Mensuration. By T. A. O'DONAHUE, M.E., First-Class Certificated Colliery Manager. 3/6

Mineralogy,

Rudiments of. By A. RAMSAY, F.G.S. Fourth Edition, revised and enlarged. Woodcuts and Plates 3/6

Coal and Coal Mining,

A Rudimentary Treatise on. By the late Sir WARINGTON W. SMYTH, F.R.S. Eighth Edition, revised by T. FORSTER BROWN 3/6

Metallurgy of Iron.

Containing Methods of Assay, Analyses of Iron Ores, Processes of Manufacture of Iron and Steel, &c. By H. BAUERMAN, F.G.S. With numerous Illustrations. Sixth Edition, revised and enlarged 5/0

The Mineral Surveyor and Valuer's Complete Guide.

By W. LINTERN. Fourth Edition, with an Appendix on Magnetic and Angular Surveying 3/6

Slate and Slate Quarrying:

Scientific, Practical, and Commercial. By D. C. DAVIES, F.G.S. With numerous Illustrations and Folding Plates. Fourth Edition . . 3/0

A First Book of Mining and Quarrying,

With the Sciences connected therewith, for Primary Schools and Self-Instruction. By J. H. COLLINS, F.G.S. Second Edition . . . 1/6

Subterraneous Surveying,

With and without the Magnetic Needle. By T. FENWICK and T. BAKER, C.E. Illustrated 2/6

Mining Tools.

Manual of. By WILLIAM MORGANS, Lecturer on Practical Mining at the Bristol School of Mines 2/6

Mining Tools, Atlas

Of Engravings to Illustrate the above, containing 235 Illustrations of Mining Tools, drawn to Scale. 4to 4/6

Physical Geology,

Partly based on Major-General PORTLOCK'S "Rudiments of Geology." By RALPH TATE, A.L.S., &c. Woodcuts. 2/0

Historical Geology,

Partly based on Major-General PORTLOCK'S "Rudiments." By RALPH TATE, A.L.S., &c. Woodcuts 2/6

Geology, Physical and Historical.

Consisting of "Physical Geology," which sets forth the Leading Principles of the Science; and "Historical Geology," which treats of the Mineral and Organic Conditions of the Earth at each successive epoch. By RALPH TATE, F.G.S. 4/6

MECHANICAL ENGINEERING.

The Workman's Manual of Engineering Drawing.
By JOHN MAXTON, Instructor in Engineering Drawing, Royal Naval College, Greenwich. Eighth Edition. 300 Plates and Diagrams . **3/6**

Fuels: Solid, Liquid, and Gaseous.
Their Analysis and Valuation. For the Use of Chemists and Engineers. By H. J. PHILLIPS, F.C.S., formerly Analytical and Consulting Chemist to the Great Eastern Railway. Third Edition **2/0**

Fuel, Its Combustion and Economy.
Consisting of an Abridgment of "A Treatise on the Combustion of Coal and the Prevention of Smoke." By C. W. WILLIAMS, A.I.C.E. With Extensive Additions by D. K CLARK, M.Inst.C.E. Fourth Edition . **3/6**

The Boilermaker's Assistant
In Drawing, Templating, and Calculating Boiler Work, &c. By J. COURTNEY, Practical Boilermaker. Edited by D. K. CLARK, C.E. . **2/0**

The Boiler-Maker's Ready Reckoner,
With Examples of Practical Geometry and Templating for the Use of Platers, Smiths, and Riveters. By JOHN COURTNEY. Edited by D. K. CLARK, M.I.C.E. Fifth Edition **4/0**

⁂ *The last two Works in One Volume, half-bound, entitled* "THE BOILER-MAKER'S READY-RECKONER AND ASSISTANT." By J. COURTNEY and D. K. CLARK. *Price* **7/0.**

Steam Boilers:
Their Construction and Management. By R. ARMSTRONG, C.E. Illustrated **1/6**

Steam and Machinery Management.
A Guide to the Arrangement and Economical Management of Machinery. By M. POWIS BALE, M.Inst.M.E. **2/6**

Steam and the Steam Engine,
Stationary and Portable. Being an Extension of the Treatise on the Steam Engine of Mr. J. SEWELL. By D. K. CLARK, C.E. Fourth Edition **3/6**

The Steam Engine,
A Treatise on the Mathematical Theory of, with Rules and Examples for Practical Men. By T. BAKER, C.E. **1/6**

The Steam Engine.
By Dr. LARDNER. Illustrated **1/6**

Locomotive Engines.
By G. D. DEMPSEY, C.E. With large Additions treating of the Modern Locomotive, by D. K. CLARK, M.Inst.C.E. . . . **3/0**

Locomotive Engine-Driving.
A Practical Manual for Engineers in charge of Locomotive Engines. By MICHAEL REYNOLDS. Eleventh Edition. 3*s.* 6*d.*; cloth boards . **4/6**

Stationary Engine-Driving.
A Practical Manual for Engineers in charge of Stationary Engines. By MICHAEL REYNOLDS. Seventh Edition. 3*s.* 6*d.*; cloth boards . **4/6**

The Smithy and Forge.
Including the Farrier's Art and Coach Smithing. By W. J. E. CRANE. Fourth Edition **2/6**

Modern Workshop Practice,
As applied to Marine, Land, and Locomotive Engines, Floating Docks, Dredging Machines, Bridges, Ship-building, &c. By J. G. WINTON. Fourth Edition, Illustrated **3/6**

Mechanical Engineering.
Comprising Metallurgy, Moulding, Casting, Forging, Tools, Workshop Machinery, Mechanical Manipulation, Manufacture of the Steam Engine, &c. By FRANCIS CAMPIN, C.E. Third Edition **2/6**

Details of Machinery.
Comprising Instructions for the Execution of various Works in Iron in the Fitting-Shop, Foundry, and Boiler-Yard. By FRANCIS CAMPIN, C.E. **3/0**

Elementary Engineering:
A Manual for Young Marine Engineers and Apprentices. In the Form of Questions and Answers on Metals, Alloys, Strength of Materials, &c. By J. S. Brewer. Fifth Edition 1/6

Power in Motion:
Horse-power Motion, Toothed-Wheel Gearing, Long and Short Driving Bands, Angular Forces, &c. By James Armour, C.E. Third Edition 2/0

Iron and Heat,
Exhibiting the Principles concerned in the Construction of Iron Beams, Pillars, and Girders. By J. Armour, C.E. 2/6

Practical Mechanism,
And Machine Tools. By T. Baker, C.E. With Remarks on Tools and Machinery, by J. Nasmyth, C.E. 2/6

Mechanics:
Being a concise Exposition of the General Principles of Mechanical Science, and their Applications. By Charles Tomlinson, F.R.S. . . . 1/6

Cranes (The Construction of),
And other Machinery for Raising Heavy Bodies for the Erection of Buildings, &c. By Joseph Glynn, F.R.S. 1/6

NAVIGATION, SHIPBUILDING, ETC.

The Sailor's Sea Book:
A Rudimentary Treatise on Navigation. By James Greenwood, B.A. With numerous Woodcuts and Coloured Plates. New and enlarged Edition. By W. H. Rosser 2/6

Practical Navigation.
Consisting of The Sailor's Sea-Book, by James Greenwood and W. H. Rosser; together with Mathematical and Nautical Tables for the Working of the Problems, by Henry Law, C.E., and Prof. J. R. Young. 7/0

Navigation and Nautical Astronomy,
In Theory and Practice. By Prof. J. R. Young. New Edition. 2/6

Mathematical Tables,
For Trigonometrical, Astronomical, and Nautical Calculations; to which is prefixed a Treatise on Logarithms. By H. Law, C.E. Together with a Series of Tables for Navigation and Nautical Astronomy. By Professor J. R. Young. New Edition 4/0

Masting, Mast-Making, and Rigging of Ships.
Also Tables of Spars, Rigging, Blocks; Chain, Wire, and Hemp Ropes, &c., relative to every class of vessels. By Robert Kipping, N.A. . 2/0

Sails and Sail-Making.
With Draughting, and the Centre of Effort of the Sails. By Robert Kipping, N.A. 2/6

Marine Engines and Steam Vessels.
By R. Murray, C.E. Eighth Edition, thoroughly revised, with Additions by the Author and by George Carlisle, C.E. . . . 4/6

Naval Architecture:
An Exposition of Elementary Principles. By James Peake . . 3/6

Ships for Ocean and River Service,
Principles of the Construction of. By Hakon A. Sommerfeldt . 1/6

Atlas of Engravings
To Illustrate the above. Twelve large folding Plates. Royal 4to, cloth 7/6

The Forms of Ships and Boats.
By W. Bland. Tenth Edition, with numerous Illustrations and Models 1/6

ARCHITECTURE AND THE BUILDING ARTS.

Constructional Iron and Steel Work,

As applied to Public, Private, and Domestic Buildings. By FRANCIS CAMPIN, C.E. 3/6

Building Estates:

A Treatise on the Development, Sale, Purchase, and Management of Building Land. By F. MAITLAND. Third Edition 2/0

The Science of Building:

An Elementary Treatise on the Principles of Construction. By E. WYNDHAM TARN, M.A. Lond. Fourth Edition 3/6

The Art of Building:

General Principles of Construction, Strength, and Use of Materials, Working Drawings, Specifications, &c. By EDWARD DOBSON, M.R.I.B.A. . 2/0

A Book on Building,

Civil and Ecclesiastical. By Sir EDMUND BECKETT, Q.C. (Lord GRIMTHORPE). Second Edition 4/6

Dwelling-Houses (The Erection of),

Illustrated by a Perspective View, Plans, and Sections of a Pair of Villas, with Specification, Quantities, and Estimates. By S. H. BROOKS, Architect 2/6

Cottage Building.

By C. BRUCE ALLEN. Twelfth Edition, with Chapter on Economic Cottages for Allotments, by E. E. ALLEN, C.E. 2/0

Acoustics in Relation to Architecture and Building:

The Laws of Sound as applied to the Arrangement of Buildings. By Professor T. ROGER SMITH, F.R.I.B.A. New Edition, Revised . . 1/6

The Rudiments of Practical Bricklaying.

General Principles of Bricklaying; Arch Drawing, Cutting, and Setting; Pointing; Paving, Tiling, &c. By ADAM HAMMOND. With 68 Woodcuts 1/6

The Art of Practical Brick Cutting and Setting.

By ADAM HAMMOND. With 90 Engravings . . 1/6

Brickwork:

A Practical Treatise, embodying the General and Higher Principles of Bricklaying, Cutting and Setting; with the Application of Geometry to Roof Tiling, &c. By F. WALKER 1/6

Bricks and Tiles,

Rudimentary Treatise on the Manufacture of; containing an Outline of the Principles of Brickmaking. By E. DOBSON, M.R.I.B.A. Additions by C. TOMLINSON, F.R.S. Illustrated 3/0

The Practical Brick and Tile Book.

Comprising: BRICK AND TILE MAKING, by E. DOBSON, M.INST.C.E.; Practical BRICKLAYING, by A. HAMMOND; BRICK-CUTTING AND SETTING, by A. HAMMOND. 550 pp. with 270 Illustrations, half-bound . . 6/0

Carpentry and Joinery—

THE ELEMENTARY PRINCIPLES OF CARPENTRY. Chiefly composed from the Standard Work of THOMAS TREDGOLD, C.E. With Additions, and TREATISE ON JOINERY, by E. W. TARN, M.A. Eighth Edition 3/6

Carpentry and Joinery—Atlas

Of 35 Plates to accompany and Illustrate the foregoing book. With Descriptive Letterpress. 4to 6/0

A Practical Treatise on Handrailing;
Showing New and Simple Methods. By GEO. COLLINGS. Third Edition, including a TREATISE ON STAIRBUILDING. With Plates . . . 2/6

Circular Work in Carpentry and Joinery.
A Practical Treatise on Circular Work of Single and Double Curvature. By GEORGE COLLINGS. Third Edition 2/6

Roof Carpentry:
Practical Lessons in the Framing of Wood Roofs. For the Use of Working Carpenters. By GEO. COLLINGS 2/0

The Construction of Roofs of Wood and Iron;
Deduced chiefly from the Works of Robison, Tredgold, and Humber. By E. WYNDHAM TARN, M.A., Architect. Fourth Edition . . . 1/6

The Joints Made and Used by Builders.
By WYVILL J. CHRISTY, Architect. With 160 Woodcuts . . 3/0

Shoring
And its Application: A Handbook for the Use of Students. By GEORGE H. BLAGROVE. With 31 Illustrations 1/6

The Timber Importer's, Timber Merchant's, and Builder's Standard Guide.
By R. E. GRANDY 2/0

Plumbing:
A Text-Book to the Practice of the Art or Craft of the Plumber. With Chapters upon House Drainage and Ventilation. By WM. PATON BUCHAN. Ninth Edition, with 512 Illustrations 3/6

Ventilation:
A Text Book to the Practice of the Art of Ventilating Buildings. By W. P. BUCHAN, R.P., Author of "Plumbing," &c. With 170 Illustrations 3/6

The Practical Plasterer:
A Compendium of Plain and Ornamental Plaster Work. By W. KEMP 2/0

House Painting, Graining, Marbling, & Sign Writing.
With a Course of Elementary Drawing, and a Collection of Useful Receipts. By ELLIS A. DAVIDSON. Eighth Edition. Coloured Plates . . 5/0

⁂ *The above, in cloth boards, strongly bound,* 6/0

A Grammar of Colouring,
Applied to Decorative Painting and the Arts. By GEORGE FIELD. New Edition, enlarged, by ELLIS A. DAVIDSON. With Coloured Plates . 3/0

Elementary Decoration
As applied to Dwelling Houses, &c. By JAMES W. FACEY. Illustrated 2/0

Practical House Decoration.
A Guide to the Art of Ornamental Painting, the Arrangement of Colours in Apartments, and the Principles of Decorative Design. By JAMES W. FACEY 2/6

⁂ *The last two Works in One handsome Vol., half-bound, entitled* "HOUSE DECORATION, ELEMENTARY AND PRACTICAL," *price* 5/0.

Portland Cement for Users.
By HENRY FAIJA, A.M.Inst.C.E. Third Edition, Corrected . . 2/0

Limes, Cements, Mortars, Concretes, Mastics, Plastering, &c.
By G. R. BURNELL C.E. Fifteenth Edition 1/6

Masonry and Stone-Cutting.
The Principles of Masonic Projection and their application to Construction. By Edward Dobson, M.R.I.B.A. 2/6

Arches, Piers, Buttresses, &c.:
Experimental Essays on the Principles of Construction. By W. Bland. 1/6

Quantities and Measurements,
In Bricklayers', Masons', Plasterers', Plumbers', Painters', Paperhangers', Gilders', Smiths', Carpenters' and Joiners' Work. By A. C. Beaton. 1/6

The Complete Measurer:
Setting forth the Measurement of Boards, Glass, Timber and Stone. By R. Horton. Sixth Edition 4/0

Guide to Superficial Measurement:
Tables calculated from 1 to 200 inches in length, by 1 to 108 inches in breadth. For the use of Architects, Surveyors, Engineers, Timber Merchants, Builders, &c. By James Hawkings. Fifth Edition . . 3/6

Light:
An Introduction to the Science of Optics. For the Use of Students of Architecture, Engineering, and other Applied Sciences. By E. W. Tarn, M.A. 1/6

Hints to Young Architects.
By George Wightwick, Architect. Sixth Edition, revised and enlarged by G. Huskisson Guillaume, Architect 3/6

Architecture—Orders:
The Orders and their Æsthetic Principles. By W. H. Leeds. Illustrated. 1/6

Architecture—Styles:
The History and Description of the Styles of Architecture of Various Countries, from the Earliest Period. By T. Talbot Bury . 2/0

⁂ Orders and Styles of Architecture, *in One Vol.*, 3/6.

Architecture—Design:
The Principles of Design in Architecture, as deducible from Nature and exemplified in the Works of the Greek and Gothic Architects. By Edw. Lacy Garbett, Architect. Illustrated 2/6

⁂ *The three preceding Works in One handsome Vol., half-bound, entitled* "Modern Architecture," *price* 6/0.

Perspective for Beginners.
Adapted to Young Students and Amateurs in Architecture, Painting, &c. By George Pyne 2/0

Architectural Modelling in Paper.
By T. A. Richardson. With Illustrations, engraved by O. Jewitt 1/6

Glass Staining, and the Art of Painting on Glass.
From the German of Dr. Gessert and Emanuel Otto Fromberg. With an Appendix on The Art of Enamelling 2/6

Vitruvius—The Architecture of.
In Ten Books. Translated from the Latin by Joseph Gwilt, F.S.A., F.R.A.S. With 23 Plates 5/0

N.B.—This is the only Edition of Vitruvius *procurable at a moderate price.*

Grecian Architecture,
An Inquiry into the Principles of Beauty in. With an Historical View of the Rise and Progress of the Art in Greece. By the Earl of Aberdeen. 1/0

⁂ *The two preceding Works in One handsome Vol., half-bound, entitled* "Ancient Architecture," *price* 6/0.

INDUSTRIAL AND USEFUL ARTS.

Cements, Pastes, Glues, and Gums.
A Guide to the Manufacture and Application of Agglutinants. With 900 Recipes and Formulæ. By H. C. Standage 2/0

Clocks, Watches, and Bells for Public Purposes.
A Rudimentary Treatise. By Edmund Beckett, Lord Grimthorpe, LL.D., K.C., F.R.A.S. Eighth Edition, with new List of Great Bells and an Appendix on Weathercocks. [*Just published.* 4/6
*** *The above, handsomely bound, cloth boards,* 5/6.

Electro-Metallurgy,
Practically Treated. By Alexander Watt. Tenth Edition . 3/6

The Goldsmith's Handbook.
Containing full Instructions in the Art of Alloying, Melting, Reducing, Colouring, Collecting and Refining, Recovery of Waste, Solders, Enamels, &c., &c. By George E. Gee. Sixth Edition 3/0

The Silversmith's Handbook,
On the same plan as the Goldsmith's Handbook. By G. E. Gee. 3/0
*** *The last two Works, in One handsome Vol., half-bound,* 7/0.

The Hall-Marking of Jewellery.
Comprising an account of all the different Assay Towns of the United Kingdom; with the Stamps and Laws relating to the Standards and Hall Marks at the various Assay Offices. By George E. Gee . . . 3/0

French Polishing and Enamelling.
Numerous Recipes for making Polishes, Varnishes, &c. By R. Bitmead. 1/6

Practical Organ Building.
By W. E. Dickson, M.A. Second Edition, Revised, with Additions 2/6

Coach-Building:
A Practical Treatise. By James W. Burgess. With 57 Illustrations 2/6

The Cabinet-Maker's Guide
To the Entire Construction of Cabinet-Work. By R. Bitmead 2/6

The Brass Founder's Manual:
Instructions for Modelling, Pattern Making, &c. By W. Graham . 2/0

The Sheet-Metal Worker's Guide.
For Tinsmiths, Coppersmiths, Zincworkers, &c. By W. J. E. Crane. 1/6

Sewing Machinery:
Its Construction, History, &c. By J. W. Urquhart, C.E. . . . 2/0

Gas Fitting:
A Practical Handbook. By John Black. New Edition . . . 2/6

Construction of Door Locks.
From the Papers of A. C. Hobbs. Edited by C. Tomlinson, F.R.S. 2/6

The Model Locomotive Engineer, Fireman, and Engine-Boy.
By Michael Reynolds 3/6

The Art of Letter Painting made Easy.
By J. G. Badenoch. With 12 full-page Engravings of Examples . 1/6

The Art of Boot and Shoemaking.
Measurement, Last-fitting, Cutting-out, Closing, &c. By J. B. Leno. 2/0

Mechanical Dentistry:
By Charles Hunter. Fourth Edition 3/0

Wood Engraving:
A Practical and Easy Introduction to the Art. By W. N. Brown . 1/6

Laundry Management.
A Handbook for Use in Private and Public Laundries 2/0

AGRICULTURE, GARDENING, ETC.

Draining and Embanking:
A Practical Treatise. By Prof. JOHN SCOTT. With 68 Illustrations 1/6

Irrigation and Water Supply:
A Practical Treatise on Water Meadows, Sewage Irrigation, Warping, &c.; on the Construction of Wells, Ponds, Reservoirs, &c. By Prof. JOHN SCOTT. With 34 Illustrations 1/6

Farm Roads, Fences, and Gates:
A Practical Treatise on the Roads, Tramways, and Waterways of the Farm; the Principles of Enclosures; and the different kinds of Fences, Gates, and Stiles. By Prof. JOHN SCOTT. With 75 Illustrations . 1/6

Farm Buildings:
A Practical Treatise on the Buildings necessary for various kinds of Farms, their Arrangement and Construction, with Plans and Estimates. By Prof. JOHN SCOTT, With 105 Illustrations 2/0

Barn Implements and Machines:
Treating of the Application of Power and Machines used in the Threshing-barn, Stockyard, Dairy, &c. By Prof. J. SCOTT. With 123 Illustrations. 2/0

Field Implements and Machines:
With Principles and Details of Construction and Points of Excellence, their Management, &c. By Prof. JOHN SCOTT. With 138 Illustrations . 2/0

Agricultural Surveying:
A Treatise on Land Surveying, Levelling, and Setting-out; with Directions for Valuing Estates. By Prof. J. SCOTT. With 62 Illustrations . 1/6

Farm Engineering.
By Professor JOHN SCOTT. Comprising the above Seven Volumes in One, 1,150 pages, and over 600 Illustrations. Half-bound 12/0

Outlines of Farm Management.
Treating of the General Work of the Farm; Stock; Contract Work; Labour, &c. By R. SCOTT BURN 2/6

Outlines of Landed Estates Management.
Treating of the Varieties of Lands, Methods of Farming, Setting-out of Farms, Roads, Fences, Gates, Drainage, &c. By R. SCOTT BURN . 2/6

Soils, Manures, and Crops.
(Vol. I. OUTLINES OF MODERN FARMING.) By R. SCOTT BURN . 2/0

Farming and Farming Economy.
(Vol. II. OUTLINES OF MODERN FARMING.) By R. SCOTT BURN 3/0

Stock: Cattle, Sheep, and Horses.
(Vol. III. OUTLINES OF MODERN FARMING.) By R. SCOTT BURN 2/6

Dairy, Pigs, and Poultry.
(Vol. IV. OUTLINES OF MODERN FARMING.) By R. SCOTT BURN 2/0

Utilization of Sewage, Irrigation, and Reclamation of Waste Land.
(Vol. V. OUTLINES OF MODERN FARMING.) By R. SCOTT BURN . 2/6

Outlines of Modern Farming.
By R. SCOTT BURN. Consisting of the above Five Volumes in One, 1,250 pp, profusely Illustrated, half-bound 12/0

Book-keeping for Farmers and Estate Owners.

A Practical Treatise, presenting, in Three Plans, a system adapted for all classes of Farms. By J. M. WOODMAN. Fourth Edition . **2/6**

Ready Reckoner for the Admeasurement of Land.

By A. ARMAN. Revised and extended by C. NORRIS. Fifth Edition **2/0**

Miller's, Corn Merchant's, and Farmer's Ready Reckoner.

Second Edition, revised, with a Price List of Modern Flour Mill Machinery, by W. S. HUTTON, C.E. **2/0**

The Hay and Straw Measurer.

New Tables for the Use of Auctioneers, Valuers, Farmers, Hay and Straw Dealers, &c. By JOHN STEELE **2/0**

Meat Production.

A Manual for Producers, Distributors, and Consumers of Butchers' Meat. By JOHN EWART **2/6**

Sheep:

The History, Structure, Economy, and Diseases of. By W. C. SPOONER, M.R.V.S. Fifth Edition, with fine Engravings **3/6**

Market and Kitchen Gardening.

By C. W. SHAW, late Editor of "Gardening Illustrated" . . . **3/6**

Kitchen Gardening Made Easy.

Showing the best means of Cultivating every known Vegetable and Herb, &c., with directions for management all the year round. By GEORGE M. F. GLENNY. Illustrated **1/6**

Cottage Gardening:

Or Flowers, Fruits, and Vegetables for Small Gardens. By E. HOBDAY, **1/6**

Garden Receipts.

Edited by CHARLES W. QUIN **1/6**

Fruit Trees,

The Scientific and Profitable Culture of. From the French of M. DU BREUIL. Fifth Edition, carefully Revised by GEORGE GLENNY. With 187 Woodcuts **3/6**

The Tree Planter and Plant Propagator:

With numerous Illustrations of Grafting, Layering, Budding, Implements, Houses, Pits, &c. By SAMUEL WOOD **2/0**

The Tree Pruner:

A Practical Manual on the Pruning of Fruit Trees, Shrubs, Climbers, and Flowering Plants. With numerous Illustrations. By SAMUEL WOOD **1/6**

⁂ *The above Two Vols. in One, handsomely half-bound, price* **3/6**.

The Art of Grafting and Budding.

By CHARLES BALTET. With Illustrations **2/6**

MATHEMATICS, ARITHMETIC, ETC.

Descriptive Geometry,
An Elementary Treatise on; with a Theory of Shadows and of Perspective, extracted from the French of G. MONGE. To which is added a Description of the Principles and Practice of Isometrical Projection. By J. F. HEATHER, M.A. With 14 Plates 2/0

Practical Plane Geometry:
Giving the Simplest Modes of Constructing Figures contained in one Plane and Geometrical Construction of the Ground. By J. F. HEATHER, M.A. With 215 Woodcuts 2/0

Analytical Geometry and Conic Sections,
A Rudimentary Treatise on. By JAMES HANN. A New Edition, re-written and enlarged by Professor J. R. YOUNG 2/0

Euclid (The Elements of).
With many Additional Propositions and Explanatory Notes; to which is prefixed an Introductory Essay on Logic. By HENRY LAW, C.E. . 2/6

*** *Sold also separately, viz:—*

Euclid. The First Three Books. By HENRY LAW, C.E. 1/6

Euclid. Books 4, 5, 6, 11, 12. By HENRY LAW, C.E. 1/6

Plane Trigonometry,
The Elements of. By JAMES HANN. 1/6

Spherical Trigonometry,
The Elements of. By JAMES HANN. Revised by CHARLES H. DOWLING, C.E. 1/0

*** *Or with "The Elements of Plane Trigonometry," in One Volume,* 2/6

Differential Calculus,
Elements of the. By W. S. B. WOOLHOUSE, F.R.A.S., &c. . . . 1/6

Integral Calculus.
By HOMERSHAM COX, B.A. 1/6

Algebra,
The Elements of. By JAMES HADDON, M.A. With Appendix, containing Miscellaneous Investigations, and a Collection of Problems . . . 2/0

A Key and Companion to the Above.
An extensive Repository of Solved Examples and Problems in Algebra. By J. R. YOUNG 1/6

Commercial Book-keeping.
With Commercial Phrases and Forms in English, French, Italian, and German. By JAMES HADDON, M.A. 1/6

Arithmetic,
A Rudimentary Treatise on. With full Explanations of its Theoretical Principles, and numerous Examples for Practice. For the Use of Schools and for Self-Instruction. By J. R. YOUNG, late Professor of Mathematics in Belfast College. Thirteenth Edition 1/6

A Key to the Above.
By J. R. YOUNG 1/6

Equational Arithmetic,
Applied to Questions of Interest, Annuities, Life Assurance, and General Commerce; with various Tables by which all Calculations may be greatly facilitated. By W. HIPSLEY 1/6

Arithmetic,
Rudimentary, for the Use of Schools and Self-Instruction. By JAMES HADDON, M.A. Revised by ABRAHAM ARMAN . . . 1/6

A Key to the Above.
By A. ARMAN 1/6

Mathematical Instruments:
Their Construction, Adjustment, Testing, and Use concisely Explained. By J. F. HEATHER, M.A., of the Royal Military Academy, Woolwich. Fifteenth Edition, Revised, with Additions, by A. T. WALMISLEY, M.I.C.E. Original Edition, in 1 vol., Illustrated 2/0

*** *In ordering the above, be careful to say "Original Edition," or give the number in the Series* (32), *to distinguish it from the Enlarged Edition in* 3 *vols. (as follows)—*

Drawing and Measuring Instruments.
Including—I. Instruments employed in Geometrical and Mechanical Drawing, and in the Construction, Copying, and Measurement of Maps and Plans. II. Instruments used for the purposes of Accurate Measurement, and for Arithmetical Computations. By J. F. HEATHER, M.A. . 1/6

Optical Instruments.
Including (more especially) Telescopes, Microscopes, and Apparatus for producing copies of Maps and Plans by Photography. By J. F. HEATHER, M.A. Illustrated 1/6

Surveying and Astronomical Instruments.
Including—I. Instruments used for Determining the Geometrical Features of a portion of Ground. II. Instruments employed in Astronomical Observations. By J. F. HEATHER, M.A. Illustrated. 1/6

*** *The above three volumes form an enlargement of the Author's original work, "Mathematical Instruments," price* 2/0. *(Described at top of page.)*

Mathematical Instruments:
Their Construction, Adjustment, Testing and Use. Comprising Drawing, Measuring, Optical, Surveying, and Astronomical Instruments. By J. F. HEATHER, M.A. Enlarged Edition, for the most part entirely re-written. The Three Parts as above, in One thick Volume. 4/6

The Slide Rule, and How to Use It.
Containing full, easy, and simple Instructions to perform all Business Calculations with unexampled rapidity and accuracy. By CHARLES HOARE, C.E. With a Slide Rule, in tuck of cover. Eighth Edition . . 2/6

Logarithms.
With Mathematical Tables for Trigonometrical, Astronomical, and Nautical Calculations. By HENRY LAW, C.E. Revised Edition 3/0

Compound Interest and Annuities (Theory of).
With Tables of Logarithms for the more Difficult Computations of Interest, Discount, Annuities, &c., in all their Applications and Uses for Mercantile and State Purposes. By FEDOR THOMAN, Paris. Fourth Edition . 4/0

Mathematical Tables,
For Trigonometrical, Astronomical, and Nautical Calculations; to which is prefixed a Treatise on Logarithms. By H. LAW, C.E. Together with a Series of Tables for Navigation and Nautical Astronomy. By Professer J. R. YOUNG. New Edition 4/0

Mathematics,
As applied to the Constructive Arts. By FRANCIS CAMPIN, C.E., &c. Third Edition 3/0

Astronomy.
By the late Rev. ROBERT MAIN, F.R.S. Third Edition, revised and corrected to the Present Time. By W. T. LYNN, F.R.A.S. 2/0

Statics and Dynamics,
The Principles and Practice of. Embracing also a clear development of Hydrostatics, Hydrodynamics, and Central Forces. By T. BAKER, C.E. Fourth Edition 1/6

BOOKS OF REFERENCE AND MISCELLANEOUS VOLUMES.

A Dictionary of Painters, and Handbook for Picture Amateurs.

Being a Guide for Visitors to Public and Private Picture Galleries, and for Art-Students, including Glossary of Terms, Sketch of Principal Schools of Painting, &c. By PHILIPPE DARYL, B.A. **2/6**

Painting Popularly Explained.

By T. J. GULLICK, Painter, and JOHN TIMBS, F.S.A. Including Fresco, Oil, Mosaic, Water Colour, Water-Glass, Tempera Encaustic, Miniature, Painting on Ivory, Vellum, Pottery, Enamel, Glass, &c. Sixth Edition **5/0**

A Dictionary of Terms used in Architecture, Building, Engineering, Mining, Metallurgy, Archæology, the Fine Arts, &c.

By JOHN WEALE. Sixth Edition. Edited by R. HUNT, F.R.S. . **5/0**

Music:

A Rudimentary and Practical Treatise. With numerous Examples. By CHARLES CHILD SPENCER **2/6**

Pianoforte,

The Art of Playing the. With numerous Exercises and Lessons. By CHARLES CHILD SPENCER **1/6**

The House Manager.

A Guide to Housekeeping, Cookery, Pickling and Preserving, Household Work, Dairy Management, Cellarage of Wines, Home-brewing and Winemaking, Gardening, &c. By AN OLD HOUSEKEEPER . . **3/6**

Manual of Domestic Medicine.

By R. GOODING, M.D. Intended as a Family Guide in all cases of Accident and Emergency. Third Edition, carefully revised . . **2/0**

Management of Health.

A Manual of Home and Personal Hygiene. By Rev. JAMES BAIRD **1/0**

Natural Philosophy,

For the Use of Beginners. By CHARLES TOMLINSON, F.R.S. . . **1/6**

The Elementary Principles of Electric Lighting.

By ALAN A. CAMPBELL SWINTON, M.INST.C.E., M.I.E.E. Fifth Edition . . . . [*Just Published* **1/6**

The Electric Telegraph,

Its History and Progress. By R. SABINE, C.E., F.S.A., &c. . **3/0**

Handbook of Field Fortification.

By Major W. W. KNOLLYS, F.R.G.S. With 163 Woodcuts . . **3/0**

Logic,

Pure and Applied. By S. H. EMMENS **1/6**

Locke on the Human Understanding,

Selections from. With Notes by S. H. EMMENS **1/6**

The Compendious Calculator

(*Intuitive Calculations*). Or Easy and Concise Methods of Performing the various Arithmetical Operations required in Commercial and Business Transactions; together with Useful Tables, &c. By DANIEL O'GORMAN. Twenty-eighth Edition, carefully revised by C. NORRIS . **2/6**

Measures, Weights, and Moneys of all Nations.
With an Analysis of the Christian, Hebrew, and Mahometan Calendars. By W. S. B. WOOLHOUSE, F.R.A.S., F.S.S. Seventh Edition . 2/6

Grammar of the English Tongue,
Spoken and Written. With an Introduction to the Study of Comparative Philology. By HYDE CLARKE, D.C.L. Fifth Edition. 1/6

Dictionary of the English Language.
As Spoken and Written. Containing above 100,000 Words. By HYDE CLARKE, D.C.L. 3/6

Composition and Punctuation,
Familiarly Explained for those who have neglected the Study of Grammar. By JUSTIN BRENAN. Nineteenth Edition. 1/6

French Grammar.
With Complete and Concise Rules on the Genders of French Nouns. By G. L. STRAUSS, Ph.D. 1/6

English-French Dictionary.
Comprising a large number of Terms used in Engineering, Mining, &c. By ALFRED ELWES 2/0

French Dictionary.
In two Parts—I. French-English. II. English-French, complete in One Vol. 3/0

French and English Phrase Book.
Containing Introductory Lessons, with Translations, Vocabularies of Words, Collection of Phrases, and Easy Familiar Dialogues 1/6

German Grammar.
Adapted for English Students, from Heyse's Theoretical and Practical Grammar, by Dr. G. L. STRAUSS 1/6

German Triglot Dictionary.
By N. E. S. A. HAMILTON. Part I. German-French-English. Part II. English-German-French. Part III. French-German-English 3/0

German Triglot Dictionary.
(As above). Together with German Grammar, in One Volume . 5/0

Italian Grammar.
Arranged in Twenty Lessons, with Exercises. By ALFRED ELWES. 1/6

Italian Triglot Dictionary,
Wherein the Genders of all the Italian and French Nouns are carefully noted down. By ALFRED ELWES. Vol. I. Italian-English-French. 2/6

Italian Triglot Dictionary.
By ALFRED ELWES. Vol. II. English-French-Italian . . . 2/6

Italian Triglot Dictionary.
By ALFRED ELWES. Vol. III. French-Italian-English 2/6

Italian Triglot Dictionary.
(As above). In One Vol. 7/6

Spanish Grammar.
In a Simple and Practical Form. With Exercises. By ALFRED ELWES 1/6

Spanish-English and English-Spanish Dictionary.
Including a large number of Technical Terms used in Mining, Engineering, &c., with the proper Accents and the Gender of every Noun. By ALFRED ELWES 4/0

⁂ *Or with the* GRAMMAR, 6/0.

Portuguese Grammar,

In a Simple and Practical Form. With Exercises. By ALFRED ELWES. **1/6**

Portuguese-English and English-Portuguese Dictionary.

Including a large number of Technical Terms used in Mining, Engineering, &c., with the proper Accents and the Gender of every Noun. By ALFRED ELWES. Third Edition, revised **5/0**

*** *Or with the* GRAMMAR, **7/0**.

Animal Physics,

Handbook of. By DIONYSIUS LARDNER, D.C.L. With 520 Illustrations. In One Vol. (732 pages), cloth boards **7/6**

*** *Sold also in Two Parts, as follows:—*

ANIMAL PHYSICS. By Dr. LARDNER. Part I., Chapters I.—VII. **4/0**

ANIMAL PHYSICS. By Dr. LARDNER. Part II., Chapters VIII.—XVIII. **3/0**

BRADBURY, AGNEW & CO., LD., PRINTERS, LONDON AND TONBRIDGE.